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Empirical investigation of a nonlinear automated vehicle controller in lane-free traffic.

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by

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- Abstract -

Designing a vehicle movement strategy that is both safe and effective in lane-free traffic is a challenging topic. Several methodological approaches may be deployed, including multi-agent systems, nonlinear feedback control, and optimal control-based techniques. In this thesis, we will concentrate on the practical implementation and simulation of the nonlinear controller scheme developed recently in the TrafficFluid group. The proposed control scheme attempts to drive the system to the desired state which is consisted by some key elements. Meaning that the vehicles should reach their desired speed at some reasonable time and remain within a safe distance from their adjacent vehicles. While at the same time, the convenience of the driver is taken into account. The proposed vehicle movement strategy is employed for traffic simulation using a recently developed microscopic simulator called TrafficFluid-Sim, which builds on the SUMO simulation infrastructure.

Ο σχεδιασμός μιας ασφαλούς και αποτελεσματικής στρατηγικής κίνησης οχημάτων σε κυκλοφορία χωρίς λωρίδες είναι ένα σύνθετο πρόβλημα. Υπάρχουν πολλές προσεγγίσεις προς αυτό το πρόβλημα που περιλαμβάνουν βέλτιστο έλεγχο, μη γραμμικό έλεγχο με ανάδραση καθώς και συστήματα πολλαπλών πρακτόρων. Σε αυτή τη διπλωματική εργασία, θα επικεντρωθούμε στην υλοποίηση και προσομοίωση ενός μη γραμμικού ελεγκτή που αναπτύχθηκε πρόσφατα από την ομάδα του TrafficFluid. Το προτεινόμενο σύστημα ελέγχου επιχειρεί να οδηγήσει το σύστημα (των οχημάτων) σε μια επιθυμητή κατάσταση που έχει ορισμένα χαρακτηριστικά. Κατά πρώτον, τα οχήματα θα πρέπει να φτάσουν την επιθυμητή ταχύτητά τους σε ρεαλιστικό χρόνο, κατά δεύτερον, να παραμείνουν σε ασφαλή απόσταση από τα γειτονικά τους οχήματα και τρίτον, θα πρέπει να ληφθεί υπόψιν η άνεση του οδηγού (μείωση των διακυμάνσεων). Η προτεινόμενη στρατηγική προσομοιώνεται με έναν πρόσφατα αναπτυγμένο περιβάλλον μικροσκοπικής προσομοίωσης, το TrafficFluid-Sim, το οποίο βασίζεται στον ευρέως γνωστό προσομοιωτή SUMO.

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CONTENTS

Chapter 1: INTRODUCTION

Large cities throughout the world face a huge issue with traffic congestion on highways and arterials, which results in delays, increased fuel waste, negative environmental impact, and decreased on-road safety. If implemented appropriately, traffic control strategies using conventional actuators may postpone or even prevent the development of congestion. Additionally, innovative vehicle automation and communication technologies that are currently advancing, should be used to create cutting-edge solutions that can be implemented inside an intelligent infrastructure. [7]

Vehicle automation systems can be used to produce innovative solutions that can be integrated into a smart road infrastructure. They can range from various types of driver support systems to highly or entirely automated driving. (i.e., SAE levels 4 and 5 vehicles). In the meanwhile, vehicle communication enables vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication, which may support a variety of applications. A significant amount of work has been dedicated over the past ten years by vehicle manufacturers, some technology firms, and research organizations to develop and implement a variety of vehicle automation and communication systems that will revolutionize the abilities of vehicles. [7]

The TrafficFluid concept [5], which was recently introduced, is a new idea for vehicular traffic that is suitable for high degrees of vehicle automation and communication. The idea of TrafficFluid is based on two laws. At first, lane-free traffic, where, unlike in a conventional traffic, vehicles are not restricted to travel in predefined lanes. Secondly, vehicle nudging, in which vehicles can "nudge" other adjacent vehicles in front as well as around them. To maximize the coverage of the available infrastructure, vehicles in a lane-free environment are organized into dynamically changing two dimensional clusters, according to traffic parameters and the utilized movement strategies. [7]

Chapter 2: BACKGROUND

In this domain we will discuss about some essential elements that are needed to better understand the idea of this thesis.

2.1 Nonlinear Control

The primary goal of control theory and engineering is to drive a dynamic system in a desired state, to achieve this a *controller* is employed to correct the dynamic system's behavior. This controller monitors the controlled *process variable*, and compares it with the reference or *set point*. The difference between actual and desired value of the process variable, is called the *error signal*, and it is applied as feedback to generate a control action to bring the controlled process variable to the same value as the set point. A dynamic system is consisted by a set of differential equations which can be linear or *nonlinear*. A system of nonlinear equations is a nonlinear dynamic system and requires a nonlinear control approach to be controlled. Nonlinear control theory is applied in systems for which linear models are not adequate [1]. To deal with these systems there are several mathematical approaches developed, such as limit cycle theory, Poincaré maps, *Lyapunov stability theorem*, and describing functions.

2.2 Lyapunov Stability Theorem

In this thesis the controller under investigation is a product of the Lyapunov stability theorem (analyzed in [2]). In the Lyapunov function, that will be discussed in a following domain (2.6), the desired state that the dynamic system should be driven to, is defined. After applying certain mathematical techniques the result of the proof is the dynamics of a controller which is proven to do the task that was defined before. The goal of this thesis is to prove whether this controller will be able to respond outside the mathematical world and inside the real-world constraints.

The dynamic system under control is a system of vehicles. Each vehicle is described by its position and velocity (longitudinally and laterally) exactly as an elementary kinematic model. The model used for this dynamic system is described in the following domain (2.3).

2.3 The Double Integrator Model

Let us define an object moving in a single dimensional line with the ability to accelerate. The dynamics of this vehicle movement are described by:

$$\dot{x} = v \tag{2.3.1}$$
$$\dot{v} = F \tag{2.3.2}$$

Where x the position v the velocity and F the acceleration of the object. These are the dynamics of a double integrator model depicted in state-space representation. Generalize that, in a way that the object is able to move in a two dimensional area. This will result in the following dynamics:

$$\dot{x} = v \tag{2.3.3}$$
$$\dot{v} = F \tag{2.3.4}$$
$$\dot{y} = w \tag{2.3.5}$$
$$\dot{w} = u \tag{2.3.6}$$

Where y the lateral position w the lateral velocity and u the lateral acceleration of the object. Every vehicle inside the traffic will follow the dynamics described above (*double integrator model*) [3]. However in a simulation the time does not change continuously, but discreetly. So it will be needed to discretize this model in order to use it. If we consider that the dynamic system changes only every T units of time then the discreet time model will be:

$$x(t+T) = x(t) + v(t)T + \frac{1}{2}F(t) \cdot T^2 \tag{2.3.7}$$

$$v(t+T) = v(t) + F(t) \cdot T \tag{2.3.8}$$

$$y(t+T) = y(t) + w(t)T + \frac{1}{2}u(t) \cdot T^2 \tag{2.3.9}$$

$$w(t+T) = w(t) + u(t) \cdot T \tag{2.3.10}$$

This will be the *plant* that the investigated controller will attempt to control. In reality a moving object can also rotate (steer) in an angle of θ. In this model the θ is considered negligible which is an accurate approximation if we consider that in sufficiently high speeds in a straight road the angle of steering of a vehicle is very small. In other situations such as the roundabout the θ angle cannot be neglected so we will need a model that involves it. A model that involves vehicle rotation is called a *bicycle kinematic model* [4] and it will not be discussed in this thesis.

2.4 Traffic Fluid elements

In order to understand better how the investigated controller is produced we will need to discuss the basic elements of the *Traffic Fluid* project [5] . In general, in this project the behavior of fluids is imitated to handle the behavior of a vehicular traffic. Fluid molecules exert intermolecular forces when they exist in a sufficiently close distance. We imitate these forces in a two dimensional space between vehicles, so the vehicles obtain a safe distance from each other. This is the first element of the traffic fluid project. Those forces are called *nudging* and *repulsion* [4,5]. If one vehicle is considered as the (0,0) of our frame of reference this vehicle will be called the *ego* vehicle, all of the vehicles with a longitudinal position greater than the ego vehicle's are called the *downstream* vehicles, and all of the vehicles with a longitudinal position smaller than the ego vehicle's are called the *upstream* vehicles.

If a virtual force is exerted from the ego vehicle to a downstream vehicle this will be called a *nudging* force. If a virtual force is exerted from the ego vehicle to an upstream vehicle this will be called a *repulsion* force. The desired state of the system is one that the sum of all forces for every vehicle is zero. This might drive the vehicles in a position and velocity which does not look like an ordinary lane-based traffic. This is the reason that the second element of the project is the *lane free* [5] element. This element releases the traffic from the constraint that keeps it aligned in lanes. This will result in the vehicles having more space to move laterally and longitudinally. By using this element the vehicles are more likely to utilize the full capacity of the road and as a result reach their desired speed more easily.

2.5 Lane Free vs Lane Based Comparison

In lane based traffic [5], lanes of the same direction are divided by dashed lines. While lanes of the opposite direction are divided by parallel lines. Dashed lines are intended to warn the drivers to avoid parallel vehicles, while parallel lanes are intended to warn the drivers to avoid collisions with the opposite direction. This strategy makes it easier for the human mind to calculate the appropriate steering and acceleration for its vehicle. The lane based traffic, is much more simplified than a lane free traffic but the vehicles cannot be at their optimal position, every time, as the lines might not allow it. This restriction might create congestion to the traffic.

In contrast, the lane free traffic [5] does not involve any lines, which releases the vehicles from the obligation to drive in a fixed rectangle. This feature enables vehicles to drive at their optimal position, velocity and acceleration. The lane free traffic is mostly intended to involve automated vehicles. Automated vehicles are using sophisticates sensors and cameras to recognize their adjacent vehicles. Moreover, sophisticated algorithms are used to calculate their trajectories and accelerations. Movement strategies like this, can be deployed inside an infrastructure or a vehicle's software in order to calculate the above. In addition, lane free strategies are proven to increase the average speed of a traffic and as a result its flow. Due to more space for wider vehicles, the space distribution is improved in comparison with a lane based traffic. Last but not least, lane free traffic consisted by CAVs might result in far less accidents than the lane based traffic.

2.6 Nonlinear Controller under Investigation

Following the predefined principles and more particularly the Lyapunov stability theorem (LST) a new controller scheme occurs (the following mathematical proof of this controller scheme is done in [4,10,11]). Using the Lyapunov function there is a demand of 4 properties from the dynamic system in the equilibrium state (in order):

1) Every vehicle reaches its desired speed.
2) No vehicle moves laterally.
3) The forces applied from the boundaries are zero.
4) The virtual forces described in the introduction sum up to zero.

The Lyapunov function:

$$H(z) = \frac{1}{2}\sum_{i=1}^{n}(v_i - v^*)^2 \; + \sum_{i=1}^{n} w_i{}^2 + \sum_{i=1}^{n} U_i(y_i) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i} V_{i,j}\left(d_{i,j}\right)$$

Where:
i : the index of each vehicle
v_i : the longitudinal velocity
v_i^* : the desired longitudinal velocity
w_i : the lateral velocity
U_i : the force applied by the boundaries
$V_{i,j}$: the magnitude of force between every pair of vehicles i,j.

Let $V_{i,j}: (L_{i,j}, +\infty) \to R_+$ that satisfies the properties:

$$V_{i,j}: (L_{i,j}, +\infty) \to R_+ \tag{2.6.1}$$

$$\lim_{d_{i,j}\to L} V_{i,j}\left(d_{i,j}\right) = +\infty \tag{2.6.2}$$

$$V_{i,j}\left(d_{i,j}\right) = 0, \quad d_{i,j} \geq \lambda \tag{2.6.3}$$

Additionaly, let $U_i: (-a, a) \to R_+$ that satisfies the properties:

$$\lim_{y \to -a} U_i(y_i) = +\infty, \tag{2.6.4}$$
$$\lim_{y \to a} U_i(y_i) = +\infty \tag{2.6.5}$$
$$U_i(0) = 0 \tag{2.6.6}$$

The functions g_1, g_2 should have: $g_1' > 0,\ g_2' > 0, \forall x \in R$

The function $\kappa_{i,j}$ should satisfy the properties:

$$\kappa_{i,j}(d_{i,j}) = 0\ , \forall d_{i,j} > \lambda \tag{2.6.7}$$
$$\kappa_{i,j}(d_{i,j}) = \kappa_{j,i}(d_{j,i})\ , \forall d_{i,j} \leq \lambda \tag{2.6.8}$$

By applying the (LST), the following dynamics are obtained [4,10,11]:

$$F_i = -k_i(z)(v_i - v^*) + \Lambda_i(z) \tag{2.6.8}$$

$$u_i = -L_i(z)w_i - G_i(z) \tag{2.6.9}$$

$$\Lambda_i(z) = -\sum_{j \neq i} V'_{i,j}\left(d_{i,j}\right)\frac{(x_i - x_j)}{d_{i,j}} + \sum_{j \neq i} \kappa_{i,j}\left(d_{i,j}\right)(g_1\left(v_j\right) - g_1(v_i)) \tag{2.6.10}$$

$$\mathrm{G}_i(z) = U'_i(y_i) + \sum_{j \neq i} p_{i,j} V'_{i,j}\left(d_{i,j}\right)\frac{(y_i - y_j)}{d_{i,j}} - \sum_{j \neq i} \kappa_{i,j}\left(d_{i,j}\right)(g_2\left(w_j\right) - g_2(w_i)) \tag{2.6.11}$$

$$k_i(z) = \mu_1 + \frac{\Lambda_i}{v^*} + \frac{v_{max}}{v^*(v_{max} - v^*)} r(-\Lambda_i(z)) \tag{2.6.12}$$

$$L_i(z) = \mu_2 + \frac{c}{w_{max}} G_i^2(z) \tag{2.6.13}$$

The F and u terms are the longitudinal and lateral acceleration respectively, μ_1 μ_2 and c are designing parameters.

2.6.1 V-potential function

It is the primary repulsion/nudging driver, generating forces with respect to the distance between vehicles [4,10,11].

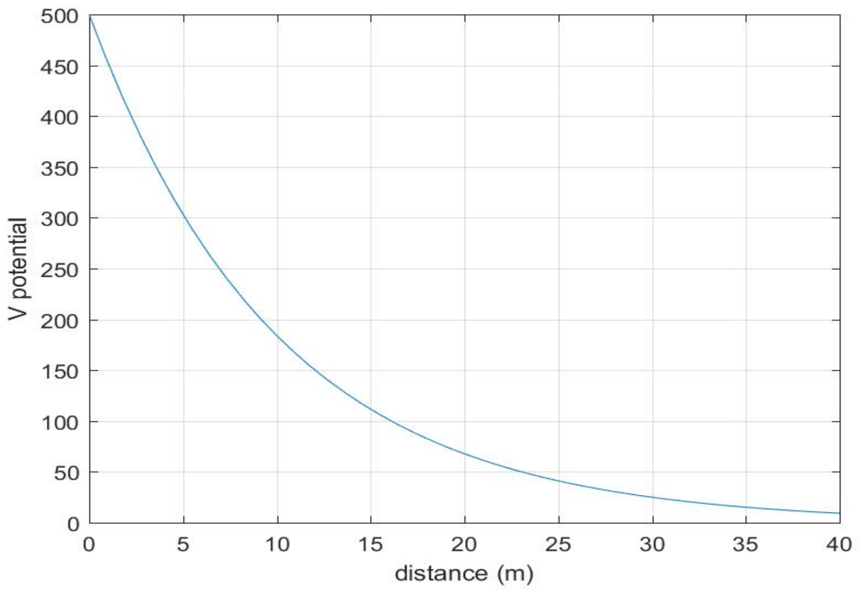

Figure 2.6.1: V-Potential function

2.6.2 U-potential function

It generates virtual force when a vehicle approximates the boundaries. The implementation of the U potential [4,10,11] is unrealistic because it generates infinite amounts of force for a sufficiently close distance to the boundaries. As a result it is eliminated (U = 0) from the dynamics and it is replaced by a boundary controller [7] (BC) which will be discussed in a following domain (3.3).

2.6.3 κ-function

The *κ(d)* function imitates the viscosity property of a fluid, in the vehicular stream.

(2.6.3.1)

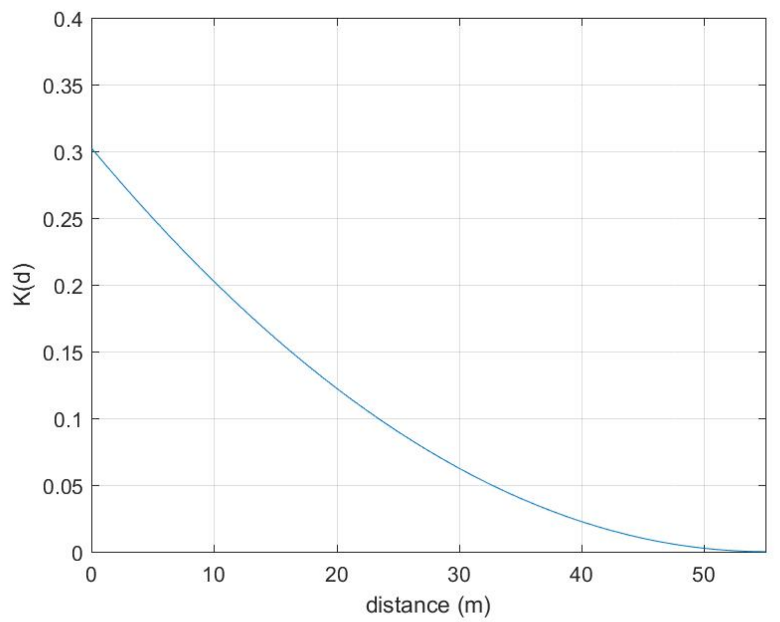

Figure 2.6.3.1: Viscosity Function

Chapter 3: MOVEMENT STRATEGY

3.1 Modifications

It is generally desired not to modify the model far from its initial form so we attempt to only apply changes necessary to conform with reality. There are also modifications that are allowed to be done by the proof. The LST guarantees that the dynamic system will at some point reach the equilibrium state.

However in reality there are several real-world constraints that make this harder. For example the V and U potential function which was discussed earlier, are the primary nudging/repulsion (forces) generators. The V potential is meant to generate infinite force when two vehicles approximate very close to each other and the U potential is meant to generate infinite force when a vehicle approximates very close to the boundaries.

However real world vehicles can only generate finite amounts of acceleration. This is the reason we replace these functions with more realistic ones.

The original V potential [4,10,11]:

$$V_{i,j}(d_{i,j}) = \begin{cases} \dfrac{q_1(\lambda - d_{i,j})^3}{d_{i,j} - L_{i,j}}, & L_{i,j} < d_{i,j} \leq \lambda \\ 0 \;, & d_{i,j} > \lambda \end{cases} \tag{3.1.1}$$

The replacement of V potential:

$$V_{i,j}(d_{i,j}) = \begin{cases} \frac{A}{B} e^{-Bd_{i,j}}, & d_{i,j} \leq \lambda \\ 0 \quad , & d_{i,j} > \lambda \end{cases} \tag{3.1.2}$$

The U term is eliminated (U=0), instead a closed loop feedback controller [7] (BC) is used to prevent from the collisions with the boundaries.

3.2 Aura

The aura is a two dimensional place used in movement strategies to define where the vehicles will interact with each other. When an adjacent vehicle enters the aura of the ego vehicle both start to apply forces to each other. In this movement strategy the aura is an ellipse around the center of the ego vehicle. The controller proof demands the distance between vehicles is calculated by the following metric [4,10,11]:

$$d = \sqrt{\Delta x^2 + p \cdot \Delta y^2}, \quad p \geq 1 \tag{3.2.1}$$

Where Δx, Δy are the longitudinal and lateral distances, respectively, between two vehicles i, j. Let us now define a constant $\lambda \in [0{,}1000]$. If $d \leq \lambda$ then the adjacent vehicle j is inside the aura of the ego vehicle i. In our case $p = 4$ and $\lambda = 55$ is used for the calculation.

3.3 Prevention of boundary collisions

As discussed earlier instead of using the U potential function, it is preferred to replace it with a secondary boundary controller (BC) [7]. This change gives the movement strategy two favorable properties. At first it guarantees that the vehicles will not exceed the boundaries of the road, at second it forces the vehicles to utilize the full lateral dimension of the road. The boundary controller is developed/discussed here: [7].

The BC gets as an input the lateral acceleration outputted from the primary NCC.

Let $\hat{y}$ and $\hat{v}$ to be defined as the reference values of the lateral position and velocity respectively. Then the error functions will be:

$$e_1(t) = y(t) - \hat{y} \tag{3.3.1}$$

$$e_2(t) = v(t) - \hat{v} \tag{3.3.2}$$

Consequently the state-feedback boundary controller for the lateral acceleration of the vehicle will be:

$$u(t) = -k_1 e_1(t) - k_2 e_2(t) \tag{3.3.3}$$

Where k_1 and k_2 are feedback gains. From the previous equations we get the following state equation for the regulation error:

$$\begin{bmatrix} e_1(t+T) \\ e_2(t+T) \end{bmatrix} = \begin{bmatrix} 1 - \frac{k_1 T^2}{2} & T - \frac{k_2 T^2}{2} \\ -k_1 T & 1 - k_2 T \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \tag{3.3.4}$$

The characteristic equation for is:

$$Z^2 + \left(Tk_2 - 2 + \frac{k_1 T^2}{2}\right) Z + (1 - k_2 T + \frac{k_1 T^2}{2}) = 0 \tag{3.3.5}$$

For stability and asymptotic response [7], the gains are defined such that the characteristic equation has two equal real non-negative stable roots. So, the following are the resulting necessary conditions for the gains:

$$\left(Tk_2 - 2 + \frac{k_1T^2}{2}\right) - 4(1 - k_2T + \frac{k_1T^2}{2}) = 0 \quad (3.3.6)$$

$$0 < 2 - Tk_2 - \frac{k_1T^2}{2} \quad (3.3.7)$$

By simplification we get:

$$k_2 = 2\sqrt{k_1} - \frac{k_1T}{2} \quad (3.3.7)$$

$$0 < k_1 < \frac{1}{T^2} \quad (3.3.8)$$

$$0 < k_2 < \frac{3}{2T} \quad (3.3.9)$$

The output of the boundary controller is the following:

$$u = \begin{cases} -k_1\,(y - \frac{w_{id}}{2} - y_{dn}) - k_2\,w, & if\ u < -k_1\,(y - \frac{w_{id}}{2} - y_{dn}) - k_2\,w \\ -k_1\,\left(y - \frac{w_{id}}{2} - y_{up}\right) - k_2\,w, & if\ u > -k_1\,(y - \frac{w_{id}}{2} - y_{up}) - k_2\,w \end{cases}$$

(3.3.10)

where k_1,k_2 are feedback gains, y_{dn},y_{up} are the positions of lower and upper boundaries respectively (working as reference values) and w_{id} is the width of the ego vehicle.

3.4 Parametric Design

In order to reach the desirable trajectory results, we have to finely tune the following parameters: μ_1, μ_2, k_1, k_2, c, λ, ρ. The μ_i, c parameters are constants of the feedback gains of the nonlinear controller, the k gains refer to the boundary controller, λ,ρ are the constants defining the ego vehicle's aura.

3.4.1 Nonlinear Controller Parameters

All parameters of the nonlinear controller were tuned through trial and error. It is still not clear what is their effect on the traffic. The main idea is tuning the parameters in a way that the resulting acceleration is within a reasonable range. The μ_i parameters should have an order of magnitude of 10^{-2} to 10^{-3} :

$$\mu_1 = 1.4 \cdot 10^{-2}$$
$$\mu_2 = 10^{-3}$$

The c parameter is by the proof (greater than or equal to) the inverse of μ_2:

$$c = 1000$$

3.4.2 Boundary Controller Parameters

The boundary controller parameters k_1, k_2 can be wherever inside the predefined range (3.3.8, 3.3.9) , although for a sufficiently big value of them, the boundary controller interferes with the nonlinear controller, in a way that forces the vehicles to form lines very close to the boundaries. To mitigate this problem we decrease the k_1, k_2 gains but they still remain in the predefined range. So:

$$k_1 = 10^{-3}$$
$$k_2 = 6.31 \cdot 10^{-2}$$

where parameter k_2 is produced by (3.3.7).

3.5 Local Density

The density of a road ρ equals the amount of vehicles per unit of distance (kilometers). If a certain region congests with a sufficiently high density the probability of an accident is higher. This problem is mitigated by implementing the local density function.

Let us define a variable $\omega \in [0{,}450]$, ω is the amount of vehicles existing in a predefined area at a specific moment t_1. This predefined area, is a rectangle around the center of the ego vehicle. Specifically ω equals:

$$\omega = \begin{cases} \rho(x-250, x+250), \;\; x \in [250{,}750] \\ \rho(x-250{,}1000) + \rho(0, x-750), \;\; x \geq 750 \\ \rho(0, x+250) + \rho(x+750{,}1000), \;\; x \leq 250 \end{cases} \tag{3.3.1}$$

Where $\rho(x_1, x_2)$ is the density in a segment of the road that starts at x_1 and ends at x_2, and x the position of the ego vehicle. Then a function $f(\omega) \in [0{,}1]$ is selected. The product $f(\omega) \cdot v^* = v_2^*$, where v_2^* is the desired speed of an ego vehicle in a road of density over 250. The selected *f(ω)* is:

$$f(\omega) = 1.83 - 0.004\omega, \qquad \omega \in [250{,}450] \tag{3.3.2}$$

If the density of the road is smaller than 250 vehicles the local density does not affect the desired speed:

$$f(\omega) = 1, \qquad \omega \in [0\,, 250] \tag{3.3.3}$$

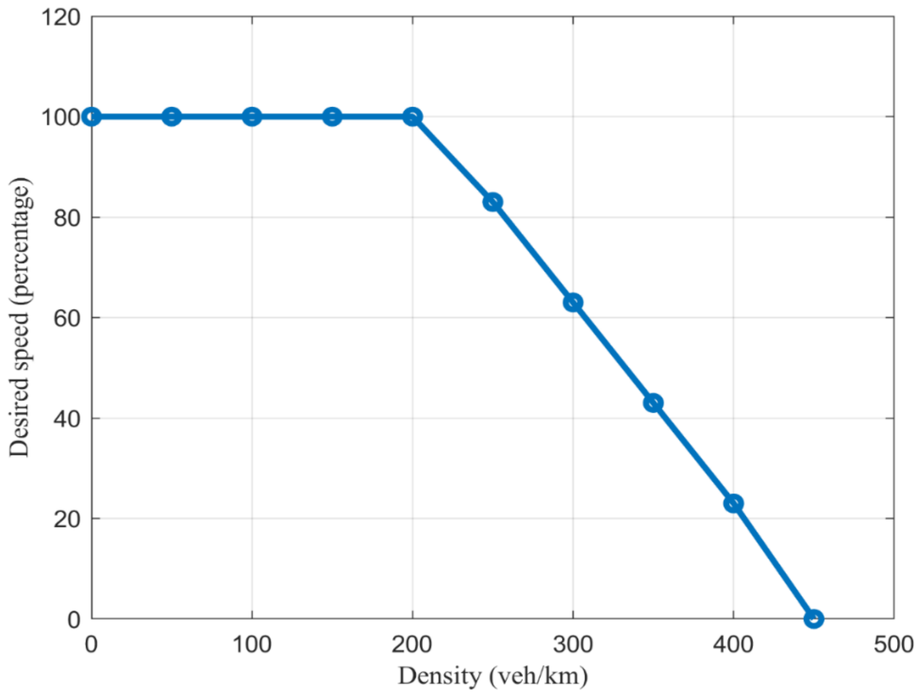

Figure 3.5.1:Local Density Function

3.6 Simulation Environment and Technical Specifications

The proposed vehicle movement strategy is employed for traffic simulation using a recently developed microscopic simulator called TrafficFluid-Sim [8] (developed here: [8]), which builds on the SUMO simulation infrastructure. There are 7 functions required to set up the simulation. Three of them signal the initialization, step, and finalization of the simulation and the other four signal events inside the simulation such as collision with boundaries and vehicles, exits and entrances of vehicles. More specifically:

simulation_initialize: Executed once before the first time-step, intended for initialization of variables, memory, files, etc.

simulation_step: Executed once in every time-step. Within this function, users can control the vehicles, and monitor the traffic environment.

simulation_ finalize: Executed once before exiting the simulation, intended for deallocation of memory, saving log files, etc.

event_vehicle_enter: Executed when a vehicle enters, intended to announce the entrance of a vehicle and/or set the circular movement of the vehicle.

event_vehicle_exit: Executed when a vehicle exits, intended to announce the exit of a vehicle.

event_vehicle_collide: Executed when a pair of vehicles collide, intended to announce the collision to the user.

event_vehicles_out_of_bounds: Executed when a vehicle gets out of bounds, intended to announce any interaction with the boundary.

Each time step updates the position, velocity and acceleration of all the vehicles in the road. The duration of the time step is *T=0.1s*. The equivalence of the simulation time to the actual time changes as the density increases. In each time step numerical values of the position, velocity and acceleration are saved in external files that are processed to extract essential graphs of the above variables with respect to time *(trajectories)*. The processing of the trajectories (4.1) is done via MATLAB 2016a. Additionally, virtual devices are placed inside the road to measure the amount of vehicles passing per unit of time (vehicles / hour). These devices are called *detectors* and their measurement is called the *flow*. The output of the detector is also saved in an external file. In the equilibrium state the flow is considered constant, so it is a common practice in traffic science to compare the flow of a traffic with respect to its density. By plotting a graph of the flow with respect to the density we obtain the *fundamental diagram*, which is the most important graph in a movement strategy investigation.

There are several road types used to investigate movement strategies. In this investigation the *ring road* is used. The primary property of the ring road is that every vehicle exiting is immediately re-entering, also no other vehicles enter or exit from any ramps, so as a consequence the density remains constant. The dimensions of the ring road are 10.2 meters of width and 1000 meters of length.

Each vehicle is manually inserted in the ring road from the simulation_initialize function. Its position can be selected exactly or randomly, but to conform with reality it selected randomly. We will discuss about that in the initial condition domain (3.7). There are six size classes of vehicles randomly selected (uniform distribution) by the simulator in the initialization. The following table [8] reveals their width and length.

Dim/Class	1	2	3	4	5	6
Length(m)	3.2	3.9	4.25	4.55	4.6	5.15
Width(m)	1.6	1.7	1.8	1.82	1.77	1.84

Each vehicle is initialized with a *desired speed* within the range of $[25{,}35]\ m/s$. The controller attempts to stabilize the vehicle in its desired speed. The desired speed can be unintentionally exceeded but there is a maximum speed limit that the simulator does not allow vehicles to exceed. This is the speed limit of 40 m/s. The acceleration is bounded between $[-5{,}5]\ m/s^2$. The desired lateral speed is zero, because it is desired for the vehicle not to move laterally, and the lateral speed bounds are $[-1{,}1]\ m/s$. The lateral acceleration bounds exist inside $[-2{,}2]\ m/s^2$.

3.7 Initial Conditions

Let us now define the set $\{x_i(0), y_i(0)\}$ of points that the vehicles are spawned at $t = 0$. Let x_i and y_i be the longitudinal and lateral position of i vehicle, l be the length of the road and n be the amount of vehicles in the road. $Y \sim \mathrm{U}(0,1)$ be a random variable following the uniform distribution.

The initial position will be:

$$x_i(0) = n_x \cdot \frac{3l}{n}, \qquad n_x \in N, \qquad 0 < x_i(0) < l \tag{3.5.1}$$

$$y_i(0) = n_y \cdot \frac{5}{2} + Y, \qquad n_y \in N, \qquad 0 < y_i(0) < y_{up} \tag{3.5.2}$$

The position of each vehicle has some level of randomness, this is done to ensure that the system is insensitive enough to respond in different initial conditions. It is worth noting that the system is deterministic, which means there is no disturbance and no unpredictable events are allowed to occur. The position and velocity of every vehicle can be predicted for every $t > 0$.

Chapter 4: RESULTS

The empirical investigations have shown that the described control scheme is safe (crash free), which means no collision between vehicles was observed. No vehicles were observed to trespass or contact the boundary.

The described control scheme is implemented and simulated inside the TrafficFluid-sim (in SUMO) and the produced trajectories and fundamental diagram is plotted using MATLAB 2016a.

4.1 Trajectories

In the following subchapter the trajectories of a chosen ego vehicle are depicted. The variables of position, velocity and acceleration are depicted with the longitudinal variables being in the left and the lateral being in the right. The ego vehicle depicted was randomly chosen and any other vehicle is expected to present a similar behavior.

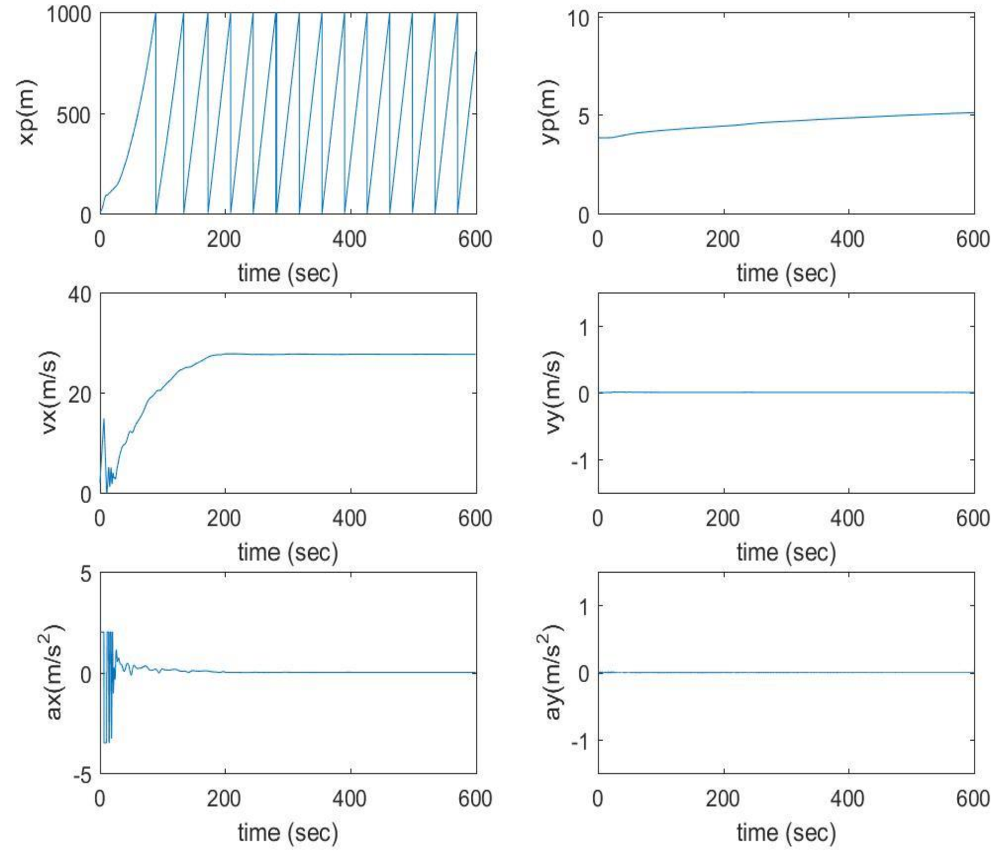

Figure: 4.1.1 Trajectories at density 50 veh/km

The trajectories of a vehicle at density 50 vehicles per kilometer and an average flow of 4970 vehicles per hour.

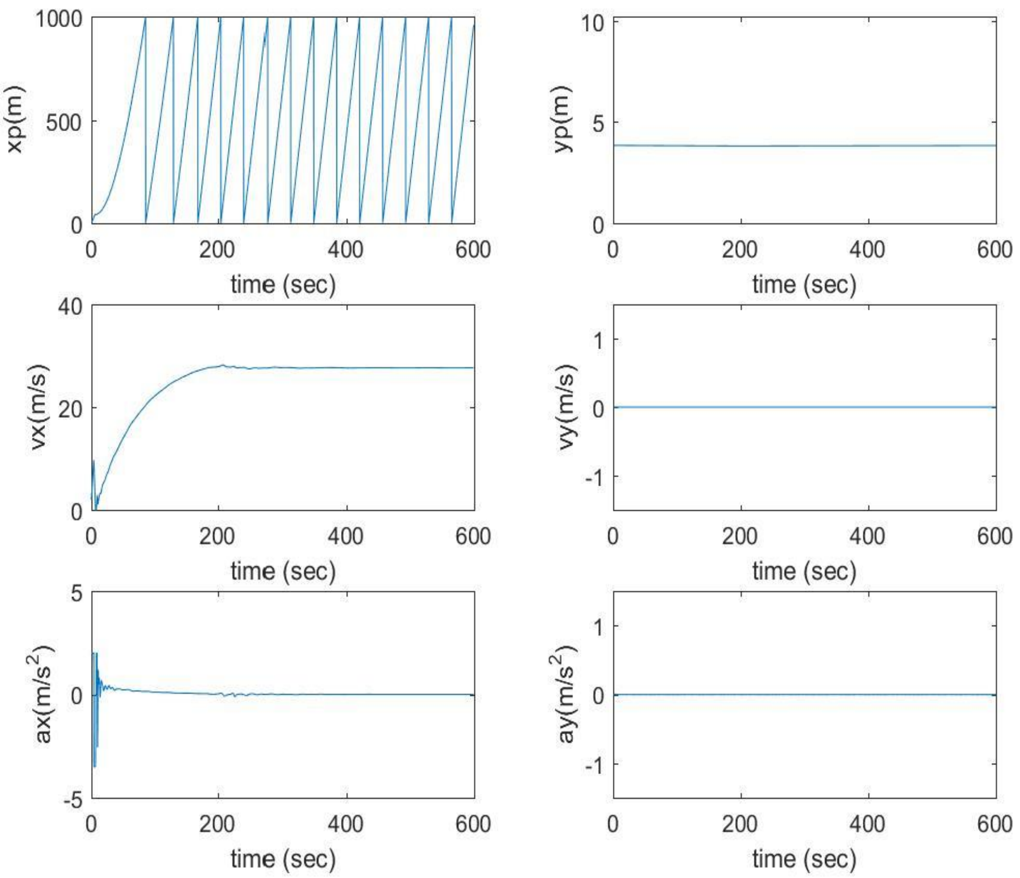

Figure: 4.1.2 Trajectories at density 100 veh/km

The trajectories of a vehicle at density 100 vehicles per kilometer and an average flow of 9963 vehicles per hour.

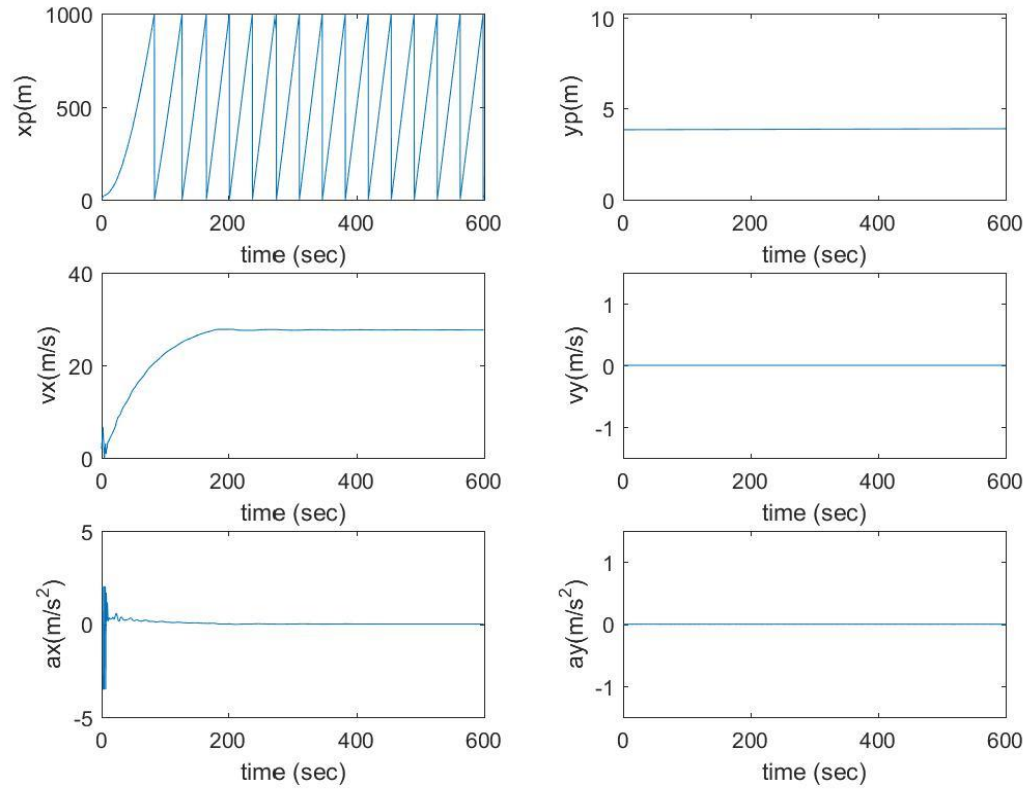

Figure: 4.1.3 Trajectories at density 150 veh/km

The trajectories of a vehicle at density 150 vehicles per kilometer and an average flow of 14911 vehicles per hour.

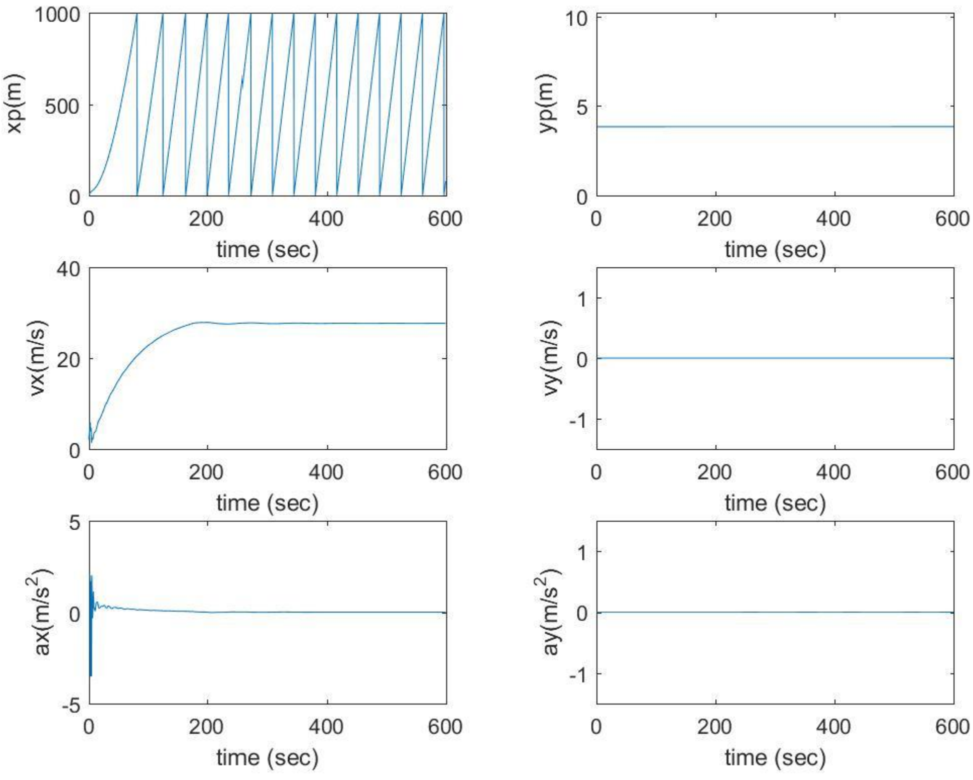

Figure: 4.1.4 Trajectories at density 200 veh/km

The trajectories of a vehicle at density 200 vehicles per kilometer and an average flow of 19905 vehicles per hour.

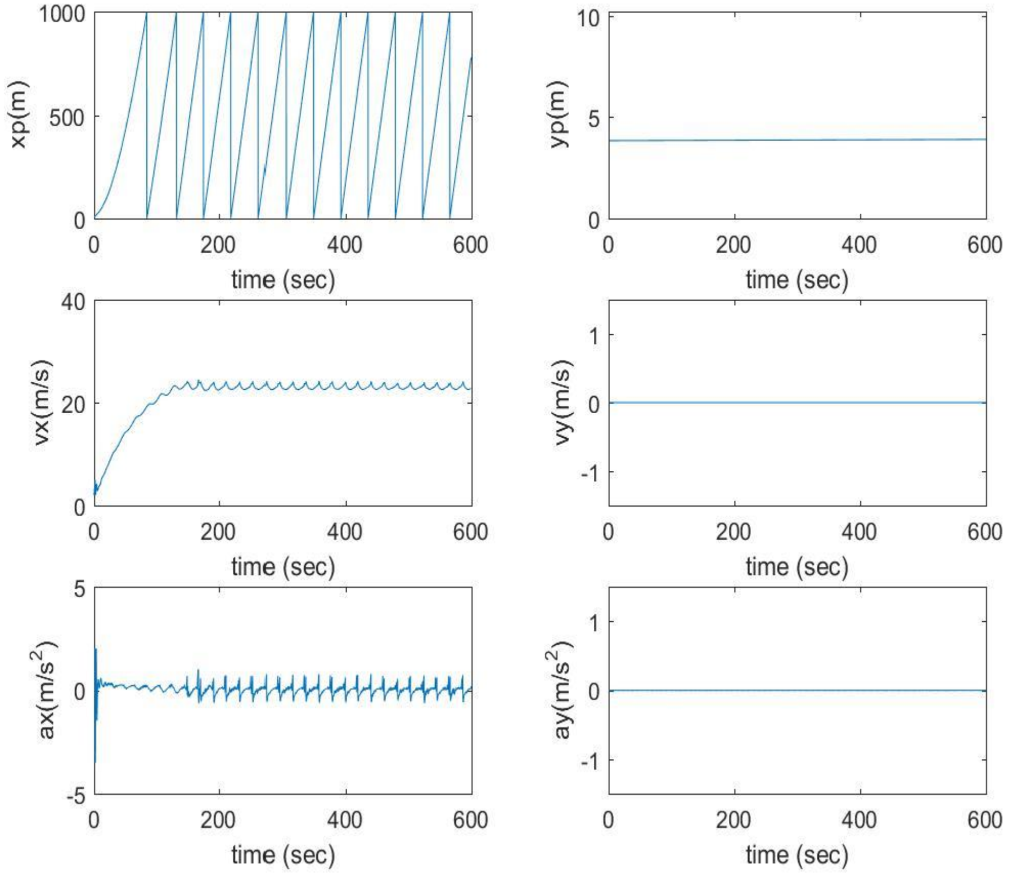

Figure: 4.1.4 Trajectories at density 250 veh/km

The trajectories of a vehicle at density 250 vehicles per kilometer and an average flow of 20694 vehicles per hour.

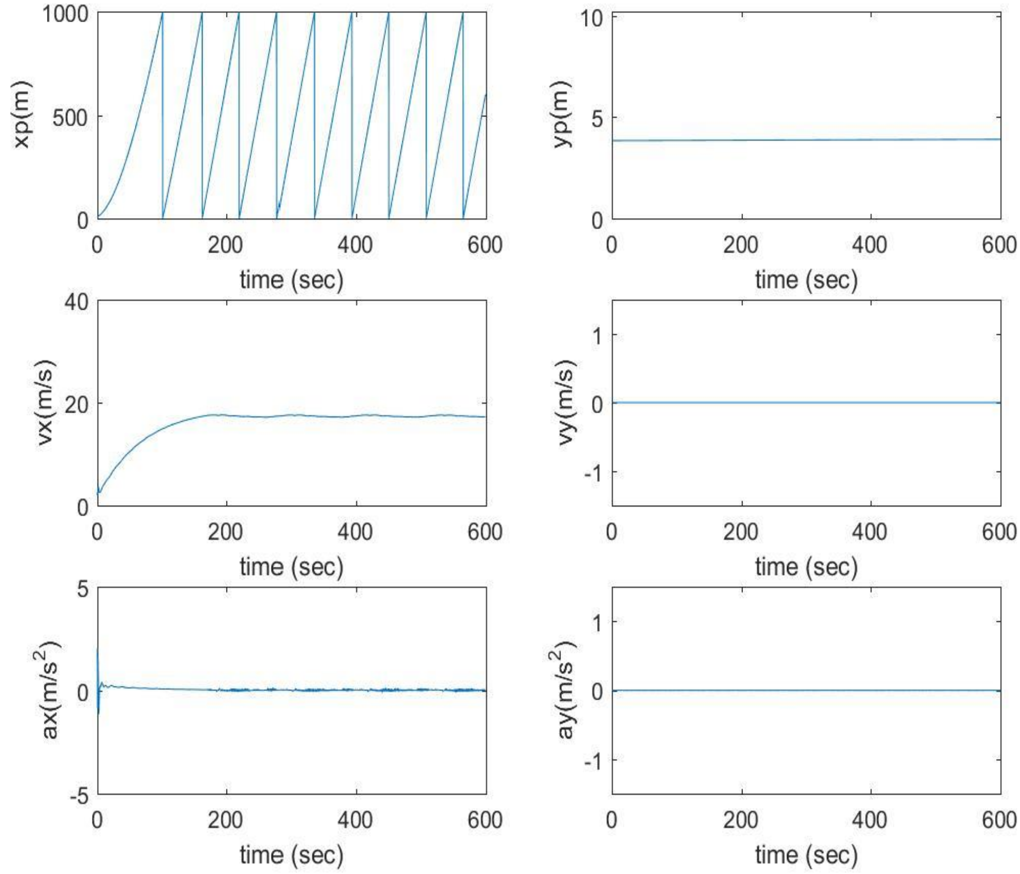

Figure: 4.1.5 Trajectories at density 300 veh/km

The trajectories of a vehicle at density 300 vehicles per kilometer and an average flow of 18743 vehicles per hour.

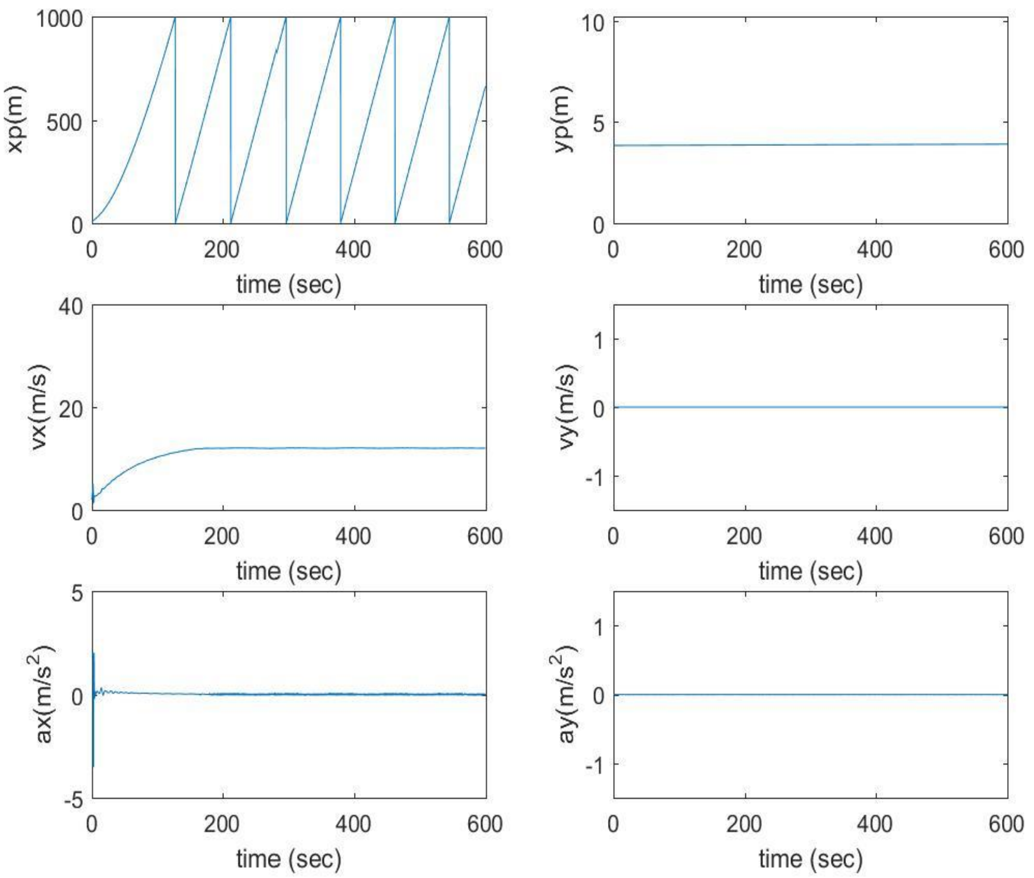

Figure: 4.1.6 Trajectories at density 350 veh/km

The trajectories of a vehicle at density 350 vehicles per kilometer and an average flow of 15166 vehicles per hour.

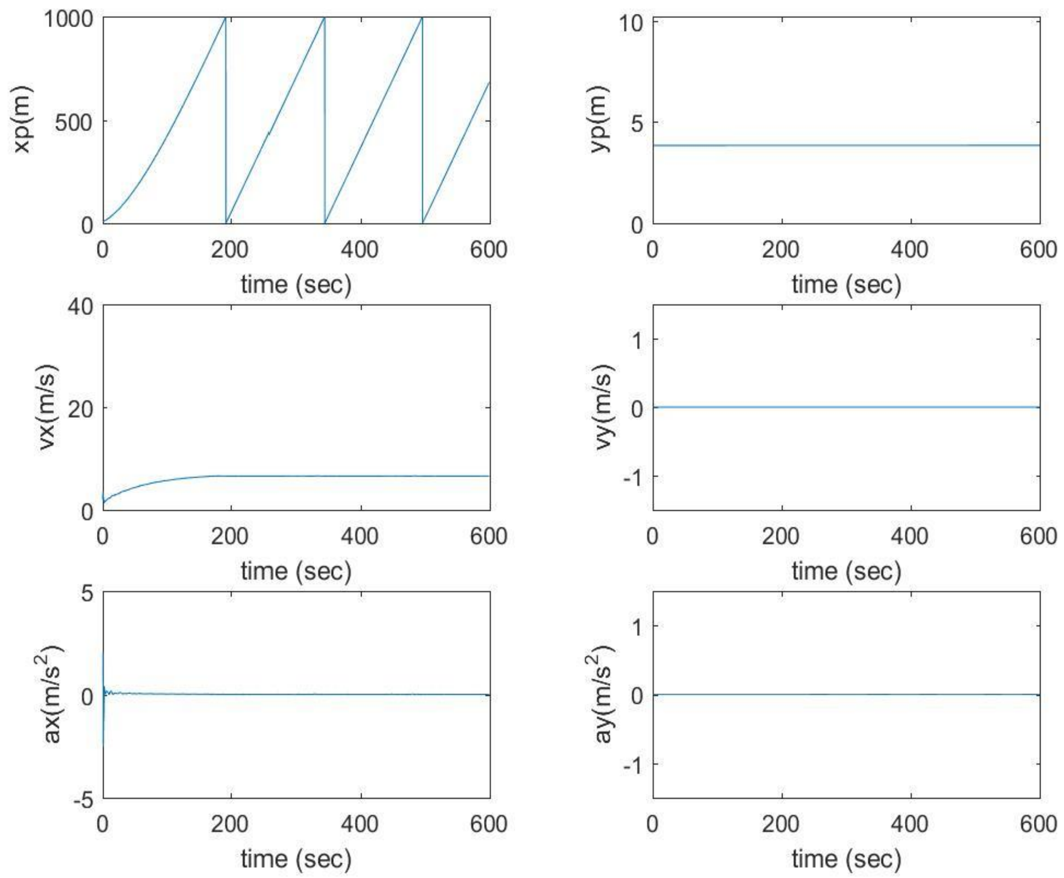

Figure: 4.1.7 Trajectories at density 400 veh/km

The trajectories of a vehicle at density 400 vehicles per kilometer and an average flow of 9476 vehicles per hour.

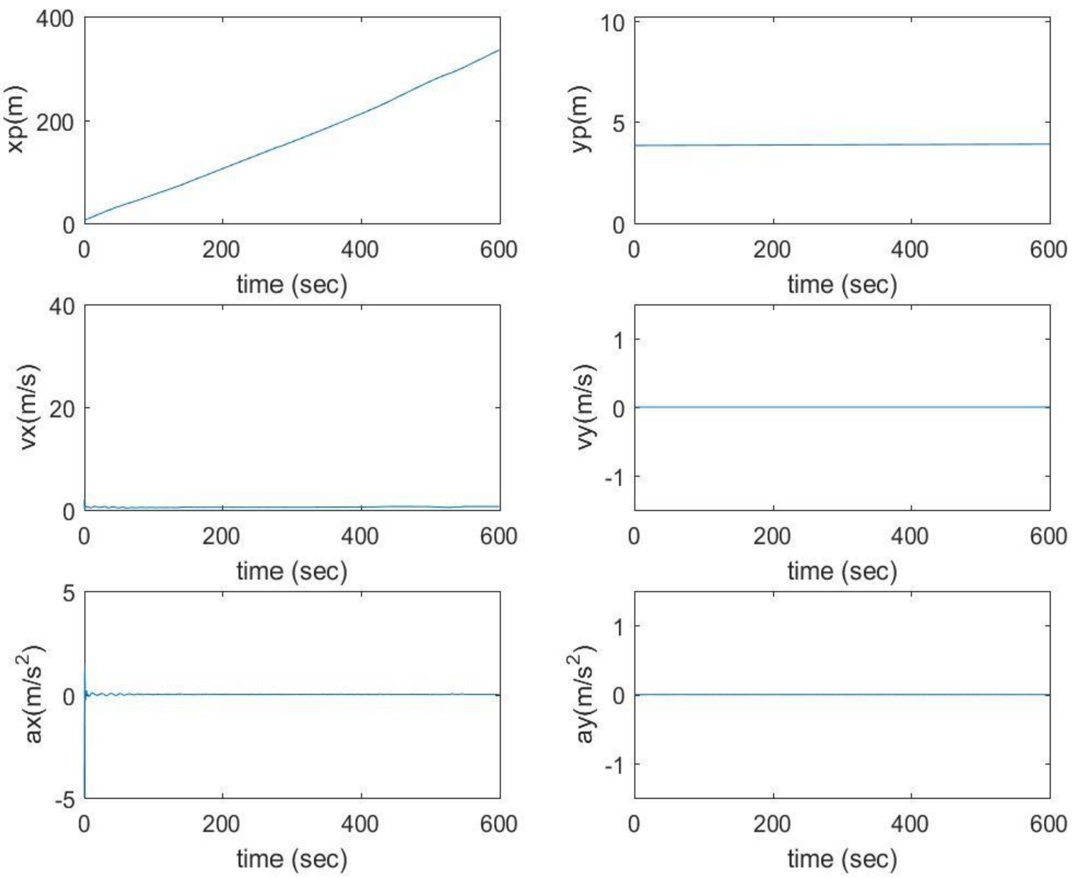

Figure: 4.1.8 Trajectories at density 450 veh/km

The trajectories of a vehicle at density 450 vehicles per kilometer and an average flow of 1021 vehicles per hour.

As it is shown the velocity and acceleration are driven to v^* and zero respectively with insignificant fluctuation. This fact proves that the nonlinear controller is effective. Additionally, fluctuation is eliminated after the first 100 seconds of simulation.

4.2 Fundamental Diagram

The fundamental diagram (FD) is the most important diagram in a movement strategy investigation. It compares the density (vehs / km) of the road with its flow (vehs / hr). The maximum point of the FD corresponds to the critical density and the maximum flow. On the critical point there are no free flow conditions which means congestion appears. In this movement strategy the FD would be linear if the *f(ω)* function was not implemented.

The *f(ω)* function artificially forces the traffic to slow down so the common form of the FD occurs. In the equilibrium state the following equation is true:

$$Q(p) = \bar{v}(p) \cdot p \qquad (4.2.1)$$

Where $Q(p), \bar{v}(p), p$ equals to the flow, average speed and density respectively. By calculating the expected average desired speed given the predefined conditions we get $\bar{v}(p) = 28\text{m}/s$. By applying the $f(\omega)$ function we get the expected (theoretical) FD which is depicted below.

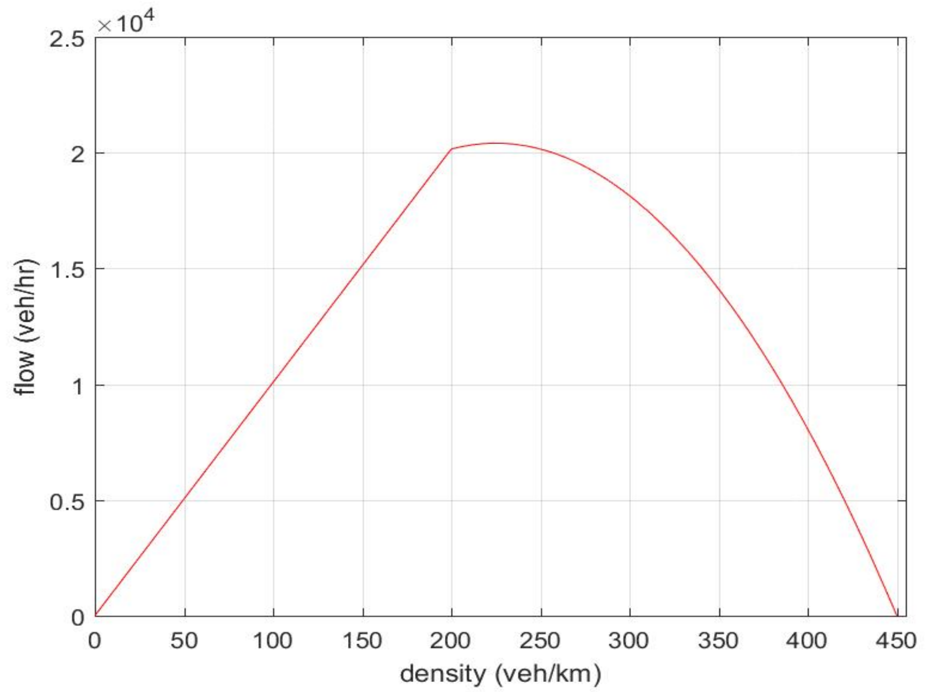

Figure 4.2.1: Theoretical Fundamental Diagram

The detectors implemented inside the simulator measure the flow in a given time frame by comparing the measured flow (in equilibrium) to the corresponding density we get the practical fundamental diagram. The practical FD should be as close as possible to the theoretical.

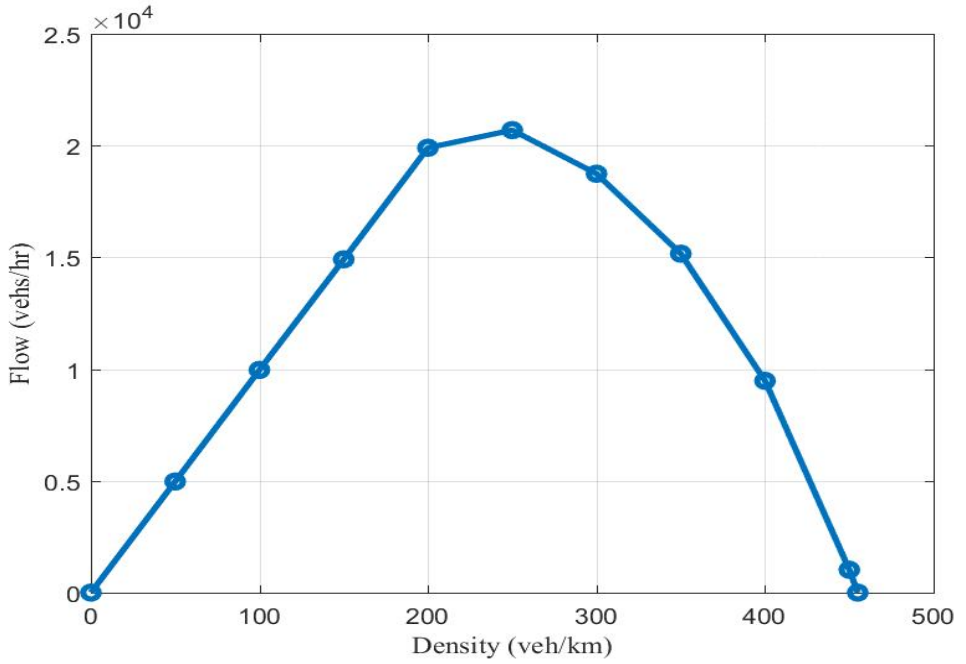

Figure 4.2.2: The Fundamental Diagram

The obtained FD has a maximum flow of 20694 vehicles per hour and a jam density of 455 vehicles per kilometer.

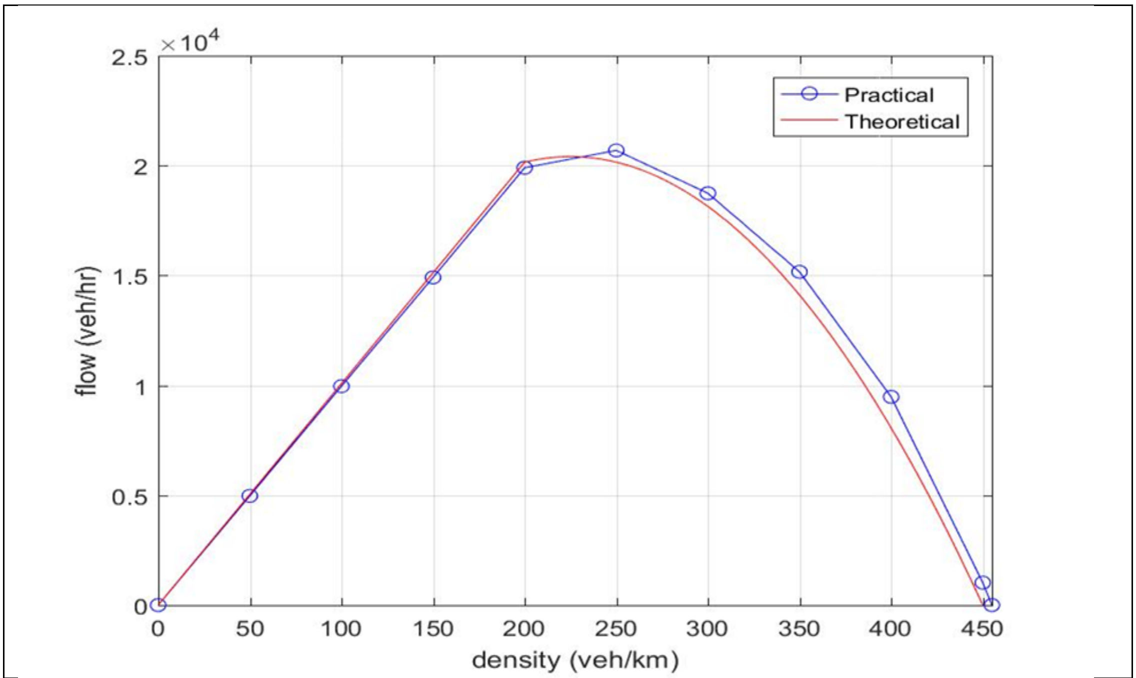

Figure 4.2.3: Theoretical and Practical FD Comparison

As it is shown above the obtained FD is almost identical with theoretical FD.

Chapter 5: CONCLUSIONS

5.1 Safety

In a simulation time frame of 600 seconds no collisions between the vehicles was observed. After the equilibrium state the vehicles remain in a safe distance longitudinally and laterally. No accidents are expected to occur at any time frame greater than 600 seconds. Moreover, vehicles keep a safe distance from the boundaries. No trespassing or contact was observed with the boundaries. So the nonlinear controller successfully cooperated with the boundary controller.

5.2 Efficacy

As discussed in subchapter 3.6, vehicles acquire their desired speed at $t = 0$. The nonlinear controller successfully drove each vehicle to its desired speed. After the equilibrium state, which comes approximately at 150 seconds, the vehicles remain stable at their desired speed without fluctuation.

5.3 Convenience

Fluctuation was eliminated from all the state variables (velocity and acceleration). Which means the drivers will not experience any unpleasant abrupt accelerations/decelerations during their travelling.

5.4 Flow

As shown in figure 4.2.3, the controller achieved a maximum flow of 20694 vehicles per hour at a density of 250 vehicles per kilometer. This result is by far higher from a lane based scenario. Additionally the predicted (theoretical) fundamental diagram (4.2.1) was almost identical with the FD obtained by the simulations.

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