

TECHNICAL UNIVERSITY OF CRETE
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING



**Synchronization techniques for non-ideal
communication channels**

by

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Abstract

This thesis investigates synchronization techniques in non-ideal wireless communication channels, also referred to as wideband channels. In these channels, several non-ideal conditions occur, like, for example, multipath propagation resulting into intersymbol interference, and carrier frequency offset. Thus, in order to ensure precise symbols recovery at the receiver, we must use advanced synchronization methods.

The study begins with the examination of the structure of the transmitter, highlighting the creation of the symbol packets via the transmitter's filter. Concepts such as training symbols, square root raised cosine filter, intersymbol interference and low pass equivalent of input signal are analyzed in Chapter 2.

In Chapter 3, the thesis delves into the characteristics of the communication channel, exploring the scenario of multipath propagation. It emphasizes the impact of delay spread, a critical parameter in multipath channels, which introduces intersymbol interference.

Chapter 4 is about the operation of the receiver. Synchronization techniques are addressed in detail, including methods for estimating and correcting the Carrier Frequency Offset (CFO) and the discrete equivalent channel. Finally, in order to counteract the channel's effects, we implement a zero-forcing equalizer, ensuring that the overall telecommunication system approximates a simple delay.

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Chapter 1

Introduction

In modern telecommunications, the efficiency and reliability of data transmission are paramount. As the demand for high speed wireless communications continues to escalate, the need to use non-ideal channels increases. This is their main advantage over ideal channels as they cover a larger bandwidth and are therefore called wideband channels. These channels, are characterized by multipath propagation, fading, and various forms of interference, which can significantly degrade the performance of communication systems. This thesis delves into synchronization techniques designed for several aspects of non-ideal channels. The following chapters provide a detailed examination of the transmitter and receiver operations, the nature of the communication channel and the impact of additive noise. Special emphasis is placed on the synchronization process, including techniques for estimating and correcting CFO (carrier frequency offset) and the implementation of channel estimation and equalization methods to mitigate the effect of ISI (intersymbol interference).

The transmitter's primary function in a communication system is to send packets of symbols through a channel to the receiver. A key component in the packet are the training symbols, which are crucial for synchronization, channel estimation and linear equalization, operations that will be implemented by the receiver. The transmitter's output is generated by convolving the input signal with an SRRC (square root raised cosine) filter. This baseband representation of the input signal is important for the next stages of the telecommunication system. The mathematical processes involved in creating the low pass equivalent signal and every operation of the transmitter are detailed in Chapter 2.

Chapter 3 provides an analysis of the channel's effect on signal transmission. The signal reaches the receiver through environmental reflections. This type of propagation is called multipath. A significant parameter in multipath propagation is delay spread. It is the time difference between the paths that the transmission signal follows. Channels are classified based on the delay spread as small delay spread channels and large delay spread channels. For small delay spread channels, the equivalent baseband output is described by a simplified model that does not introduce ISI. In contrast, large delay spread channels exhibit frequency selective fading, resulting in ISI and requiring more complex models to describe their behavior. Chapter 3 also explores the impact of additive white Gaussian noise on the transmitted signal. The noise is added to the signal during transmission and is measured using SNR (signal to noise ratio).

Chapter 4 explores the functionality of the receiver in a wireless telecommunication system,

emphasizing the processes of channel estimation, synchronization, and equalization. The receiver is tasked with processing the incoming signal, which involves several critical steps to ensure accurate data recovery. First of all, the signal is convolved with an SRRC pulse, identical to the transmitter's filter, to form the composite impulse response. A significant parameter is the CFO (carrier frequency offset), which is created during demodulation. The CFO estimation and correction are performed after filtering and synchronization. Synchronization is essential for aligning the received signal with the transmitted symbols. This is achieved by sampling the signal and calculating the optimal delay. The method of cross-correlation is useful to determine the synchronization point, resulting to a "good" symbol-spaced output sequence. In Chapter 4, the concept of the discrete equivalent channel is introduced, whose estimation is performed jointly with the estimation of the CFO as a least squares problem. After that, the impact of the CFO must be corrected. Finally, a zero forcing equalizer to mitigate the channel's effects and recover the transmitted data accurately is implemented. This equalizer nullifies the channel's influence, resulting in a system approximately equivalent to a simple delay. The overall process involves detailed mathematical formulations and algorithms to ensure precise signal processing and data recovery.

Chapter 2

Transmitter

The role of a transmitter is to send fixed-size packets of symbols through a channel to the receiver. Each step of the transmitter's operation will be presented analytically.

2.1 Creation of packet

Each packet from the transmitter to the receiver contains information. More specifically, let us assume that a packet consists of N symbols. Every symbol is randomly selected from a Quadrature Amplitude Modulation (QAM) constellation. Specifically, we assume that its value is a complex number among $\pm 1 \pm j$. The symbol period is denoted as T . The mathematical representation of a symbol of a packet a , is given below

$$a_n = a_{n,I} + ja_{n,Q},$$

where $n = 0, \dots, N-1$, $a_{n,I} = \pm 1$ is the real part of the symbol, and $a_{n,Q} = \pm j$ its imaginary part. Therefore, it is easy to describe a packet of symbols as

$$a = [a_0 \ a_1 \ \dots \ a_{N-1}].$$

An alternative mathematical expression of a packet of N symbols as a function of time is

$$a(t) = \sum_{n=0}^{N-1} a_n \delta(t - nT),$$

where t represents time. It is important to mention that there N_{tr} symbols in the packet known to the receiver. These symbols, referred to as training symbols, play a crucial role in channel estimation, equalization and synchronization.

Since the receiver is aware of training symbols, they are not used to transmit information. Training symbols can be placed at the beginning, end, or interspersed among other symbols within the packet. In this thesis, the training symbols are positioned consecutively at the start of the packet.

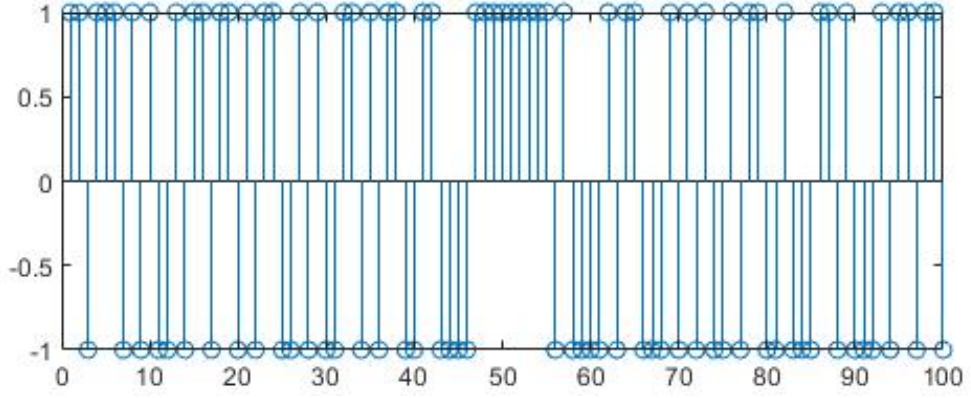


Figure 2.1: Real part of a packet with 100 symbols.

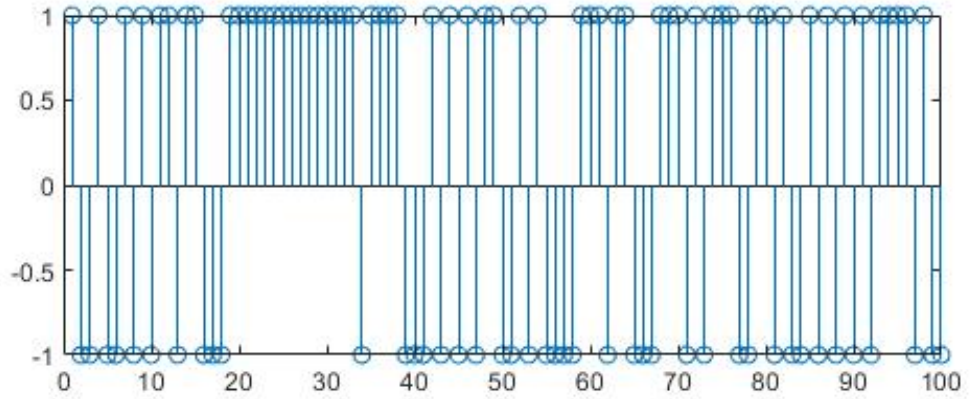
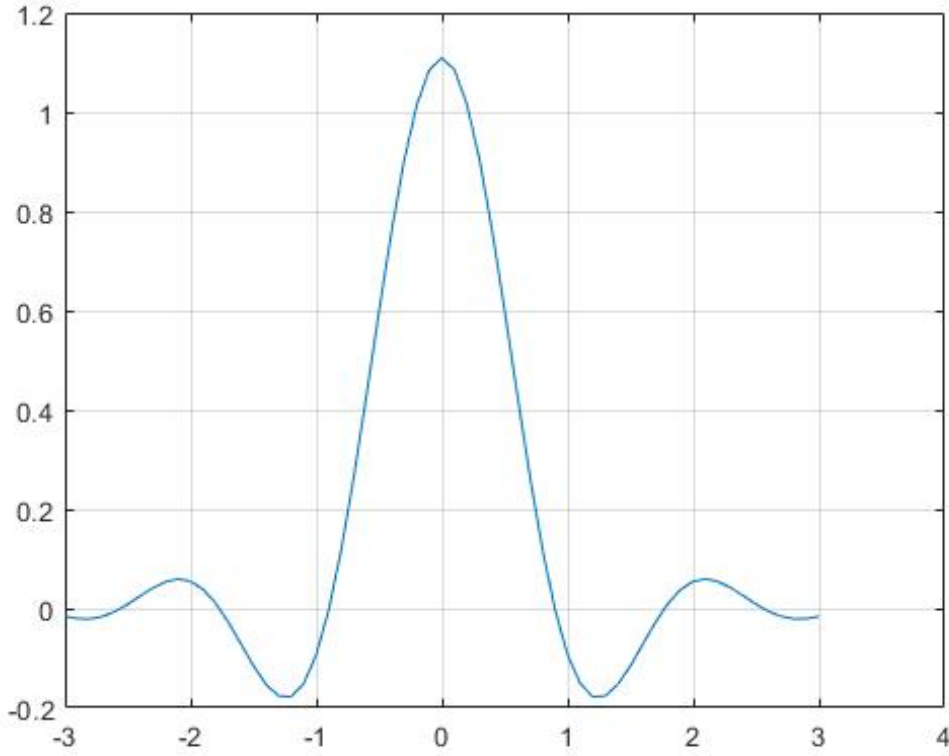


Figure 2.2: Imaginary part of a packet with 100 symbols.

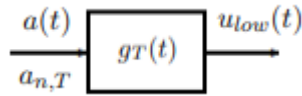
2.2 Transmission filter for ISI mitigation

Intersymbol interference (ISI) in wireless communication occurs when past symbols interfere with current symbols, leading to the degradation of the system's performance [1]. The predominant choice to eliminate the effect of ISI is to design a transmission system where the overall impulse response adheres to the Nyquist criterion. The Raised Cosine (RC) filter stands as the most popular among filter responses that fulfills the specified criterion for mitigating ISI. At the transmitter, the signal is passed through a square root raised cosine (SRRC) filter. An identical SRRC filter is utilized at the receiver. The convolution of two SRRC filters results in a composite RC filter response.

Figure 2.3: SRRC pulse with roll-off factor $b = 0.4$.

2.3 Low-pass equivalent of input signal

The output of the transmitter u_{low} , is generated by convolving the input signal a , with a SRRC pulse g_T . This process produces what is known as the low-pass equivalent of the input signal [2]. It refers to a baseband representation of the input signal and is used as input to the next stage of the telecommunication system [3].



To compute the low-pass equivalent of the channel input requires understanding the properties of convolution. The proof is given below

$$\begin{aligned} u_{low}(t) &= a(t) \otimes g_T(t) \\ &= \int_{-\infty}^{\infty} a(\tau) g_T(t - \tau) d\tau, \end{aligned}$$

where

$$a(t) = \sum_{n=0}^{N-1} a_n \delta(t - nT).$$

Thus,

$$\begin{aligned} u_{low}(t) &= \int_{-\infty}^{\infty} \left[\sum_{n=0}^{N-1} a_n \delta(\tau - nT) \right] g_T(t - \tau) d\tau \\ &= \sum_{n=0}^{N-1} a_n \int_{-\infty}^{\infty} \delta(\tau - nT) g_T(t - \tau) d\tau \end{aligned}$$

and

$$u_{low}(t) = \sum_{n=0}^{N-1} a_n g_T(t - nT).$$

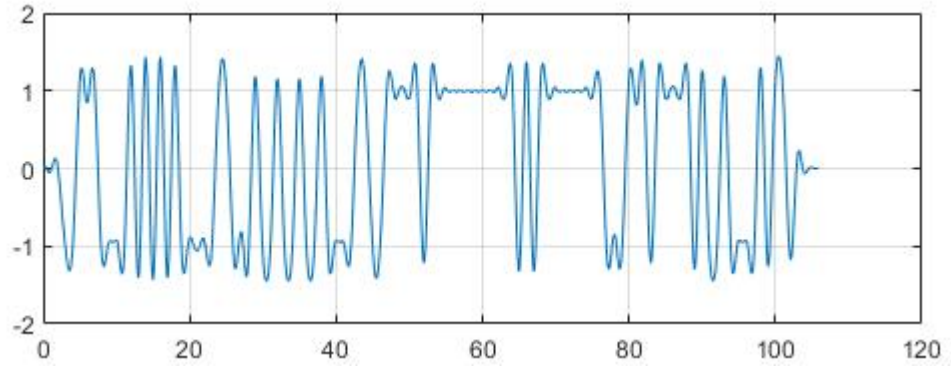


Figure 2.4: Real part of low-pass equivalent of input signal.

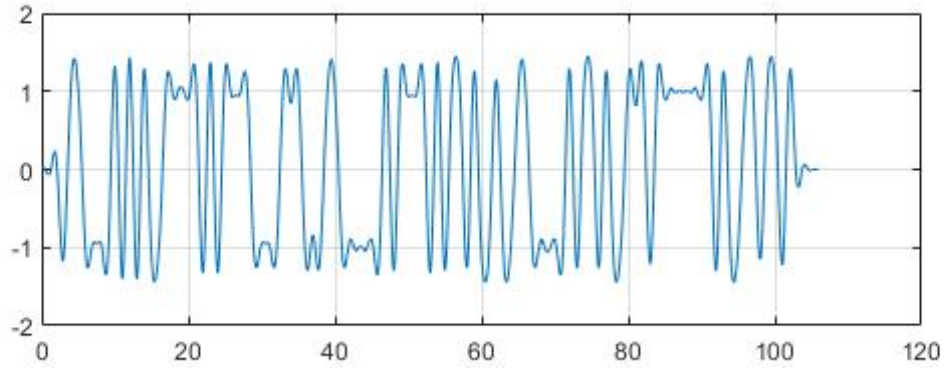


Figure 2.5: Imaginary part of low-pass equivalent of input signal.

Chapter 3

Channel

In telecommunication systems, the word channel refers to the medium through which an electromagnetic wave is transmitted from the transmitter to the receiver. In wireless telecommunications, this medium is air. However, the channel includes more than the medium itself. It also includes the specific path that the electromagnetic wave follows within this medium. This path critically influences the quality of the signal received by the receiver. More specifically, there are two cases of paths that affect the received signal differently. The first one is called line-of-sight (LOS). The signal travels in a straight line from the transmitter to the receiver and there are no obstructions between them. The second one is called multipath [4]. In this scenario the signal reaches the receiver through environmental reflections.

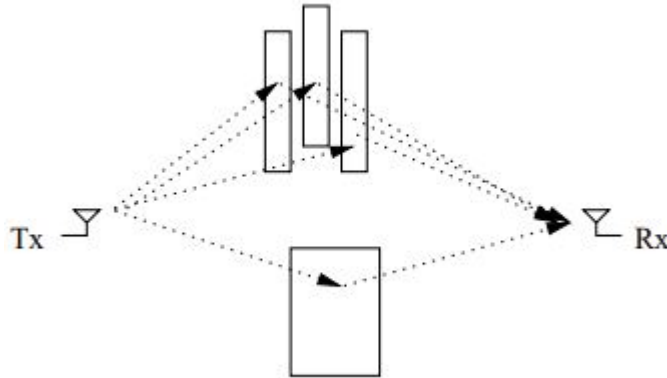


Figure 3.1: Multipath

3.1 Multipath

Let assume that the low-pass equivalent signal is

$$u_{low}(t) = \sum_k u_k g_T(t - kT),$$

where u_k the information symbols and $g_T(t)$ the SRRC pulse.

The modulated signal is

$$u(t) = \Re \left\{ u_{low}(t) e^{j2\pi f_c t} \right\}.$$

Due to the multiple paths, the received signal is given by the relation

$$r(t) = \sum_{n=0}^{N(t)} a_n(t) u(t - \tau_n(t)),$$

where $N(t)$ is the number of the distinguishable paths, $\tau(n) = \frac{r_n(t)}{c}$ ($r_n(t)$ is the length of the n -path and $c = 3 \times 10^8 m/s$ is the speed of light propagation) is the delay of the n -th path and $a_n(t)$ is the coefficient expressing the contribution of the n -th path to the received signal. For the time being, the parameter of noise is ignored.

At this point, a significant parameter is introduced, which is crucial for the study of phenomena in wireless communications. This parameter is known as delay spread and is described as the time difference between the propagation delays along the two signal paths. Delay spread is defined as

$$T_d(t) := \max_{i,j} |\tau_i(t) - \tau_j(t)|,$$

and for time invariant channels

$$T_d := \max_{i,j} |\tau_i - \tau_j|.$$

3.1.1 Small delay spread wireless channels

If the delay spread satisfies the relation

$$T_d(t) \ll \frac{1}{W} \approx T,$$

where W is the bandwidth and T the symbol's period, then

$$u_{low}(t - \tau_i(t)) \approx u_{low}(t - \tau_j(t)) \approx u_{low}(t - \tilde{\tau}(t)),$$

and $\tilde{\tau}(t)$ is the average delay at the time instant t . Combining this relation with the equation of the signal at the receiver, we get

$$\begin{aligned} r(t) &= \sum_{n=0}^{N(t)} a_n(t) u(t - \tau_n(t)) \\ &= \Re \left\{ \sum_{n=0}^{N(t)} a_n(t) u_{low}(t - \tau_n(t)) e^{j2\pi f_c(t - \tau_n(t))} \right\} \\ &= \Re \left\{ \left[\left(\sum_{n=0}^{N(t)} a_n(t) e^{-j2\pi f_c \tau_n(t)} \right) u_{low}(t - \tilde{\tau}(t)) \right] e^{j2\pi f_c t} \right\}. \end{aligned}$$

Thus, the equivalent baseband output is

$$r_{low}(t) = \left(\sum_{n=0}^{N(t)} a_n(t) e^{-j2\pi f_c \tau_n(t)} \right) u_{low}(t - \tilde{\tau}(t)).$$

Let us define

$$v(t) := \sum_{n=0}^{N(t)} a_n(t) e^{-j2\pi f_c \tau_n(t)}.$$

So, the low-pass equivalent output is

$$r_{low}(t) = v(t) u_{low}(t - \tilde{\tau}(t)).$$

If $v(t)$ and $\tau(t)$ are time invariant terms, then

$$r_{low}(t) = v u_{low}(t - \tilde{\tau}).$$

Sampling the above relation

$$r_m := r_{low}(t)|_{t=mT+\tilde{\tau}} = v u_{low}(t - \tilde{\tau})|_{t=mT+\tilde{\tau}} = v u_m.$$

We observe that the equivalent baseband output at the time instants $mT + \tilde{\tau}$, r_m is equal to the information symbol u_{low_m} multiplied by the scalar quantity v . In other words, in this case, there is no intersymbol interference (ISI). Of course, the quantity v is a crucial factor for the performance of the system in a noisy environment. In summary, assuming discrete time m , and denoting the impulse response of the time invariant system with h , the equivalent baseband of a channel with small delay spread can be described as follows:

$$r_m = h u_m + w_m,$$

where w_m the additive white Gaussian noise. In case that the channel is time variant then

$$r_m = h_m u_m + w_m.$$

The magnitude of the frequency response of the discrete channel, $|H(F)| = |h_m|$, is flat. This type of fading, which affects only the amplitude and the phase of the input symbols at the output without introducing ISI, is called flat fading and the channel h is called narrow-band channel or flat fading channel.

3.1.2 Large delay spread wireless channels

If delay spread satisfies the relation

$$T_d(t) > \frac{1}{W} \approx T,$$

then,

$$\begin{aligned} r(t) &= \sum_{n=0}^{N(t)} a_n(t) u(t - \tau_n(t)) \\ &= \Re \left\{ \sum_{n=0}^{N(t)} a_n(t) u_{low}(t - \tau_n(t)) e^{j2\pi f_c(t - \tau_n(t))} \right\} \end{aligned}$$

$$= \Re \left\{ \left(\sum_{n=0}^{N(t)} a_n(t) e^{-j2\pi f_c \tau_n(t)} u_{low}(t - \tau_n(t)) \right) e^{j2\pi f_c t} \right\}.$$

Let assume that

$$a_n^b(t) = a_n(t) e^{-j2\pi f_c \tau_n(t)}.$$

The equivalent baseband output is

$$\begin{aligned} r_{low}(t) &= \sum_n a_n^b(t) u_{low}(t - \tau_n(t)) \\ &= \sum_n a_n^b(t) \sum_k u_{low_k} g_T(t - kT - \tau_n(t)) \\ &= \sum_k u_k \sum_n a_n^b(t) g_T(t - kT - \tau_n(t)). \end{aligned}$$

Sampling the above relation

$$r_m := r_{low}(t)|_{t=mT} = \sum_k u_k \sum_n a_n^b(mT) g_T(mT - kT - \tau_n(mT)).$$

Setting $l := m - k$,

$$r_m = \sum_l u_{m-l} \sum_n a_n^b(mT) g_T(lT - \tau_n(mT)).$$

Let define $h_{m,l} := \sum_n a_n^b(mT) g_T(lT - \tau_n(mT))$, then

$$r_m = \sum_l h_{m,l} u_{m-l}.$$

If the system is time invariant ($a_n^b(t) = a_n^b$ and $\tau_n(t) = \tau_n$),

$$r_m = \sum_l h_l u_{m-l},$$

where $h_l := \sum_n a_n^b g_T(lT - \tau_n)$. The magnitude of the frequency response of the channel, $|H(F)| = |\mathcal{F}\{..., h_{-1}, h_0, h_1, ...\}|$, is not flat. This type of fading introduces intersymbol interference (ISI) and is called frequency selective fading. In this case, the channel is referred to as wideband or frequency selective channel.

3.2 Additive noise

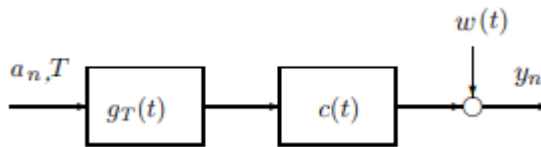


Figure 3.2: Signal before receiver.

During the transmission of the signal from the transmitter to the receiver, the presence of noise is inevitable. In this thesis, the noise is white Gaussian and is added, as shown in the figure 3.2, after the channel and before the receiver's filter. A representative measure for describing the noise is signal-to-noise ratio (SNR). The unit of measurement for the SNR is dB (decibels). The SNR is defined as follows:

$$SNR_{dB} = 10 \log_{10} \frac{P_{signal}}{P_{Noise}},$$

where P_{signal} is the power of the signal and P_{Noise} is the power of the additive white Gaussian noise. The transmission signal with the effect of the channel, $c(t)$, and after the addition of the white Gaussian noise, $w(t)$, is presented in the following figures.

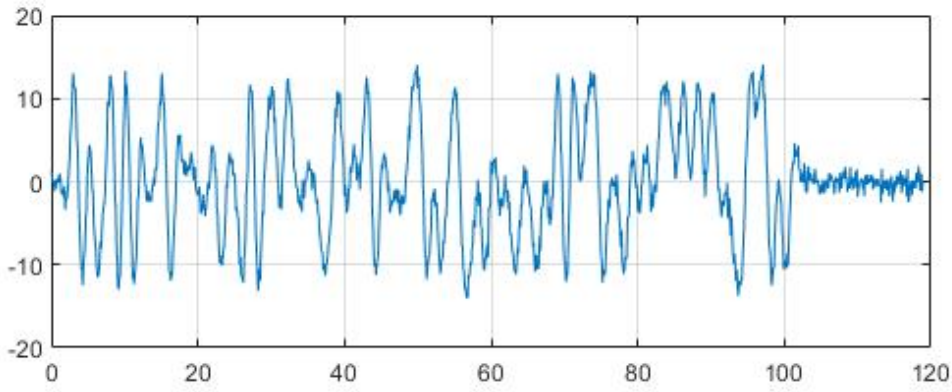


Figure 3.3: Real part of signal with noise ($SNR_{dB} = 20$).

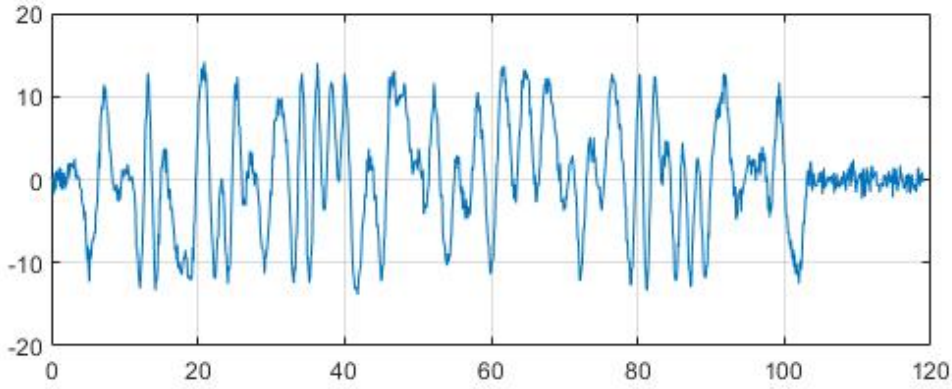


Figure 3.4: Imaginary part of signal with noise ($SNR_{dB} = 20$).

Chapter 4

Receiver

4.1 Analog Composite Channel and CFO

Once the packet arrives at the receiver, it is convolved with the square root raised cosine (SRRC) pulse in order to create, in combination with the pulse at the transmitter, the raised cosine (RC) filter. The convolution $h(t) = g_T(t) \otimes c(t) \otimes g_R(t)$ is known as composite impulse response or analog composite channel. The absolute value of the analog composite channel is presented in the following figure:

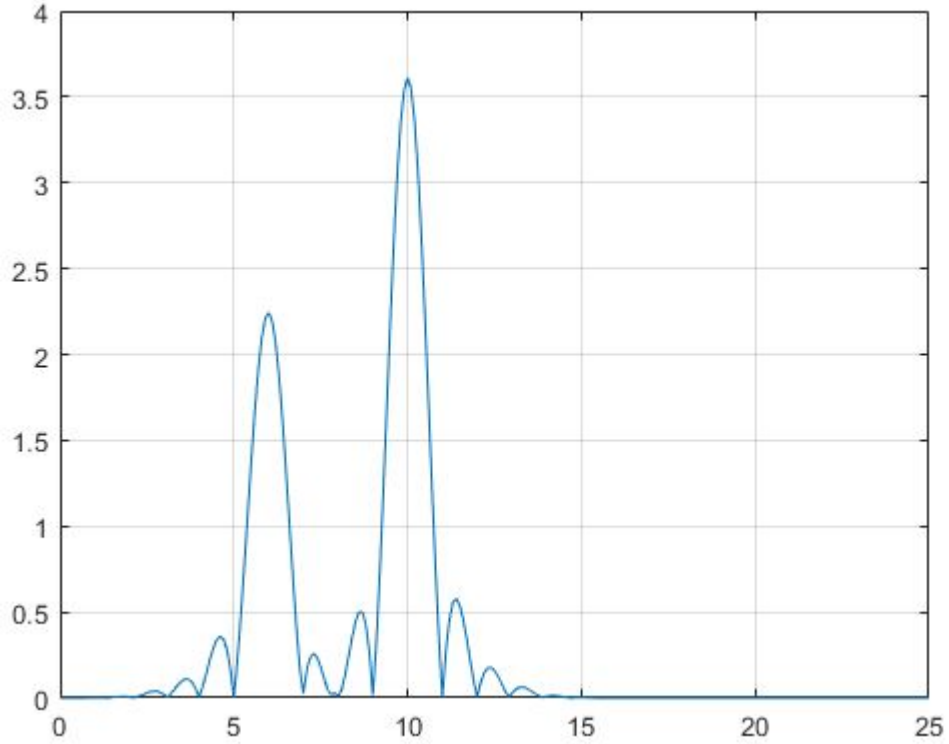


Figure 4.1: Absolute value of analog composite channel.

At this point, it is important to note the carrier frequency offset (CFO). The CFO enters into the received packet during the process of demodulation at the receiver. The receiver demodulates the packet based on the carriers

$$\cos(2\pi(f_c + \Delta F)t + \varphi), \quad -\sin(2\pi(f_c + \Delta F)t + \varphi).$$

The quantity ΔF is called CFO. Because of the fact that CFO, in this thesis, is small ($\Delta F \ll \frac{1}{T}$), its estimation and correction can be performed after filtering and synchronization. Hence, precise estimation and correction of CFO will be presented in detail later.

The signal at the receiver, with the presence of CFO, becomes

$$y(t) = e^{j(2\pi\Delta F t + \varphi)} \sum_{n=0}^{N-1} u_n h(t - nT) + w(t),$$

where u_n is the input symbol and $h(\cdot)$ the analog composite channel.

4.2 Synchronization

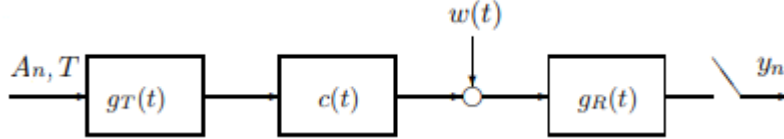


Figure 4.2: Sampled signal at the receiver.

What is sought with synchronization [5], is a “good” symbol-spaced output sequence that is correlated to the output packet. This issue will be described with great precision.

Let us assume that the analog composite channel is

$$h(t) = g_T(t) * c(t) * g_R(t),$$

where $c(t) = \sum_i c_i g(t - t_i)$ is the impulse response of the physical channel ($t_i > 0$ and $c_i \in \mathbb{C}$). If u_n , for $n = 0, \dots, N - 1$, is the sequence of the input symbols and T the symbol period, then, the noisy output of the composite impulse response (receiver’s input) is

$$y(t) = \sum_{n=0}^{N-1} u_n h(t - nT) + w(t).$$

To achieve synchronization, it is sufficient to calculate a delay d in $y(t)$, and then sample the output y at the time instants $t_k = kT + d, k = 0, \dots, N + M - 2$. More specifically,

$$\begin{aligned} y_k^d &:= y(t)|_{t=kT+d} = \sum_{n=0}^{N-1} u_n h(kT + d - nT) + w_k \\ &= \sum_{n=0}^{N-1} u_n h((k - n)T + d) + w_k \\ &= \sum_{n=0}^{N-1} u_n h_{k-n}^d + w_k \\ &= \sum_{m=0}^{M-1} u_{k-m} h_m^d + w_k. \end{aligned}$$

The quantity $h_m^d = h(mT + d)$, for $m = 0, \dots, M - 1$, is called discrete equivalent channel and its length M is known to the receiver [6]. Thus, the estimation for the output packet will be $\{y_k^d\}_{k=0}^{N+M-2}$.

Subsequently, a method for calculating the delay d when there is no noise and CFO, will be presented. This technique is called cross correlation.

Assume that the receiver obtains samples with a sampling period $T_s = \frac{T}{\text{over}}$, and that u_k , for $k = 0, \dots, N_{tr} - 1$ is the sequence with training symbols which are known to the receiver. For $d = 0, \dots, d_{max}$, the quantity $corr_d$ is calculated as follows

$$\begin{aligned}
 corr_d &:= corr(dT_s) = \sum_{k=0}^{N_{tr}-1} u_k^* y(dT_s + kT) \\
 &= \sum_{k=0}^{N_{tr}-1} u_k^* \sum_{n=0}^{N-1} u_n h(dT_s + kT - nT) \\
 &= \sum_{k=0}^{N_{tr}-1} u_k^* \sum_{n=0}^{N-1} u_n h(dT_s + (k - n)T) \\
 &= \sum_{k=0}^{N_{tr}-1} u_k^* \sum_{n=0}^{N-1} u_n h_{k-n}^d \\
 &= \sum_{k=0}^{N_{tr}-1} \sum_{n=0}^{N-1} u_k^* u_n h_{k-n}^d,
 \end{aligned}$$

where $h_k^d := h(dT_s + kT)$. If the u_k are independent of each other with a mean of 0 and a variance of σ_u^2 , then

$$\begin{aligned}
 \mathcal{E}[corr_d] &= \mathcal{E} \left[\sum_{k=0}^{N_{tr}-1} \sum_{n=0}^{N-1} u_k^* u_n h_{k-n}^d \right] \\
 &= \sum_{k=0}^{N_{tr}-1} \sum_{n=0}^{N-1} \mathcal{E}[u_k^* u_n] h_{k-n}^d \\
 &\approx \sum_{k=0}^{N_{tr}-1} \sigma_u^2 h_0^d \\
 &= N_{tr} \sigma_u^2 h_0^d \\
 &= N_{tr} \sigma_u^2 h(dT_s).
 \end{aligned}$$

Thus, an estimation for the sampled channel, with a sampling period T_s , is obtained as follows

$$corr_d \approx N_{tr} \sigma_A^2 h(dT_s) \implies h(dT_s) \approx \frac{corr_d}{N_{tr} \sigma_A^2}.$$

An estimation for the discrete equivalent channel of length M is obtained by calculating the energy of subsequences, of length M , of the sequence $\{corr_d\}_{d=0}^{d_{max}}$. The first element of the d -th subsequence is $corr_d$, and the consecutive elements of this subsequence are spaced by over positions. That is

$$E_d = \sum_{m=0}^{M-1} |corr_{d+m \cdot \text{over}}|^2,$$

and calculating

$$d^* = \operatorname{argmax}_d E_d.^1$$

The corresponding discrete equivalent channel is given by the sequence

$$\{h_{d^*}, h_{d^*+over}, \dots, h_{d^*+(M-1)over}\}$$

and the “good” symbol-spaced sequence of the output packet is

$$\{y_{d^*}, y_{d^*+over}, \dots, y_{d^*+(N+M-2)over}\}.$$

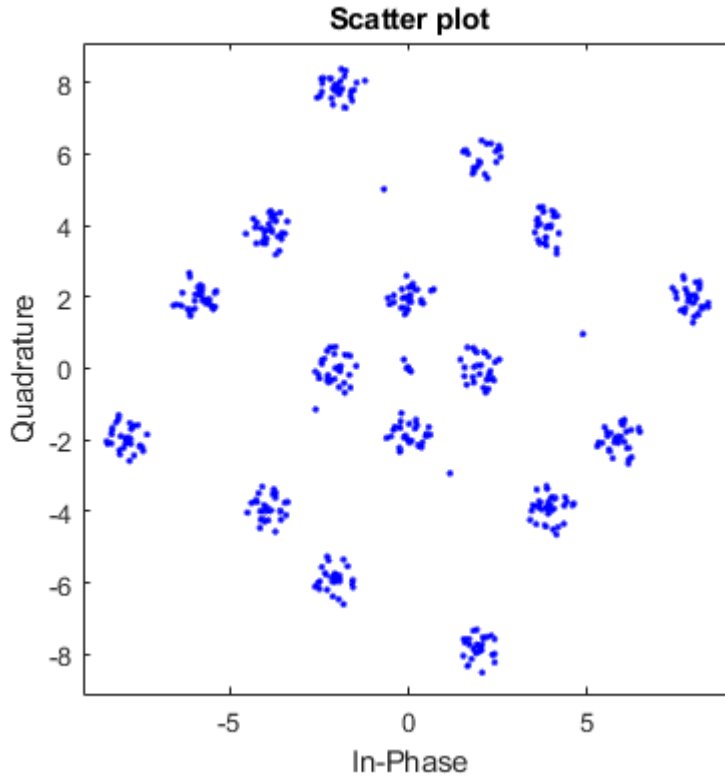


Figure 4.3: Symbols after synchronization without CFO.

If there is a small CFO in the system, with a value ΔF , then the output is given by the relation

$$y(t) = e^{j(2\pi\Delta Ft + \varphi)} \sum_{n=0}^{N-1} A_n h(t - nT).$$

¹The symbolism d^* denotes the position where the start of the packet was calculated.

Repeating the same process

$$\begin{aligned}
corr_d := corr(dT_s) &= \sum_{k=0}^{N_{tr}-1} u_k^* y(dT_s + kT) \\
&= \sum_{k=0}^{N_{tr}-1} u_k^* e^{j(2\pi\Delta F(dT_s+kT)+\varphi)} \sum_{n=0}^{N-1} u_n h(dT_s + kT - nT) \\
&= e^{j(2\pi\Delta FdT_s+\varphi)} \sum_{k=0}^{N_{tr}-1} u_k^* e^{j2\pi\Delta FkT} \sum_{n=0}^{N-1} u_n h(dT_s + (k-n)T) \\
&= e^{j(2\pi\Delta FdT_s+\varphi)} \sum_{k=0}^{N_{tr}-1} u_k^* e^{j2\pi\Delta FkT} \sum_{n=0}^{N-1} u_n h_{k-n}^d \\
&= e^{j(2\pi\Delta FdT_s+\varphi)} \sum_{k=0}^{N_{tr}-1} e^{j2\pi\Delta FkT} \sum_{n=0}^{N-1} u_k^* u_n h_{k-n}^d,
\end{aligned}$$

and if the u_k are independent of each other with a mean of 0 and a variance of σ_u^2 , then

$$\begin{aligned}
\mathcal{E}[corr_d] &= \mathcal{E} \left[e^{j(2\pi\Delta FdT_s+\varphi)} \sum_{k=0}^{N_{tr}-1} e^{j2\pi\Delta FkT} \sum_{n=0}^{N-1} u_k^* u_n h_{k-n}^d \right] \\
&= e^{j(2\pi\Delta FdT_s+\varphi)} \sum_{k=0}^{N_{tr}-1} e^{j2\pi\Delta FkT} \sum_{n=0}^{N-1} \mathcal{E}[u_k^* u_n] h_{k-n}^d \\
&\approx e^{j(2\pi\Delta FdT_s+\varphi)} \sum_{k=0}^{N_{tr}-1} e^{j2\pi\Delta FkT} \sigma_u^2 h_0^d \\
&= x^d h_0^d \\
&= x^d h(dT_s),
\end{aligned}$$

where $x^d := e^{j(2\pi\Delta FdT_s+\varphi)} \sum_{k=0}^{N_{tr}-1} e^{j2\pi\Delta FkT} \sigma_u^2$. Observing that $|x^d| = x$, for all d , an estimate of the sampled channel, with a sampling period T_s , is obtained as $|h(dT_s)| \approx \frac{|corr_d|}{x}$.

The Figure 4.4, there is a subplot in which the absolute value of the analog composite channel is shown at the top, while the bottom part displays the absolute value of the quantity $corr_d$. The two diagrams look quite similar, which indicates that the cross correlation technique has the ability to find the synchronization point, d^* , of the packet.

Similar to the case where there is no CFO, the energy of the d subsequences of length M is calculated as

$$E_d = \sum_{m=0}^{M-1} |corr_{d+m \cdot over}|^2,$$

and

$$d^* = \underset{d}{\operatorname{argmax}} E_d.$$

The “good” symbol-spaced sequence of the output packet is

$$\{y_{d^*}, y_{d^*+over}, \dots, y_{d^*+(N+M-2)over}\}.$$

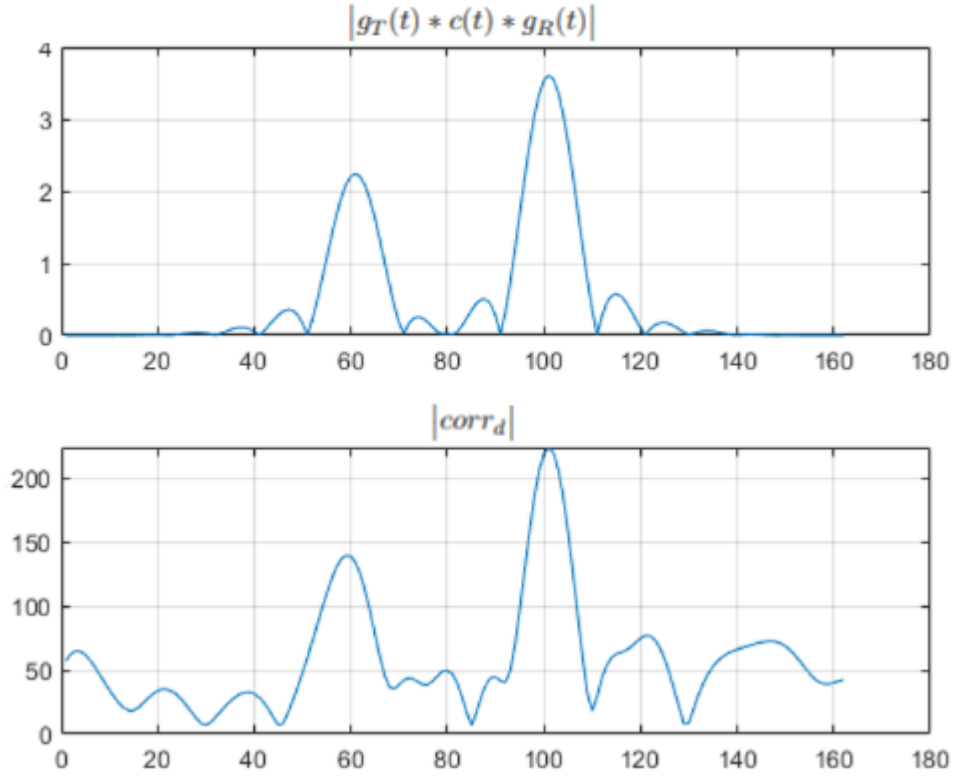


Figure 4.4: Absolute value of analog composite channel and $corr_d$.

Due to the presence of the CFO, the estimation of the discrete equivalent channel cannot be performed. This can only be done once the impact of the CFO is estimated and subsequently compensated for, as will be demonstrated later in this thesis.

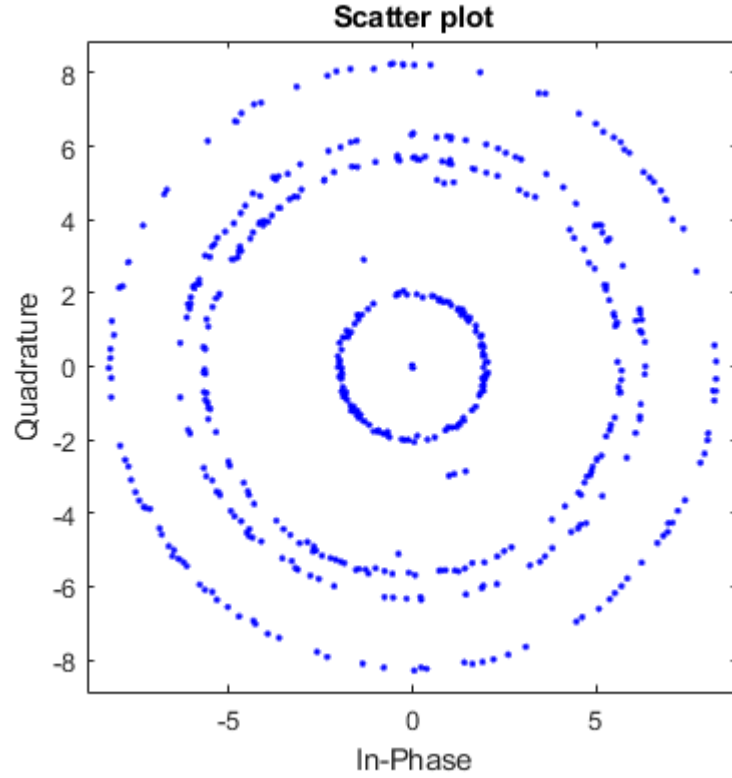


Figure 4.5: Symbols after synchronization when CFO exists.

4.3 Joint CFO and discrete equivalent channel estimation

Assuming that there is no CFO, the input-output relation of the channel is given by the convolution

$$y_k = \sum_{m=0}^{M-1} u_{k-m} h_m^o + w_k,$$

where $\{h_m^o\}_{m=0}^{M-1}$ is the discrete equivalent channel which is unknown to the receiver and must be estimated. The input packet is

$$\begin{bmatrix} u_{n_1} \cdots u_{n_2} & u_{n_2+1} \cdots u_N \end{bmatrix},$$

where $\{u_n\}_{n=n_1}^{n_2}$ the sequence with the training symbols. The output packet at the receiver after synchronization is

$$\begin{bmatrix} y_{n_1} \cdots y_{n_2} & y_{n_2+1} \cdots y_N \cdots y_{N+M} \end{bmatrix}.$$

The channel estimation will be performed using the least squares method [7]. The use of training samples is essential. The channel outputs y_n , which depend only on the training symbols $u_{n_1}, \dots, u_{n_2}$ can be expressed in vector form as follows

$$\begin{bmatrix} y_{n_1+M} \\ \vdots \\ y_{n_2} \end{bmatrix} = \begin{bmatrix} u_{n_1+M} & u_{n_1+M-1} & \cdots & u_{n_1} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n_2} & u_{n_2-1} & \cdots & u_{n_2-M} \end{bmatrix} \begin{bmatrix} h_0^o \\ \vdots \\ h_M^o \end{bmatrix} + \begin{bmatrix} w_{n_1+M} \\ \vdots \\ w_{n_2} \end{bmatrix}$$

or, with explicit definitions as

$$\mathbf{y} = \mathbf{A}\mathbf{h}^o + \mathbf{w},$$

where $\mathbf{y}$, $\mathbf{A}$ are known and $\mathbf{h}^o$, $\mathbf{w}$ are unknown to the receiver. In the search of $\mathbf{h}^o$, it is necessary to compute $\mathbf{h}_{LS}$, which minimizes the cost function

$$C(\mathbf{h}) = \|\mathbf{y} - \mathbf{A}\mathbf{h}\|_2^2 = (\mathbf{y} - \mathbf{A}\mathbf{h})^H (\mathbf{y} - \mathbf{A}\mathbf{h}),$$

where $\mathbf{h} = [h_0 \cdots h_M]^H$. This involves finding the quantity

$$\mathbf{h}_{LS} = \underset{\mathbf{h}}{\operatorname{argmin}} C(\mathbf{h}).$$

The function $C(\mathbf{h})$ is a quadratic function composed of the elements of $\mathbf{h}$, specifically, a linear combination of product terms of the form $h_i h_j$. It can be demonstrated that if the matrix $\mathbf{A}$ is full rank (which occurs when its columns are linearly independent), then the minimization problem that described, has a unique solution, $\mathbf{h}_{LS}$, provided by solving the equation

$$\frac{\partial C(\mathbf{h})}{\partial \mathbf{h}} = 0.$$

To solve this equation, the scalar function of the vector $\mathbf{h}$, $C(\mathbf{h})$, needs to be differentiated. This is accomplished using Taylor expansion. The final least squares solution is

$$\mathbf{h}_{LS} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{y}.$$

In the case where there is carrier frequency offset [5], the problem of jointly estimating the CFO and the discrete equivalent channel can be considered as a least squares problem with a cost function

$$F(\omega, \mathbf{h}) = \|\mathbf{y} - \Gamma(\omega)\mathbf{A}\mathbf{h}\|_2^2,$$

where $\omega := 2\pi\Delta F$ and $\Gamma(\omega) := \operatorname{diag}(1, e^{j\omega}, e^{j2\omega}, \dots, e^{j(K-1)\omega})$, with K equal to the length of the vector $\mathbf{y}$. Thus, for each value of ω , this is a least squares problem with the coefficient matrix

$$\tilde{\mathbf{A}} := \Gamma(\omega)\mathbf{A}.$$

So, the optimal solution for $\mathbf{h}$ is

$$\begin{aligned} \mathbf{h}_{LS}(\omega) &= (\tilde{\mathbf{A}}^H \tilde{\mathbf{A}})^{-1} \tilde{\mathbf{A}}^H \mathbf{y} \\ &= (\mathbf{A}^H \Gamma(\omega)^H \Gamma(\omega) \mathbf{A})^{-1} \mathbf{A}^H \Gamma(\omega)^H \mathbf{y} \\ &= (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \Gamma(\omega)^H \mathbf{y}. \end{aligned}$$

By substituting this value into the cost function, then

$$\begin{aligned}
 F(\omega, h(\omega)) &= \|y - \Gamma(\omega)Ah(\omega)\|_2^2 \\
 &= (y - \Gamma(\omega)Ah(\omega))^H (y - \Gamma(\omega)Ah(\omega)) \\
 &= (y - \Gamma(\omega)A(A^H A)^{-1}A^H \Gamma(\omega)^H y)^H (y - \Gamma(\omega)A(A^H A)^{-1}A^H \Gamma(\omega)^H y).
 \end{aligned}$$

Let define $P := A(A^H A)^{-1}A^H$, then

$$\begin{aligned}
 F(\omega, h(\omega)) &= (y - \Gamma(\omega)P\Gamma(\omega)^H y)^H (y - \Gamma(\omega)P\Gamma(\omega)^H y) \\
 &= y^H y - 2y^H \Gamma(\omega)P\Gamma(\omega)^H y + y^H \Gamma(\omega)P\Gamma(\omega)^H \Gamma(\omega)P\Gamma(\omega)^H y \\
 &= c - 2y^H \Gamma(\omega)P\Gamma(\omega)^H y + y^H \Gamma(\omega)P\Gamma(\omega)^H y \\
 &= c - y^H \Gamma(\omega)P\Gamma(\omega)^H y,
 \end{aligned}$$

where $c := y^H y$, $P^H = P$, $P^2 = P$ and $\Gamma(\omega)^H \Gamma(\omega) = I$. To solve the problem, a grid of equidistant values must be constructed in the interval $[-\pi, \pi)$ and the ω^* that maximizes the function $w(\omega) = y^H \Gamma(\omega)P\Gamma(\omega)^H y$, must be calculated. Subsequently, the optimal h is computed as

$$h_{LS}(\omega^*) = (A^H A)^{-1} A^H \Gamma(\omega^*)^H y$$

and the carrier frequency offset is

$$\Delta F(\omega^*) = \frac{\omega^*}{2\pi}.$$

Once the CFO is computed, its effect can subsequently be canceled by multiplying the output signal, which includes the CFO, by $e^{-j(2\pi\Delta F(\omega^*)k+\varphi)}$, for $k = 0, \dots, N + M - 2$.

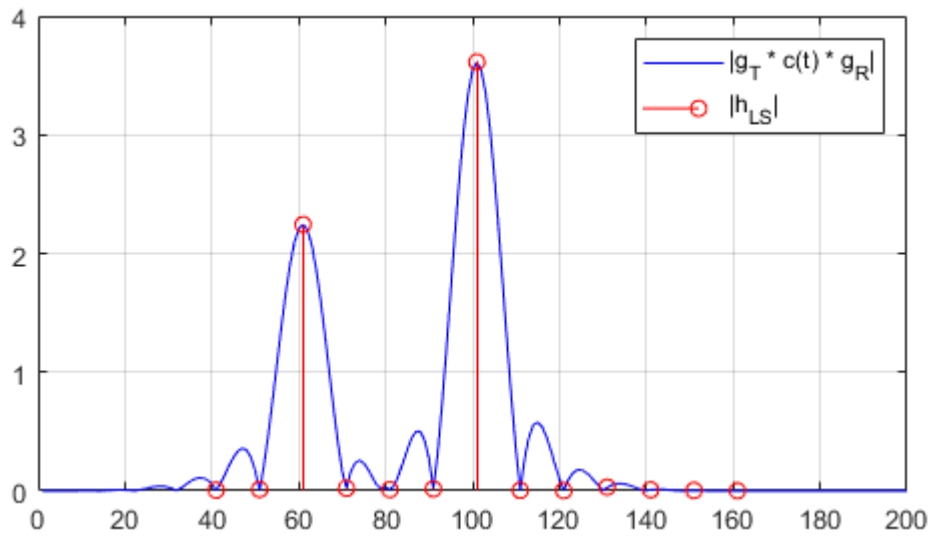


Figure 4.6: Analog composite channel and estimation of discrete equivalent channel.

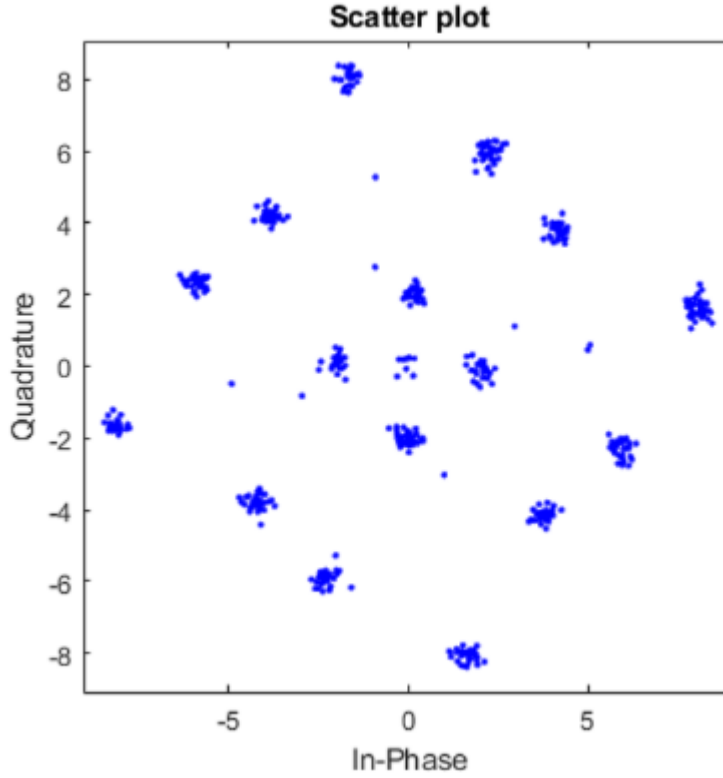


Figure 4.7: Symbols after synchronization and CFO correction.

4.4 Zero forcing equalizer

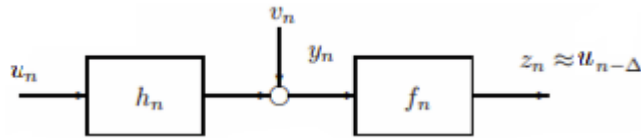


Figure 4.8: Linear equalization.

In order to complete the recovery of the input symbols, the effect of the channel must be canceled. This process is called equalization and its implementation requires the previously estimated channel. The equalizer that is implemented in this thesis is called zero forcing and its task is to cancel the effect of the channel so that the overall system is approximately equivalent to a delay [7]. More specifically

$$h_n * f_n = \delta_{n-\Delta},$$

where h the estimated channel. This convolution can be expressed in vector form as follows

$$\begin{bmatrix}
 h_0 & & & & \\
 h_1 & h_0 & & & \\
 \vdots & & \ddots & & \\
 h_M & h_{M-1} & \cdots & h_0 & \\
 & \ddots & & \ddots & \\
 & & h_M & \cdots & h_1 & h_0 \\
 & & & \ddots & \vdots & \\
 & & & & \ddots & \vdots \\
 & & & & & h_M
 \end{bmatrix}
 \begin{bmatrix}
 f_0 \\
 f_1 \\
 \vdots \\
 f_K
 \end{bmatrix}
 =
 \begin{bmatrix}
 g_0 \\
 g_1 \\
 \vdots \\
 g_M \\
 \vdots \\
 g_K \\
 \vdots \\
 \vdots \\
 g_{M+K}
 \end{bmatrix}$$

where $K > M$, or

$$Hf = g.$$

The vector f is selected to minimize the quantity $\|e_\Delta - g\|_2^2 = \|e_\Delta - Hf\|_2^2$, where e_Δ is a vector with 1 at position Δ and 0 elsewhere. The solution is given by

$$f_{LS}^{ZF} = (H^H H)^{-1} H^H e_\Delta.$$

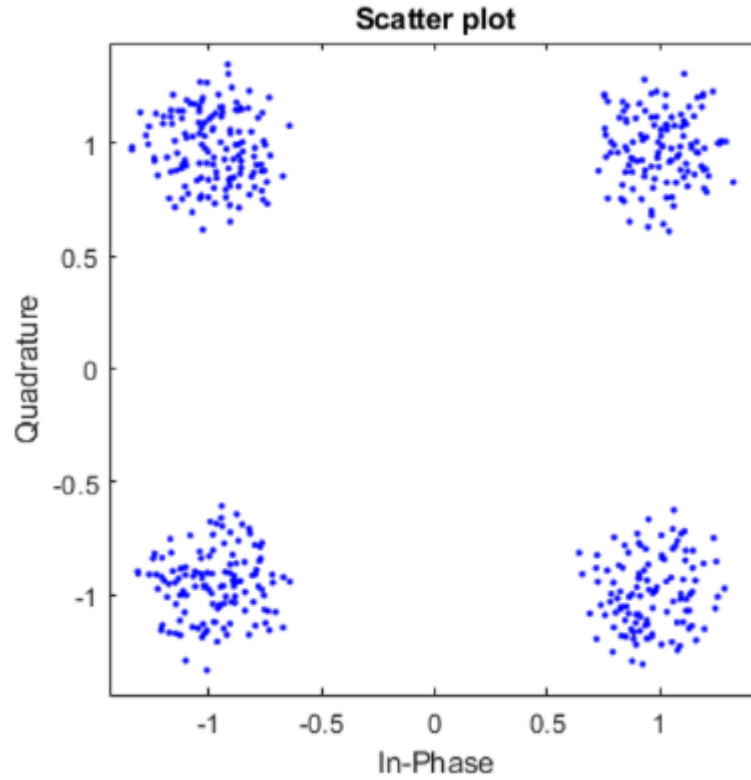


Figure 4.9: Symbols after equalization of channel.

The final step after channel equalization is the decision rule for each of the symbols. The decision rule for 4-QAM constellation is

- if $\text{real}(z[i]) > 0$ and $\text{imag}(z[i]) > 0$ then $\text{est_symbol}(i) = 1 + j$
- if $\text{real}(z[i]) > 0$ and $\text{imag}(z[i]) < 0$ then $\text{est_symbol}(i) = 1 - j$
- if $\text{real}(z[i]) < 0$ and $\text{imag}(z[i]) > 0$ then $\text{est_symbol}(i) = -1 + j$
- if $\text{real}(z[i]) < 0$ and $\text{imag}(z[i]) < 0$ then $\text{est_symbol}(i) = -1 - j$.

4.5 Experiments

Figure 4.10 shows the symbol error rate (SER) estimation of the implemented system with respect to SNR. For this particular estimation, the channel was kept constant throughout the experiment while the CFO and the additive white Gaussian noise varied randomly for each symbol packet. It is evident that as the SNR increases, the probability of symbol error decreases.

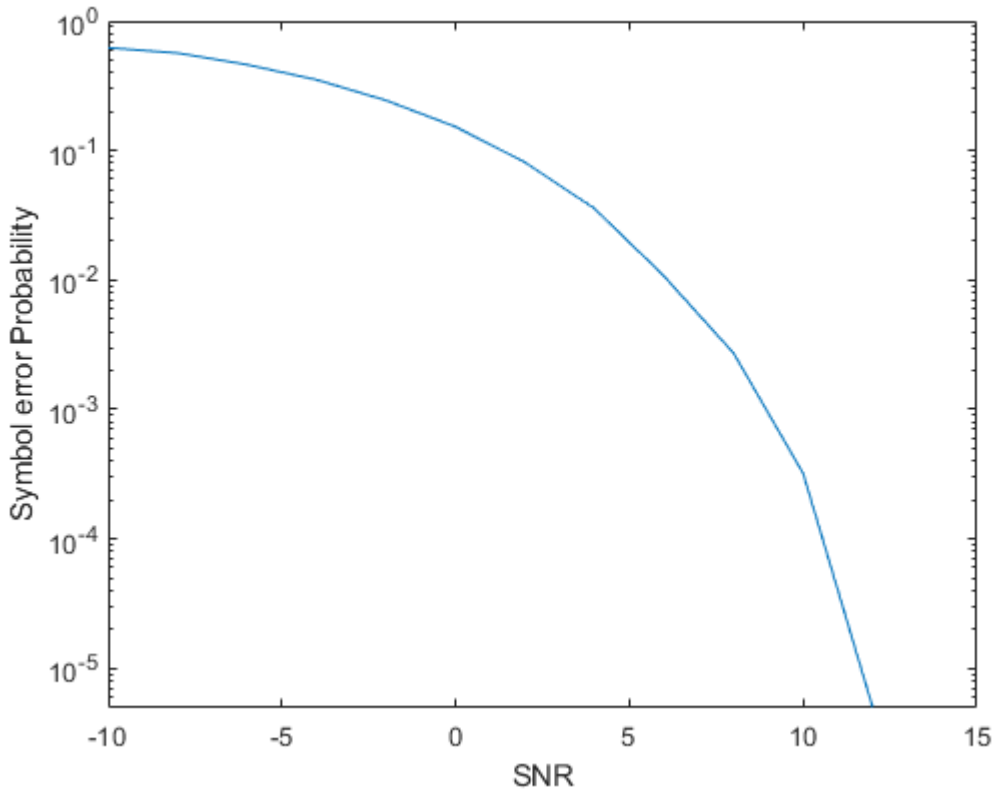


Figure 4.10: Symbol error rate estimation with respect to SNR.

Figure 4.11 shows the symbol error rate estimation of the implemented system with respect to various system delays during equalization. Just like the previous experiment, the channel was kept constant while the CFO and the noise varied randomly for each symbol packet. The experiment was performed for cases where the zero-forcing equalizer's length was 2 (yellow colour), 3 (red colour) and 4 (blue colour) times the length of the channel. In all three cases, it is evident that the probability of error increases when the selected delay is near the lower or upper end of the delay vector, e_{Δ} , which implemented in the equalizer.

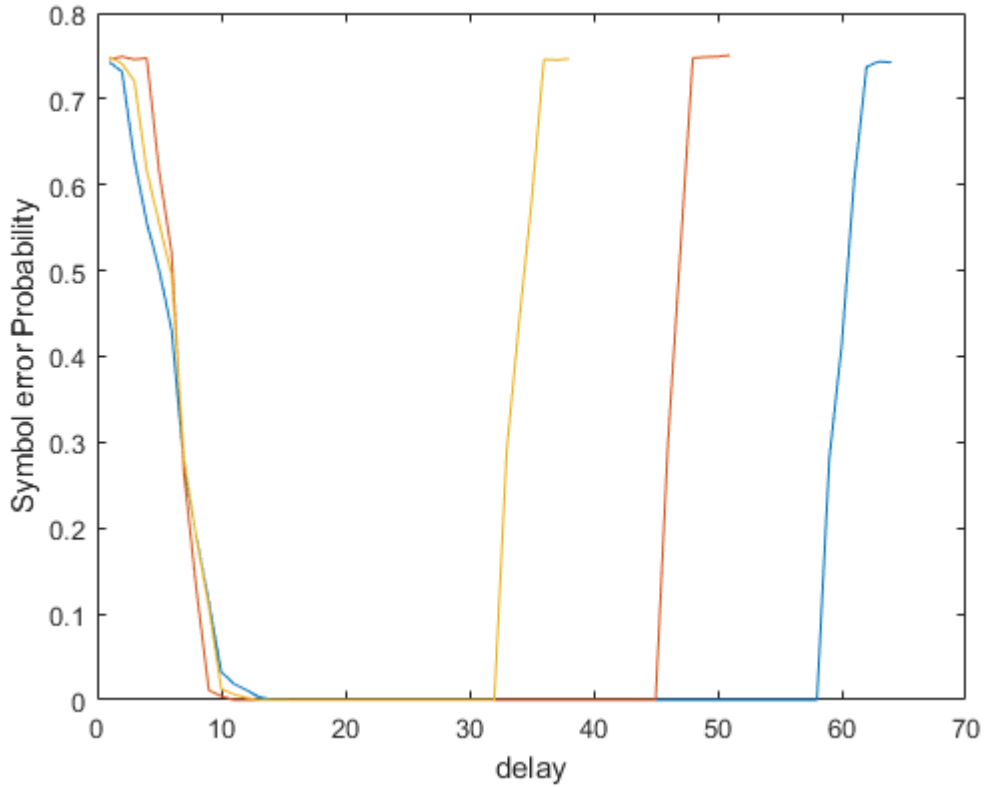


Figure 4.11: Symbol error rate estimation with respect to different delays.

Conclusion and future work

This thesis has explored synchronization techniques for frequency selective wireless telecommunications channels. The primary goal was to enhance the reliability and efficiency of data transmission from the transmitter to the receiver. In the beginning, the role of the transmitter was studied, detailing the creation of symbol packets and the implementation of transmitter's filter. Additionally, the low-pass equivalent channel was defined and implemented and the phenomenon of intersymbol interference was analyzed. Subsequently, the characteristics of the channel through which the signal is transmitted to the receiver were studied. Significant terms like multipath propagation, delay spread, flat and frequency selective fading were analyzed. The impact of the presence of additive white Gaussian noise in the channel was also emphasized. A key focus of the thesis was on the functionality of the receiver. Including filtering of the input signal, other key functions necessary for the operation of the system were implemented in the receiver. These were synchronization, the estimation and the correction of the CFO, the estimation of the discrete equivalent channel. For the elimination of the channel's impact a zero-forcing equalizer was implemented.

To enhance the relevance and modernity of the telecommunication system, it would be beneficial to incorporate and analyze more advanced modulation techniques like Orthogonal Frequency Division Multiplexing (OFDM) and Frequency Domain Equalization (FDE). Therefore, synchronization and channel estimation techniques in OFDM systems could be studied. Additionally, a comparison could be made between time-domain and frequency-domain equalization techniques under various channel conditions, including frequency selective channels.

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