

# Macroscopic modeling and control of reversible lanes on freeways

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**Abstract**—This paper proposes a macroscopic model and two control algorithms for the dynamic operation of reversible lanes on freeways. The proposed model is an extension of the second-order traffic flow model METANET. The reversible lanes are modeled like variable lane drops (taking into account that the cars in the closed/opened lanes need a certain time to leave/enter the corresponding segments). Based on this model, two kinds of dynamic controllers have been developed. The first one is an easy-to-implement logic-based controller which takes into account the congestion lengths generated by the reversible lane bottleneck and uses this information for the dynamic operation of the lanes. The second one is a discrete Model Predictive Control (MPC) which minimizes the Total Time Spent (TTS) of the modeled network within some constraints for the maximum values of the generated bottleneck queues. The discrete optimization is carried out via evaluation of the cost function for all the leafs in a reduced search tree. The proposed model and control algorithms are simulated and tested using loop detector data collected over a section of the SE-30 freeway in Seville, Spain. The modeled network includes the Centenario Bridge, which is a bottleneck with a reversible lane that creates recurrent congestion during the morning rush-hour period. The results show that the proposed model is able to reproduce traffic congestion due to the reversible lanes and that all the proposed controllers (which can be computed in a short time) substantially reduce this congestion.

**Index Terms**—Reversible Lanes, Traffic Control, MPC, METANET, Macroscopic Traffic Flow Modeling

## I. INTRODUCTION

Reversible traffic operations [1] are widely regarded as one of the most cost-effective methods to increase the capacity of an existing freeway. The principle of reversible lanes is to match available capacity to the traffic demand taking advantage of the unused capacity in the minor-flow direction lanes to increase the capacity in the major-flow direction. There are numerous examples of reversible lanes successfully implemented during the last 85 years in many countries (especially in USA, Australia, Canada and UK) [2]. The reversible lanes are generally used in bottlenecks

like bridges or tunnels but there are also examples of entire roadways routinely reversed [3]. Surprisingly, despite the long history and widespread use of reversible lanes worldwide, there have been few quantitative evaluations and research studies conducted on their performance [1]. There is also a limited number of published guidelines and standards related to their planning, design, operation, control, management, and enforcement. Therefore, most reversible lane systems have been developed and managed based primarily on experience, professional judgment, and empirical observation.

It has been reported in the literature under different conditions that dynamic traffic control is a good solution to decrease congestion [4], [5] using ramp metering, Variable Speed Limits (VSL) and other traffic control measures. In general, dynamic traffic control takes the state of the traffic into account and computes control signals that change the response of the traffic system improving its behavior. However, to the best of our knowledge, dynamic traffic control has never been applied to the control of reversible lanes.

This paper proposes the use of on-line control techniques for the reversible lane operation. For this purpose, a modification of the second-order macroscopic model METANET [6] to address reversible lanes is proposed and used for the design of control algorithms. It has to be pointed out that the model and control algorithms proposed in this paper can be equivalently applied to other macroscopic traffic models as the Cell Transmission Model (CTM) [7]).

Section II briefly introduces the general concepts of the macroscopic model METANET. Section III presents the proposed model for reversible lanes. Sections IV and V present the proposed control strategies. Finally, numerical results are presented and discussed in Sections VI and VII.

## II. TRAFFIC MODEL METANET

The METANET model [6] is a macroscopic second-order traffic flow model that provides a good trade-off between simulation speed and accuracy for online traffic control purposes [8]. The original METANET model is deterministic and can be adapted to freeway networks of arbitrary topology and characteristics taking into account the effects of the most common control actions (ramp metering, route guidance, and VSL) but not reversible lanes. The freeway is discretized in consecutive links with segments of length  $L_i$  with density  $\rho_i(k)$  and speed  $v_i(k)$  as state variables at time step  $k$ . For simplicity, this paper does not consider merge and join nodes, nor other extensions, neither does it differentiate between links and segments (See [6] for further details).

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The main equations of the METANET model are:

- **Flow equation:**

$$q_i(k) = \lambda_i \rho_i(k) v_i(k) \quad (1)$$

where  $q_i(k)$  is the flow leaving segment  $i$  and  $\lambda_i$  is the number of lanes.

- **Density equation:**

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{\lambda_i L_i} (q_{i-1}(k) - q_i(k) + q_{r,i}(k) - \beta_i(k) q_{i-1}(k)) \quad (2)$$

where  $\beta_i(k)$  is the split ratio for an off-ramp between segment  $i$  and segment  $i+1$  ( $\beta_i(k) = 0$  if there is no off-ramp),  $T$  is the model sample time, and  $q_{r,i}(k)$  is flow entering by an on-ramp at the start of segment  $i$  ( $q_{r,i}(k) = 0$  for a segment without an on-ramp).

- **Speed equation:**

$$v_i(k+1) = v_i(k) + \frac{T}{\tau_i} (V(\rho_i(k)) - v_i(k)) + \frac{T}{L_i} v_i(k) (v_{i-1}(k) - v_i(k)) - \frac{\mu_i T}{\tau_i L_i} \frac{\rho_i(k+1) - \rho_i(k)}{\rho_i(k) + K_i} \quad (3)$$

where  $K_i$ ,  $\tau_i$ , and  $\mu_i$  are model parameters and  $V(\rho_i(k))$  is the speed desired for the drivers. In the segments where there is an on-ramp, the following negative term is added to the right-hand side of equation (3):

$$\nabla_{\text{r}} v_i(k) = - \frac{\delta_i T q_{r,i}(k) v_i(k)}{L_i \lambda_i (\rho_i(k) + K_i)} \quad (4)$$

where  $\delta_i$  is a model parameter that is positive if there is an on-ramp at the end of segment  $i$ . Equivalently, in the segments where there is a lane-drop of  $\Delta_{\lambda,i}$  lanes, the following negative term is added:

$$\nabla_{\text{d}} v_i(k) = - \frac{\phi_i T \Delta_{\lambda,i} \rho_i(k) v_i^2(k)}{L_i \lambda_i \rho_{c,i}} \quad (5)$$

where  $\rho_{c,i}$  is the critical density and  $\phi_i$  is a model parameter that is positive if there is a lane-drop at the end of segment  $i$ .

- **Desired speed equation** (as proposed in [9]):

$$V(\rho_i(k)) = \min(v_{f,i} \exp\left(-\frac{1}{a_i} \left(\frac{\rho_i(k)}{\rho_{c,i}}\right)^{a_i}\right), (1 + \alpha_i) V_{c,i}(k)) \quad (6)$$

where  $a_i$ ,  $\alpha_i$  are model parameters,  $v_{f,i}$  is the free flow speed and  $V_{c,i}(k)$  is the speed limit applied to segment  $i$ .

- **Origin flow equation:**

$$q_{r,i}(k) = \min(r_i(k) C_{r,i}, D_i(k) + \frac{w_i(k)}{T}, C_{r,i} \frac{\rho_{m,i} - \rho_i(k)}{\rho_{m,i} - \rho_{c,i}}) \quad (7)$$

where origins may be on-ramps or mainstream origins;  $C_{r,i}$  is the origin capacity,  $D_i(k)$  is the origin demand,  $w_i(k)$  is the origin queue length,  $\rho_{m,i}$  is the maximum density, and  $r_i(k)$  is the ramp metering rate.

- **Queue length equation:**

$$w_i(k+1) = w_i(k) + T \cdot (D_i(k) - q_{r,i}(k)) \quad (8)$$

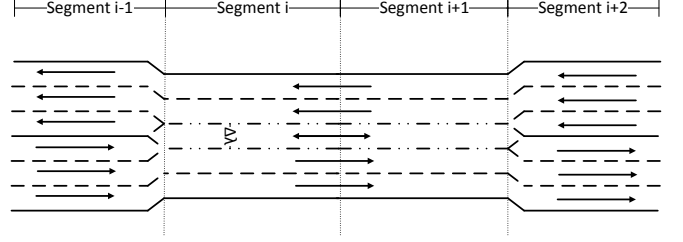


Fig. 1. Freeway stretch with one reversible lane

### III. MACROSCOPIC MODELING OF REVERSIBLE LANES ON FREEWAYS

Consider the stretch of freeway in Figure 1 with  $\bar{\lambda}_i$  lanes in each direction. For a certain number of consecutive segments, there is a bottleneck for which  $\Delta_{\lambda,i}$  lanes have to be shared between both directions. This is done by  $\Delta_{\lambda,i}$  reversible lanes in which traffic may travel in either direction depending on the current traffic conditions.

In each direction the reversible lanes may be modeled like variable lanes drop (i.e. lanes drop which could appear or disappear in a certain sample time). Different but equivalent modeling has to be used for the closing and the opening of the reversible lanes. Also different models have to be used depending of the Variable Message Signs (VMS) location.

#### A. Closing of the lanes

1) **VMS at the beginning of the reversible lane:** Consider one direction of the stretch of freeway in Figure 1 with initially  $\bar{\lambda}_i$  lanes for all the segments. Assume that there are VMS located at the beginning of the reversible lanes to inform drivers about the current status of the reversible lanes (i.e. open or closed for arriving traffic). Assume that, for a certain number of consecutive segments,  $\Delta_{\lambda,i}$  reversible lanes which were open to the arriving traffic, are closed at time step  $k_C$  creating a merging area as can be seen in Figure 2.

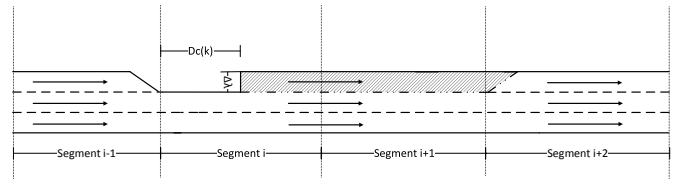


Fig. 2. One direction of freeway in Figure 1 with one lane being closed and the VMS at the beginning of the reversible lanes

For the correct modeling of the reversible lanes, it has to be taken into account that, although the lanes are closed at time step  $k_C$ , the remaining cars need a certain time to leave the corresponding segments. This effect is modeled by the definition of  $D_c(k)$ , which is an estimation of the length of the lane that is already car-free. It has to be pointed out that using the common density equation with an instantaneous change of the number of lanes would entail the violation of the conservation equation.

The distance  $D_c(k)$  can be computed according to (9) assuming that the speed of the remaining cars in the closed lanes equals the mean speed of all the lanes in the corresponding

segment.

$$D_c(k+1) = D_c(k) + T(v_j(k)) \quad (9)$$

with  $D_c(k_C) = 0$  and  $k \geq k_C$

where  $v_j(k)$  is the speed of the segment which has a reversible lane partially closed at time step  $k$ .

In the segment upstream of the lane closing ( $i-1$  in Fig. 2), the lane-drop term  $\nabla_d v_i(k)$  (5) has to be added in the speed equation (3) at sample time  $k_C$ . This term remains active as long as the lanes are closed. The density equation (2) of the upstream segment  $i-1$  is not affected by the lane closing.

In the segments affected by the lane reduction (in Fig. 2, segments  $i$  and  $i+1$ ), the traffic states per lane are modeled by defining an equivalent number of lanes  $\hat{\lambda}_i(k) \in [\bar{\lambda}_i - \Delta_{\lambda,i}, \bar{\lambda}_i]$  during the period of time that there are still cars leaving the closed lanes. The equivalent number of lanes  $\hat{\lambda}_i(k+1)$  for the first segment (in Fig. 2, segment  $i$ ) can be computed for each time step in terms of  $D_c(k+1)$  by:

$$\hat{\lambda}_i(k+1) = \begin{cases} \bar{\lambda}_i - \frac{\Delta_{\lambda,i} D_c(k+1)}{L_i} & \text{for } D_c(k+1) < L_i \\ \bar{\lambda}_i - \Delta_{\lambda,i} & \text{for } D_c(k+1) > L_i \end{cases} \quad (10)$$

In order to simplify, equation (10) can be also written as:

$$\hat{\lambda}_i(k+1) = \max(\bar{\lambda}_i - \Delta_{\lambda,i}, \bar{\lambda}_i - \frac{\Delta_{\lambda,i} D_c(k+1)}{L_i}) \quad (11)$$

This equation can be generalized for subsequent segments by:

$$\hat{\lambda}_i(k+1) = \min(\bar{\lambda}_i, \max(\bar{\lambda}_i - \Delta_{\lambda,i}, \bar{\lambda}_i - \frac{\Delta_{\lambda,i} (D_c(k+1) - \sum_{n=I}^{n=i} L_n)}{L_i})) \quad (12)$$

Where  $I$  is the first segment affected by the reversible lanes. The term  $(D_c(k+1) - \sum_{n=I}^{n=i} L_n)$  expresses the length of the corresponding lane that is already empty of cars. This term only applies when a part of segment is empty and the rest of the segment is still occupied by vehicles. If the segment is completely empty, the equivalent number of lanes is set to  $\bar{\lambda} - \Delta_{\lambda}$ . If the segment is completely occupied, the equivalent number of lanes is set to  $\bar{\lambda}$ .

The density equations (2) are modified as can be seen in (13). The first term is multiplied by  $\frac{\hat{\lambda}_i(k)}{\hat{\lambda}_i(k+1)}$  in order to adapt the previous density to the current equivalent number of lanes. The second term takes into account the flows entering and leaving the segment with respect to the equivalent number of lanes in the previous time step.

$$\rho_i(k+1) = \frac{\hat{\lambda}_i(k) \rho_i(k)}{\hat{\lambda}_i(k+1)} + \frac{T}{\hat{\lambda}_i(k) L_i} (q_{i-1}(k) - q_i(k) + q_{r,i}(k) - \beta_i(k) q_{i-1}(k)) \quad (13)$$

The speed equation (3) does not depend on the number of lanes except in the case of having an on-ramp or a lane drop. In this case, and assuming that the ramps are located at the beginning of a segment, it is necessary to instantaneously change the number of lanes (from  $\bar{\lambda}_i$  to  $\bar{\lambda}_i - \Delta_{\lambda,i}$ ) when  $D_c(k)$  reaches the corresponding segment. This change only affects to the on-ramp penalization term (4) and the lane-drop term (5) as can be seen in (14) and (15):

$$\nabla_r v_i(k) = \begin{cases} -\frac{\delta_i T q_{r,i}(k) v_i(k)}{L_i \lambda_i (\rho_i(k) + K_i)} & \text{for } D_c(k) < \sum_{n=I}^{n=i} L_n \\ -\frac{\delta_i T q_{r,i}(k) v_i(k)}{L_i (\bar{\lambda}_i - \Delta_{\lambda,i}) (\rho_i(k) + K_i)} & \text{for } D_c(k) > \sum_{n=I}^{n=i} L_n \end{cases} \quad (14)$$

$$\nabla_d v_i(k) = \begin{cases} 0 & \text{for } D_c(k) < \sum_{n=I}^{n=i} L_n \\ -\frac{\phi_i T \Delta_{\lambda,i} \rho_i(k) v_i^2(k)}{L_i (\bar{\lambda}_i - \Delta_{\lambda,i}) \rho_{c,i}} & \text{for } D_c(k) > \sum_{n=I}^{n=i} L_n \end{cases} \quad (15)$$

The traffic flow leaving the affected segments should be computed by equation (1) but using the equivalent number of lanes  $\hat{\lambda}_i(k)$  instead of a constant number of lanes.

The complete model for the closing of the lanes is composed of equations (1), (3), (6)-(9) and (12)-(15). The model of the segment downstream the lane opening (in Fig. 3, segment  $i+2$ ) is not affected in any way.

2) *VMS along the reversible lanes*: In the case of having the variable message signs (VMS) distributed along the segments of reversible lanes, it has to be considered that the cars may tend to leave the reversible lanes before they reach their end. Therefore, it would be necessary to consider a new model which takes this effect into account. However, in real applications it is not recommendable to immediately close the whole length of the reversible lanes for both safety and operational reasons. Instead, the VMS should change progressively in order to avoid unnecessary lane changes.

If the VMS are changed progressively, the traffic behavior of the closing with the VMS along the reversible lanes will be roughly the same as the one with the VMS located at the beginning. Therefore, the authors propose to use the model developed in the previous subsection for both cases.

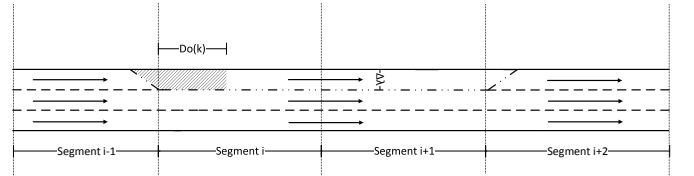


Fig. 3. VMS at the beginning of the reversible lanes: Opening of the lanes.

## B. Opening of the lanes

1) *VMS at the beginning of the reversible lane*: Consider again one direction of the stretch of freeway in Figure 1 but, in this case, initially  $\Delta_{\lambda,i}$  lanes are closed and, at time step  $k_O$ , these lanes are opened (see Fig. 3). Assuming that the VMS are located at the beginning of the lanes or that the VMS change progressively along the lanes, it has to be taken into account that the cars need a certain time to fill the new opened lanes. In the figure,  $D_o(k)$  is the length of the lane that is already occupied by cars and can be computed, as for the lane closing, according to (16):

$$D_o(k+1) = D_o(k) + T(v_j(k)) \quad (16)$$

where  $v_j(k)$  is the speed of the segment which has a reversible lane partially open at time step  $k$ .

For the segment upstream of the reversible lanes (in Fig. 3 segment  $i - 1$ ), the lane-drop term  $\nabla_d v_i(k)$  (5) is removed from the speed equation at sample time  $k_O$  so the traffic state equations (1)-(3) of this segment are the same as in the original model.

The model equations for the segments with the reversible lanes (In Fig. 3, segments  $i$  and  $i + 1$ ) are defined in terms of  $D_o(k)$  similarly as for the closing of the lanes explained in the previous subsection (the model of the segment downstream the lane opening is not affected in any way):

$$\hat{\lambda}_i(k+1) = \max(\bar{\lambda}_i - \Delta_{\lambda,i}, \min(\bar{\lambda}_i, \bar{\lambda}_i - \frac{\Delta_{\lambda,i}(D_o(k+1) - \sum_{n=I}^{n=i} L_n)}{L_i})) \quad (17)$$

$$\rho_i(k+1) = \frac{\hat{\lambda}_i(k)\rho_i(k)}{\hat{\lambda}_i(k+1)} + \frac{T}{\hat{\lambda}_i(k)L_i}(q_{i-1}(k) - q_i(k)) \quad (18)$$

$$\begin{aligned} \nabla_r v_i(k) &= \\ &= \begin{cases} -\frac{\delta_i T q_{r,i}(k) v_i(k)}{L_i(\bar{\lambda}_i - \Delta_{\lambda,i})(\rho_i(k) + K_I)} & \text{for } D_o(k) < \sum_{n=I}^{n=i} L_n \\ -\frac{\delta_i T q_{r,i}(k) v_i(k)}{L_i \bar{\lambda}_i (\rho_i(k) + K_i)} & \text{for } D_o(k) > \sum_{n=I}^{n=i} L_n \end{cases} \quad (19) \end{aligned}$$

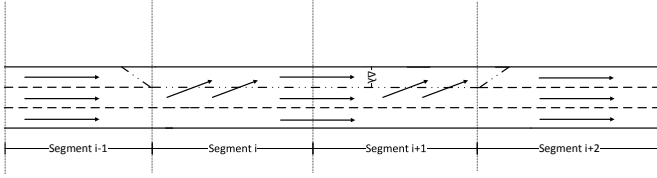


Fig. 4. VMS along the reversible lanes: Opening of the lanes.

2) *VMS along the reversible lanes*: Consider now that the VMS are distributed along the  $\Delta_{\lambda,i}$  reversible lanes which are opened at time step  $k_O$  (see Fig. 4). In this case, the cars in the non-reversible lanes are going to change to the reversible lanes as soon as they are able to (especially if the non-reversible lanes are congested before the opening of the lanes).

In contrast to the closing, it is recommendable to immediately open the whole length of the reversible lanes if the VMS are distributed along them entailing a better performance in terms of congestion without a decrease in safety.

For the modeling of this case, It is assumed that the number of lanes instantaneously changes from  $\bar{\lambda}_i - \Delta_{\lambda,i}$  to  $\bar{\lambda}_i$  at time step  $k_O$  for all the considered segments. Therefore the densities have to change instantaneously on time step  $k_O$  according to (20) in order to respect the conservation equation.

$$\rho_i(k_o) = \frac{\bar{\lambda}_i}{\bar{\lambda}_i - \Delta_{\lambda,i}} \rho_i(k_o) \quad \forall i \text{ with lanes opened at } k_o \quad (20)$$

Equation (20) assumes that the cars in the non-reversible lanes will instantaneously occupy the reversible lanes, homogenizing the densities. The common equations of the METANET model may be used for all the following time steps, including  $k_O$ .

It has to be pointed that the speed is not instantaneously affected except for the upstream segment where the lane-drop term is removed or in the case of having an on-ramp. The speed increase due to the higher number of lanes will appear in the following time steps due to the lower densities.

### C. Simulation Example

In order to analyze the performance of the previously proposed model, one direction of the stretch of freeway in Figure 1 with one reversible lane ( $\Delta_{\lambda,i} = 1$ ) has been simulated. Constant mainstream demand and typical and uniform values for model parameters have been used. Each segment has a length of 2 kilometers in order to obtain figures which a considerable portion of time with the lane closing or opening so the effects can be easily analyzed. When shorter lengths are used, results are equivalent. The reversible lanes are assumed to be open at the beginning of the simulation. Subsequently, they are closed during a period of time and opened again. The lane-drop parameter  $\phi$  is chosen to be 0 in order to analyze the behavior of the proposed model without being affected by the speed decrease due to the merging.

In the first simulation, a low enough mainstream demand was used to avoid congestion (even with the reversible lane closed). The results can be seen in Figure 5.

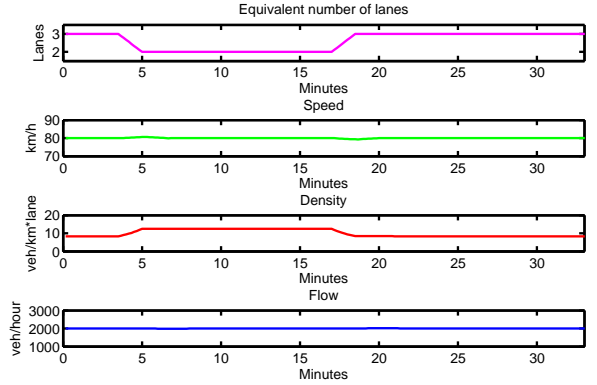


Fig. 5. Freeflow example of closing and opening a reversible lane

The results show that, as expected, the flow leaving the reversible lanes is almost unaffected by the lane closing if the traffic is totally uncongested. The density is increased during the time that the cars are leaving the reversible lane, because the same number of cars is forced to drive in 2 lanes instead of 3. The speeds are not directly affected by the lane closing but they are slightly modified in the following time steps due to the anticipation term and the change in density.

Equivalent results are obtained for the opening of the lane. The flow and speed almost do not change while the density is decreased in order to adapt to the equivalent number of lanes.

In the second simulation, a mainstream demand was used which does not create congestion with 3 lanes but, in this case, congestion appears when the lane is closed. The results can be seen in Figure 6.

As in the previous case, the density quickly increases during the transient in which the cars are leaving the reversible lane. When the density starts to approach the critical density, the speed starts to decrease with the corresponding outflow decrease. When the reversible lane is empty, the increase in density due to the closing of the lane finishes. However, the system is already in an unstable congested point so the density

keeps increasing (and the speed and flow decreasing) until the lane is opened again. Equivalent results are obtained for the lane opening.

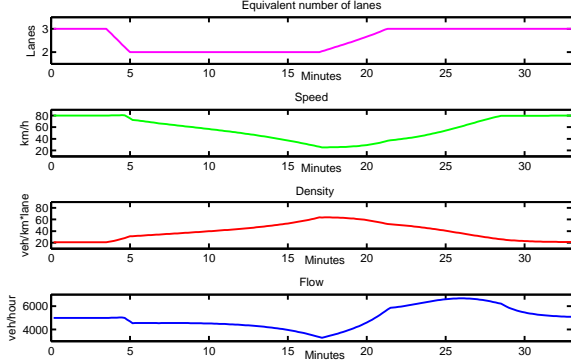


Fig. 6. Congested example of closing and opening a reversible lane

In both simulations, it has been checked that the conservation equation is respected in all the time steps (i.e. there is no loss of vehicles).

#### IV. LOGIC-BASED CONTROL FOR REVERSIBLE LANES

The first controller proposed is a logic-based controller that changes the state of the reversible lane by a simple feedback of the system state. Such a controller can be easily implemented in real applications without the need of an on-line optimization. The proposed control algorithm is designed for networks with only one reversible lane, but similar controllers could be proposed for cases of more than one reversible lanes.

The proposed controller uses a controller sample time  $T_c$  which is a multiple of the model sample time  $T$  (i.e.  $t = k \cdot T = k_c \cdot T_c$ ). This work uses a controller sample time of 2 minutes. The discrete input variable  $R(k_c)$ , which has to be computed for each controller time step, indicates if the reversible lane is open in the increasing direction (1), or in the decreasing direction (-1) or close for both directions (0).

For safety reasons, the reversible lane has to be closed during a certain period of time whenever the opened direction of the lane changes. In this work, it is assumed that the reversible lane has to be closed during one sample time during any direction change (i.e. if  $R(k_c - 1) = 1 \implies R(k_c) \neq -1$ ).

The feedback of the controller will be done with respect to the congestion lengths  $L_c(k_c)$  generated by the bottleneck with the reversible lane in both directions. The congestion lengths may be estimated for each direction by using the speed measured in the segments upstream the bottleneck:

$$L_c(k_c) = \sum_{n=Z+1}^{n=I-1} L_n \quad (21)$$

with  $v_Z(k_c) > V_{cg}$  and  $v_i(k_c) < V_{cg} \forall i \in (Z, I)$

where  $I$  is the first segment affected by the reversible lanes and  $Z$  is the first uncongested segment.  $V_{cg}$  is a speed threshold below which the system can be considered uncongested. This speed may differ from one network to another but it can

be easily estimated looking the data (for example, using the fundamental diagram). This paper uses a speed threshold ( $V_{cg}$ ) of 60 km/h.

Once we have an estimation of the congestion lengths, it is necessary to differentiate between three cases for the operation of the reversible lane:

1) *Congestion lengths are 0 for both directions:* In this case, the assignment of the reversible lane is not really critical. A reasonable strategy is to refrain from any change, that is, unless the flow in the direction currently using the reversible lane becomes much lower than in the opposite direction. This can be implemented via:

$$R(k_c + 1) = \begin{cases} 1 & \text{if } R(k_c - 1) = 1 \text{ and } \chi \cdot q_{b,I}(k_c) > q_{b,D}(k_c) \\ -1 & \text{if } R(k_c - 1) = 1 \text{ and } \chi \cdot q_{b,I}(k_c) < q_{b,D}(k_c) \\ 1 & \text{if } R(k_c - 1) = -1 \text{ and } q_{b,I}(k_c) > \chi \cdot q_{b,D}(k_c) \\ -1 & \text{if } R(k_c - 1) = -1 \text{ and } q_{b,I}(k_c) < \chi \cdot q_{b,D}(k_c) \end{cases} \quad (22)$$

where  $q_{b,I}(k_c)$  and  $q_{b,D}(k_c)$  are the flows through the bottleneck in the increasing and decreasing directions respectively, and  $\chi$  is a parameter bigger than 1 (in this paper,  $\chi = 1.3$ ). For  $\chi = 1$ , we would have the highest frequency of switching, but the reversible lane will be always open for the direction with the greatest flows.

2) *Any congestion length (or both) is (are) bigger than 0 and smaller than the maximum congestion length:* In this case, it is reasonable and equitable to attempt a balance of the congestion lengths on both directions via appropriate switchings. This may be achieved via the following switching regulator:

$$R(k_c + 1) = \begin{cases} 1 & \text{if } R(k_c - 1) = 1 \text{ and } \Lambda \cdot L_{c,I}(k_c) > L_{c,D}(k_c) \\ -1 & \text{if } R(k_c - 1) = 1 \text{ and } \Lambda \cdot L_{c,I}(k_c) < L_{c,D}(k_c) \\ 1 & \text{if } R(k_c - 1) = -1 \text{ and } L_{c,I}(k_c) > \Lambda \cdot L_{c,D}(k_c) \\ -1 & \text{if } R(k_c - 1) = -1 \text{ and } L_{c,I}(k_c) < \Lambda \cdot L_{c,D}(k_c) \end{cases} \quad (23)$$

where  $L_{c,I}(k_c)$  and  $L_{c,D}(k_c)$  are the congestion lengths in the increasing and the decreasing direction, respectively) and  $\Lambda$  is a parameter bigger than 1 (in this paper,  $\Lambda = 1.3$ ).

It should be noted that each switching decision has a cost due to the mandatory intermediate state  $R(k_c) = 0$  whereby no direction can profit from the capacity of the reversible lane. For  $\Lambda = 1$ , we would have the highest frequency of switching, but also the lowest differences in congestions lengths in both directions. Thus, the selected value of  $\Lambda$  reflects a trade-off between overall efficiency versus equity of the control system.

3) *Both congestion lengths are at or beyond their respective maximum values:* The situation is overcritical and the control strategy to be pursued should be based on a policy decision by the responsible authority. In the present study, the reversible lane will be opened in each direction after a fixed time  $T_R$  (in this paper, 15 minutes).

## V. DISCRETE MODEL PREDICTIVE CONTROL FOR REVERSIBLE LANES

The main concepts behind a MPC [10] strategy are:

- 1) The use of a prediction model to obtain the trajectories of the relevant variables of the system.
- 2) The optimization of an objective/cost function to determine the best sequence of control actions for the system.
- 3) The application of the rolling horizon procedure; from the best sequence of control actions only the first component is applied to the system and in the next control step the initial conditions are updated and the procedure is repeated again.

To formalize these concepts, consider the freeway traffic system with reversible lanes whose dynamic evolution is described in Sections II and III. The control inputs will be considered constant during one controller sample resulting in the following state-space model:

$$x(k_c + 1) = f(x(k_c), u(k_c), d(k_c)) \quad (24)$$

with  $x(k_c)$  the state vector that includes speeds, densities and queues and  $d(k_c)$  the non-controllable input vector that includes demand profiles, upstream speeds and downstream densities.  $u(k_c)$  is the discrete input vector containing a discrete variable  $R_j(k_c)$  for each reversible lane  $j$ .

In an MPC controller, the core is the optimization (25) of a cost function  $J(k_c)$ , which is used to measure the performance of the system along the prediction horizon  $N_p$  with respect to the input control sequence along the control horizon  $N_u$ :

$$\min_{R_t(k_c)} J(k_c) \quad \text{with } R_t(k_c) \in \{-1, 0, 1\} \quad (25)$$

where  $R_t(k_c) = [R_{j_1}(k_c), R_{j_1}(k_c + 1), \dots, R_{j_1}(k_c + N_u - 1), R_{j_2}(k_c), R_{j_2}(k_c + 1), \dots, R_{j_{N_R}}(k_c + N_u - 1)]$  is the set of reversible lanes and  $N_R$  the number of reversible lanes. The control inputs are kept constant after the control horizon  $N_u$ .

In this work, the MPC controller uses a cost function (26) containing one term for the TTS and another term that limits (using a soft constraint) the maximum values of the queues:

$$J(k_c) = \sum_{\ell=1}^{N_p} [T \sum_{i \in \mathbb{O}} w_i(k_c + \ell) + T \sum_{i \in \mathbb{I}_1} (\rho_i(k_c + \ell) L_i \lambda_i) + T \sum_{i \in \mathbb{I}_2} (\rho_i(k_c + \ell) L_i \hat{\lambda}_i(k_c + \ell)) + \sum_{i \in \mathbb{O}} \Omega_i(k_c + \ell)] \quad (26)$$

where  $\Omega_i(k + \ell)$  is a penalization term that is different to zero if the corresponding queue constraint is violated and  $\mathbb{O}$ ,  $\mathbb{I}_1$  and  $\mathbb{I}_2$  are the set of all the segments with an on-ramp, without reversible lanes and with reversible lanes, respectively. If it is desired to limit or reduce the frequency of reversible lane switching, an additional term penalizing each switching may be readily included in the cost function.

In order to solve optimization (25), we use a search tree including all the feasible profiles for the opening of the reversible lanes  $R_t(k_c)$ . As we are dealing with non-convex integer optimization, a direct way to obtain the global optimum of (25) is to evaluate the cost function for all the feasible points in the search tree. Limiting the number of feasible nodes in the mixed integer optimization problem is a technique that has been successfully used before in the context of freeway

traffic MPC for VSL [11]. The feasible nodes limitation can be obtained by using the following constraints:

- A reversible lane has to be closed in both directions during one controller sample time for all direction switches, in order to allow the cars to leave the corresponding lane.
- It is always suboptimal to keep lane closed in both direction during more than one controller sample time.
- If the lane is closed in both directions, it is always suboptimal to open the lane in the same direction in which it was previously open.
- For the last prediction step, it is not allowed to set the last value of the input along the control horizon as 0, because it is suboptimal to keep the reversible lane closed if it is not going to be opened during the next steps.

Fig. 7 shows an example of the search tree if only one reversible lane is used and initially the lane is opened in the increasing direction:

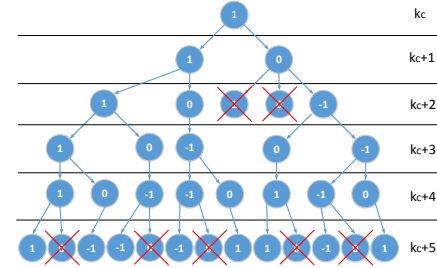


Fig. 7. Search tree for a reversible lane with  $R(k_c) = 1$  and  $N_u = 5$

The number of leafs  $N_l(N_u)$  increases with respect to the control horizon according to the one-step delayed Fibonacci series (1,2,3,5,8,13,21,34,55,89,...):

$$N_l(N_u) = \frac{\phi^{N_u+1} - (1 - \phi)^{N_u+1}}{\sqrt{5}} \quad \text{with } \phi = \frac{1 + \sqrt{5}}{2} \quad (27)$$

If the reversible lane is closed for the current controller step ( $R_t(k_c) = 0$ ), it has to be allowed to open the lane in both directions for  $k_c + 1$  due to unpredicted changes during the last controller time step  $T_c$ . Therefore, the number of leafs increases according to:

$$N_l(N_u) = 2 \cdot \frac{\phi^{N_u} - (1 - \phi)^{N_u}}{\sqrt{5}} \quad (28)$$

The a-priori knowledge of the number of leafs allows to know the maximum horizons for which the optimal solution can be computed within a controller time step. The computation time needed for obtaining each MPC solution is the number of leafs multiplied by the time needed for the simulation of the network during the prediction horizon.

For a larger freeway network with many reversible lanes, the main problem is the computation time needed for the evaluation of such a large number of possible combinations of the discrete variables. In these cases, additional constraints connecting the reversible lanes may be used. For instance, the lanes closed in one direction should be on the left hand side of the opened lanes (in the countries with left-hand drive). When even using the new constraints, it is not possible to find the optimal solution within a limited computation time available, thus genetic or distributed MPC could be used as in [5], [11].



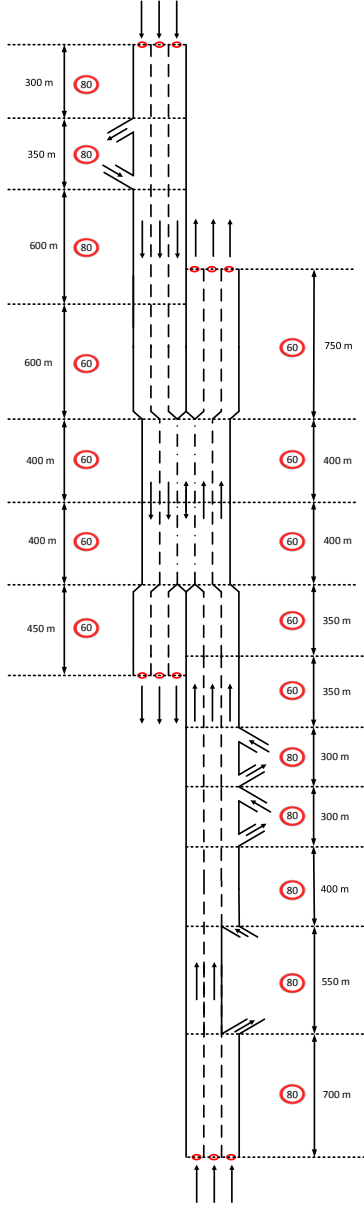


Fig. 8. Sketch of the modeled network

## VI. ANALYZED NETWORK

Fig. 8 shows the freeway stretch that has been considered as a simulation-based testbed. The modeled freeway is a subsection of the ring road SE-30 (Spain) from marker post 7/5 to 12/0 (S-N direction) and from 13/1 to 10/0 N-S direction.

The modeled network includes the Centenario bridge, which is a bottleneck that creates, during the peak hours in the morning, the biggest recurrent congestion in the region of Seville. The bridge has 2 lanes fixed in each direction and one reversible lane. The reversible lane is currently changed manually by the traffic operators looking at the cameras along the bridge. This real-time manual control is deemed to perform better than fixed control, which would operate with pre-specified switching intervals during pre-specified (peak) periods of the day. The rest of the modeled freeway has 3 lanes in each direction (except segment 2 in the S-N direction which

has two lanes). The morning rush-hour congestion usually occurs between 8 and 9 am. The congestion is created by the bottleneck on the bridge and propagates upstream for both directions. The detector measurements indicate that the traffic downstream the bridge is always uncongested in both directions and there is no other source of recurrent congestion.

This simulation uses loop detector data over the 6AM-11AM time range for ten different weekdays from four loop detectors located in the mainline at the beginning and the end of both directions (marked (red) cycles in Fig. 8). Each loop detector provides measurements of aggregated speeds and flows every 15 minutes. There is also data available of the state of the reversible lane  $R_t(k)$  indicating the time when the lane is closed or opened in one direction. For stability, the segment length and the simulation time step should satisfy  $L_i > v_{free,i}T$  for every link  $i$ . Therefore, the model sample time has to be smaller than the data sample time (15 minutes). Since this paper uses a model sample time  $T$  of 10 seconds, a zero-order interpolation was applied to the data

The ramp flow data have been directly estimated by taking the difference between the aggregated flow data in the mainline and distributing between the ramps using a-priori knowledge about the ramp flows. The second and third on-ramps in the S-N direction are modeled as only one ramp in order to avoid stability problems due to their proximity.

The process of validating the developed model consists of manually calibrating a number of parameters via repeated computer simulations similarly to the validation done in [12]. The results are compared to real data from loop detectors after each simulation, and a manual adjustment of a number of parameters is performed based on the observation of whether or not congestion is predicted accurately enough. A more detailed validation cannot be carried out due to the lack of ramp data, the aggregation of the detector data (15 minutes) and the absence of enough mainline detectors (especially in the segments with the reversible lane).

For a proper identification, the upstream end of the freeway stretch should be congestion-free; otherwise the entering flows are determined by the internal congestion. Due to a lack of measurements at the upstream end of the modeled part of the freeway (in both directions), one additional (virtual) measurement point was produced in the same way as in [13].

For the controller simulations, it is assumed that there is a sensor in each segment which provides density and speed measurements for each controller step; alternatively, an appropriate estimation scheme [14] may be employed. The control parameters were manually selected by performing some simulations with different values of the parameters.

The simulations were carried out using MATLAB and an Intel Core i5 CPU. The average time needed is 1.25 seconds for the network simulation (from 6 AM to 11 AM), 0.0003 seconds (any controller step) for the logic-based controller and 0.65 seconds (any controller step) for the MPC (with  $N_u = 5$  and  $N_p = 5$ ). It can be seen that the logic-based control can be computed almost instantaneously so to be implemented at the same time when the measures are taken. The MPC can be computed within a controller time step (i.e. implemented at  $k_c + 1$  for the measures taken at  $k_c$ ).

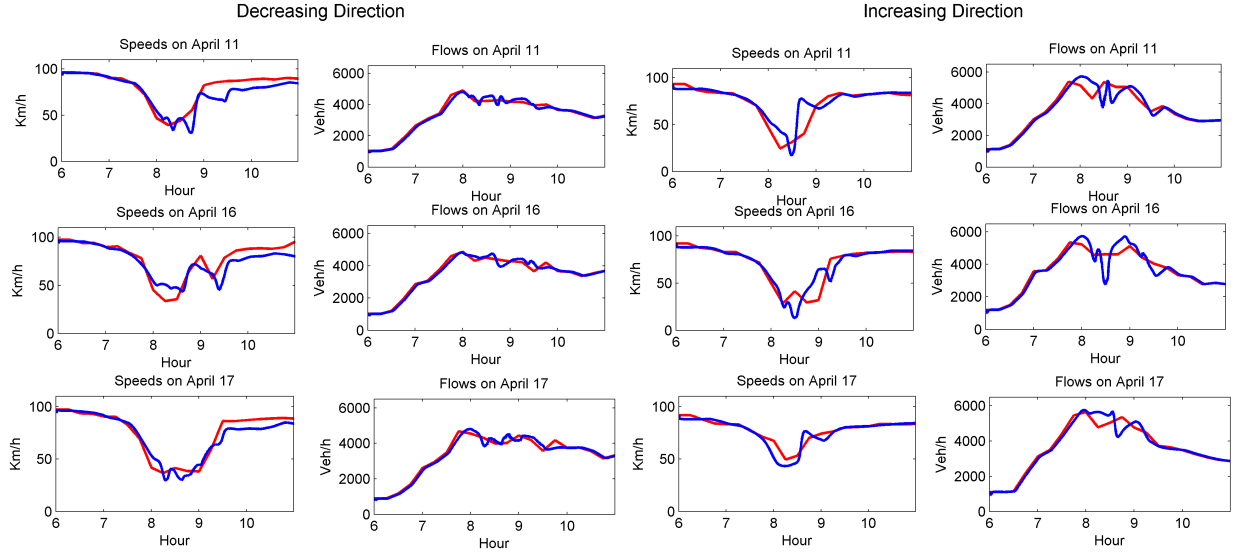


Fig. 9. Comparison between simulated speeds and flows (blue) and interpolation of 15 minutes aggregated data (red) for 11th, 16th and 17th of April 2012.

## VII. SIMULATION RESULTS

### A. Model Validation

Fig. 9 shows the speeds and flows measured upstream of the bridge for both directions and the corresponding values estimated by the proposed model. It can be seen that the model shows a relatively good speed and flow estimation for three typical days with relatively different congestion profiles.

Table I shows the mean relative errors for speeds and flows for the detectors upstream the bridge (marker post 7/5 for the S-N direction and marker post 13/1 for the N-S direction). The days used for the identification of the model parameters are 11th, 16th and 17th of April 2012. The rest of the days (9th, 10th, 12th, 13th, 18th, 19th and 20th of April 2012) are used to validate the model identification (parameter values).

TABLE I  
MODELING ERRORS

		Speed Error (%)	Flow Error (%)
Identification	S-N direction	8.47 %	3.46 %
	N-S direction	8.79 %	5.21 %
Validation	S-N direction	15.68 %	4.10 %
	N-S direction	12.90 %	6.29 %

### B. Control

Two different MPC controllers have been simulated. The first one has a constraint of a maximum of 100 vehicles waiting in the queues. The second one has a constraint of 500 vehicles which means to have in practice unconstrained queues.

The results of the reversible lane operations computed by each controller can be seen in Fig. 10. The figure shows that the MPC controllers tend to reduce the time that the reversible lane is closed by decreasing the number of direction changes in the reversible lane. For the constrained MPC, the queue

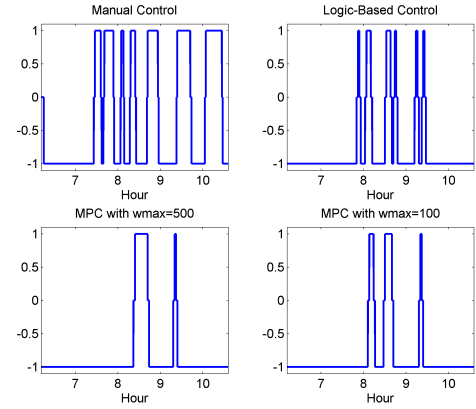


Fig. 10. Reversible lane operations on April 11, 2012

constraint causes that the reversible lane has to change the direction one more time than for the unconstrained MPC in order to keep the queues under 100 vehicles. This can be seen in Fig. 11 where the queues at the mainstream origins obtained for the different reversible lane operations are shown (the increasing queue for the Logic-Based controller is 0 during the all simulation). None of the simulations carried out create on-ramp queues. Fig. 11 shows that the constrained MPC is the only controller simulated that keeps both queues under the constraint. The logic-based controller provides an intermediate solution between the real implemented manual controller and the MPC controller proposed.

The density contour plots obtained with the different controllers are shown in Fig. 12. It can be seen that the unconstrained MPC removes the congestion completely in the N-S direction by increasing the congestion substantially in the S-N direction. This solution may be optimal in terms of cost function performance but it may be difficult to implement due to equity reasons (one direction is seen to be clearly benefitted



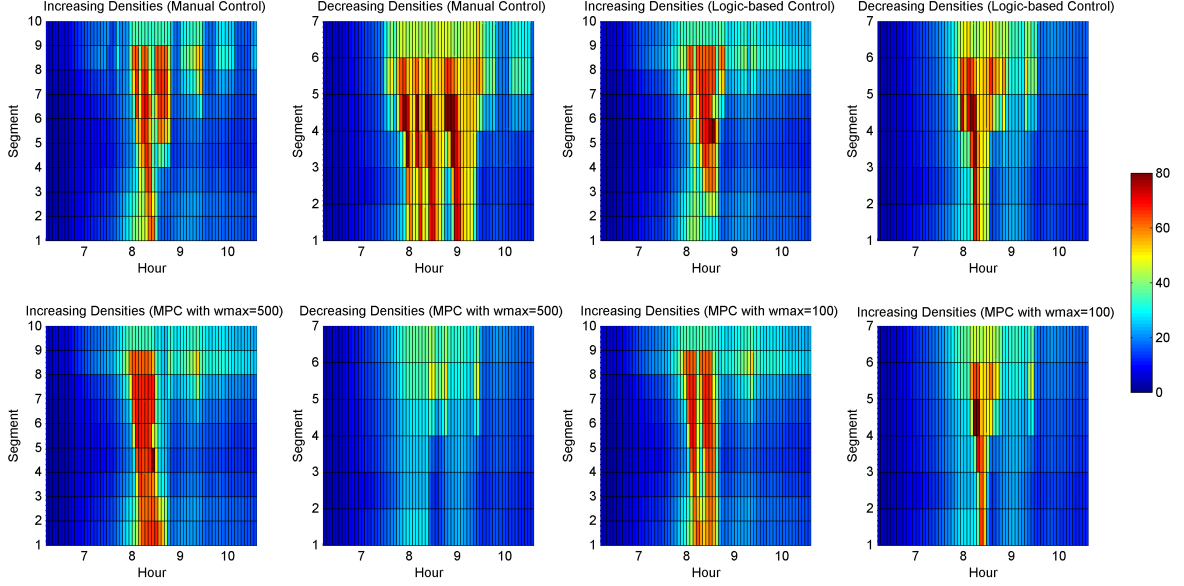


Fig. 12. Densities for the different controllers applied on April 11, 2012

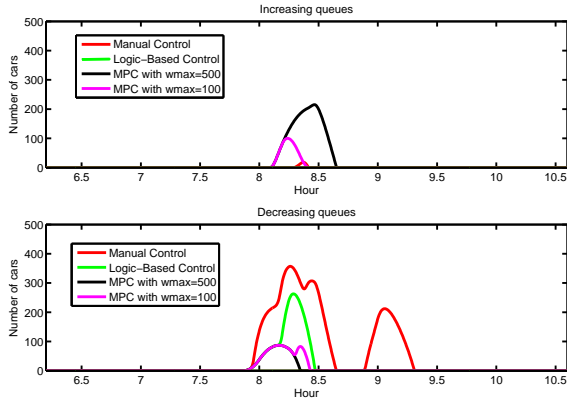


Fig. 11. Mainstream queues on April 11, 2012

versus the other). On the other hand, the constrained MPC and the logic-based controller provide control inputs that tend to equalize the congestion in both directions. Therefore, these controllers may be good solutions for real implementations.

In the numerical results on Table II, it can be seen how the three proposed controllers substantially reduce the TTS with respect to the manually controlled case. The TTS reductions obtained for the three controllers are very close to each other. However, it can be seen that unconstrained MPC results in a higher TTS reduction than the other proposed controllers.

The TTS reduction shown covers both directions. This reduction is obtained by decreasing the TTS in one direction while increasing the TTS in the opposite direction or by decreasing the TTS for both directions. For example, for April 17 the TTS is decreased for the N-S direction with all the used controllers (38.95 % with the logic-based controller, 40.58 % with the unconstrained MPC and 46.61 % with the constrained

MPC). However, for the N-S direction the TTS is increased with two controllers (1.35 % with the logic-based controller and 6.39 % with constrained MPC) and decreased with one controller (6.85 % with the unconstrained MPC).

TABLE II  
CONTROLLER PERFORMANCES IN TERMS OF TTS REDUCTION (%)

	April 11	April 16	April 17
Manually controlled system	0 %	0 %	0 %
Logic-based controller	8.84 %	19.97 %	22.68 %
Discrete MPC with $w_{max} = 500$	11.94 %	22.24 %	26.97 %
Discrete MPC with $w_{max} = 100$	9.43 %	19.82 %	25.22 %

## VIII. CONCLUSION

This paper has proposed a macroscopic model and two control algorithms for the dynamic operation of reversible lanes on freeways. The proposed extension of model METANET, which uses the concept of an equivalent number of lanes, has been validated with real data over the Centenario Bridge of the SE-30 freeway in Seville, Spain. The results show that the proposed model is able to reproduce traffic congestion due to the reversible lanes with a mean error on the identification days of 8.63% and 4.33% for speed and flows, respectively. The errors are 14.29% and 5.19% for the days used for validation.

Based on this model, two kinds of dynamic controllers have been developed. The first one is an easy-to-implement logic-based controller, which takes into account the congestion lengths generated by the reversible lane bottleneck. The second one is a discrete Model Predictive Control (MPC) where the discrete optimization carried out is via evaluation of the cost function for all the leafs in a reduced search tree. The main advantages of the proposed MPC are larger TTS

reductions, reduction of the number reversible lane switching and possibility of using constraints. The main advantages of the logic-based controller are ease of real implementation, intuitive tuning and equity for opposite directions. All the proposed controllers show a substantial reduction of the TTS and can be computed in a short time.

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