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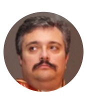

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Shape Control of Smart Beams by Genetic Optimization

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Abstract-- This paper deals with the shape control of beams under general loading conditions, using piezoelectric patch actuators that are surface bonded onto beams to provide the control forces. To formulate the model, we use beam super-convergent finite elements based on two mathematical models, namely the shear-deformable (Timoshenko) and the shear-indeformable (Euler-Bernoulli) model. The optimal values for the locations of the piezo-actuators are determined. Using genetic algorithm, optimal voltages for shape matching are obtained for cantilever beams.

Index terms—piezoelectric smart beams, finite element analysis, shape control, displacement control.

I. INTRODUCTION

Due to the increasing demand of high structural requirements, the modeling and control of flexible structures have received considerable interest among the research community.[1-3] Recently, considerable attention has been focused on the development of advanced structures with integrated distributed control and self-monitoring capabilities These structures are frequently classified as "smart" or "intelligent" structures [4]. The smart structures are primarily employed to control the static and dynamic responses of distributed parameter systems operating under variable service conditions. Piezoelectric materials belong to a class of materials being used as sensor or actuator element in smart structures. These materials respond to mechanical forces/pressures and generate an electric charge/voltage. This phenomenon is called the *direct piezoelectric effect.* Conversely, electric charge/field applied to the piezoelectric material induces mechanical stresses or strains, and this phenomenon is called the *converse piezoelectric effect* [5]. In smart piezoelectric structures, the *direct* effect is used for structural measurements (*sensor*) and the *converse* effect is used for active shape and vibration controls (*actuator*). Most of the past research and development efforts have been directed toward controlling the vibration characteristics of structures. Relatively lesser investigations have made on the shape control resulting from bending, despite its practical importance. Readjusting the shapes and the focal points of space antennas and the contours of aircraft's, spacecraft's, and ship surfaces are some examples requiring structural shape control. In designing smart structures integrated with piezoelectric actuators, engineers have to select the appropriate type of actuators, their locations on the structure, and the amount of voltages to be applied to the actuators. Investigations in determining the input voltages for piezoelectric actuators used in shape control of structural elements was first reported by Koconis et al [6-7]. The analyses were based on classical structural theories in which the effect of transverse shear deformation was neglected. Although numerous researchers [8-9] have well established a mathematical shear-indeformable model, modeling of the adaptive structures by shear deformable theory is limited [10-12]. In situations where very precise displacement control is required, such as MEM structures, or when dealing with shear deformable beams where the effect of transverse shear on deformation is significant, it may be necessary to use the more accurate shear deformation Timoshenko beam theory. In this study, detailed shear-deformable (Timoshenko) and shear-indeformable (Euler-Bernoulli) models are established for laminated beam and their application to distributed measurement and shape control is investigated. A finite element formulation is presented for both models.

II. THE MATHEMATICAL MODEL

A slender beam with rectangular cross section having length L, width b and thickness h is considered. A pair of piezoelectric patches with thickness h_s and h_A is symmetrically bonded at the top and the bottom surfaces of the beam, as shown in Figure 1. The top layer acts like a sensor and the bottom one as an actuator.

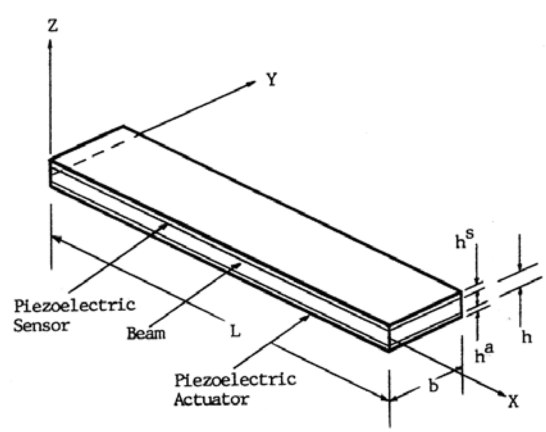

Figure. 1. A laminated beam with piezoelectric sensor/actuator

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2.1. Piezoelectric Equations

The constitutive relations for a piezoelectric material are given by [13]

$$\{\sigma\} = [Q]\left(\{\varepsilon\} - [d]^T \{E\}\right) \tag{2.1}$$

$$\{D\} = [d][Q]\{\varepsilon\} + [\xi]\{E\} \tag{2.2}$$

where $\{\sigma\}$ is the stress tensor, $\{\varepsilon\}$ is the strain tensor, $\{D\}$ is the electric displacement, $\{E\}$ is the electric field, $[Q]$ is the elastic stiffness matrix, $[d]$ is the piezoelectric matrix and $[\xi]$ is the permittivity matrix.

Equation (2.1) describes the inverse piezoelectric effect and equation (2.2) describes the direct piezoelectric effect.

In order to derive the basic equations for piezoelectric sensors and actuators we assume that

- The piezoelectric layers are bonded perfectly on the host beam and are much thinner than the host beam.
- The piezoelectric material is homogeneous and polarized in z-direction and exhibit transverse isotropic properties in xy-plane.

Under these assumptions the set of equations (2.1) and (2.2) is reduced as follows

$$\begin{Bmatrix} \sigma_x \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_x \\ \gamma_{xz} \end{Bmatrix} - \begin{bmatrix} d_{31} \\ 0 \end{bmatrix} E_z \right) \tag{2.3}$$

$$D_z = Q_{11} d_{31} \varepsilon_x + \xi_{33} E_z \tag{2.4}$$

The electric field intensity E_z can be expressed as

$$E_z = \frac{V}{h_A} \tag{2.5}$$

where V is the applied voltage across the thickness direction of an actuator and h_A is the thickness of the actuator.

2.2. Equations of motion.

The length, width and thickness of the host beam are denoted by L, b and h.

The equations of motion of a beam with bonded S/As are derived based on the following assumptions:

i) The beam centroidal and elastic axis coincide with the x-axis so that no bending-torsion coupling is considered.

ii) The axial vibration of the beam centerline is considered negligible and the displacement field $\{u\}$ is assumed to have the form (Timoshenko assumption):

$$\begin{aligned} u_1(x,y,z) &\approx z\ \psi(x,t), \\ u_2(x,y,z) &\approx 0, \\ u_3(x,y,z) &\approx w(x,t), \end{aligned} \tag{2.6}$$

where ψ is the rotation of the beam cross section about the positive y-axis and w is the transverse displacement of the point of the centroidal axis $(y = z = 0)$.

The strain-displacement relation can be expressed as

$$\varepsilon_{xx} = z\frac{\partial \psi}{\partial x}, \quad \gamma_{xz} = \psi + \frac{\partial w}{\partial x} \tag{2.7}$$

In order to derive the equations of motion of the beam we use Hamilton's principle:

$$\delta \int_{t_1}^{t_2} (T - U + W)\, dt = 0 \tag{2.8}$$

where $\delta(\bullet)$ is the first variation operator, T is the kinetic energy, U is the potential energy and W is the work done by the external loads or moments.

The kinetic energy is given

$$T = \frac{1}{2}\int_V \rho \{\dot{u}\}^T \{\dot{u}\}\, dV = b\int_0^L \int_{-h/2-h_A}^{h/2+h_S} \rho\left[(z\dot{\psi})^2 + \dot{w}^2\right] dz dx \tag{2.9}$$

The strain (potential) energy is given by

$$U = \frac{1}{2}\int_V \{\varepsilon\}^T \{\sigma\}\, dV = \frac{1}{2}\int_V \left[\sigma_x \varepsilon_x + \tau_{xz}\gamma_{xz}\right] dV$$

$$= \frac{b}{2}\int_0^L \int_{-h/2-h_A}^{h/2+h_S} \left[Q_{11}\left(z\frac{\partial \psi}{\partial x}\right)^2 + Q_{55}\left(\psi + \frac{\partial w}{\partial x}\right)^2 \right] dz dx \tag{2.10}$$

If the only loading consists of moments induced by piezoelectric actuators and since the structure has no bending-twisting couple then the first variation of the work has the form

$$\delta W = b\int_0^L M^A \delta\left(\frac{\partial \psi}{\partial x}\right) dx \tag{2.11}$$

where M^A is the moment per unit length induced by the actuator layer and is given by

$$M^A = \int_{-h/2-h_A}^{-h/2} z\sigma_x^A dz = \int_{-h/2-h_A}^{-h/2} z Q_{11} d_{31} E_z^A dz \tag{2.12}$$

2.3. Finite Element Formulation

Consider a beam element of length L_e which has two mechanical degrees of freedom (d.o.f.) at each node: one translational d.o.f. $w_1(w_2)$ in direction Z and one rotational d.o.f. $\psi_1(\psi_2)$, as shown in Fig. 2.

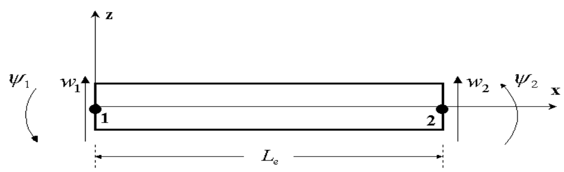

Figure. 2. Beam element

The nodal variable $\mathbf{X}^e$ is defined as

$$\mathbf{X}^e = \begin{bmatrix} w_1 & \psi_1 & w_2 & \psi_2 \end{bmatrix}^T \tag{2.13}$$

The beam element's transverse deflection and the beam element rotation are approximated by

$$w(x,t) = \sum_{i=1}^{4} H_i^w(x)\ X_i^e(t) \tag{2.14}$$

$$\psi(x,t) = \sum_{i=1}^{4} H_i^\psi(x)\ X_i^e(t) \tag{2.15}$$

where H_i^w is a cubic shape function and H_i^ψ is a quadratic shape function. (Superconvergent Element) [14]

Using interpolation functions (2.14), (2.15) into the Hamilton's principle (2.8) and assembling for the entire system, the equations of motion for the discretized structure read [15]:

$$M\ \ddot{X} + D\ \dot{X} + K\ X = F_m + F_e \tag{2.16}$$

where vector ***X*** contains the states of the system (vertical transverse deflection and rotations of the nodes), $(\ddot{\ })$ stands for the second time derivative. ***M*** is the mass matrix, ***K*** is the stiffness matrix. $\boldsymbol{F}_m$ is a mechanical point forces vector added *a posteriori* to the discretized system, $\boldsymbol{F}_e$ is the generalized electrical load vector provided by the applied voltages and proportional to them. ***D*** is a viscous damping matrix added *a posteriori* and $\dot{\mathbf{X}}$ is a velocity vector.

III. SHAPE CONTROL

For static problems, like shape control, the state vector ***X*** of the system is time-independent. Consequently, the equations of motion (2.16) take the following simple form for these problems.

$$K\ X = F_m + F_e \tag{3.1}$$

With fixed, time-independent value of the electric potential at the various actuators one is able to control the shape of the composite smart beam. This way partial or total alleviation of the deflections due, for instance, to the external loading is possible.
Let us assume a beam with the following data: The host beam is made of T300/976 graphite/epoxy and the piezoceramic actuators and sensors are PZT G1195N. The adhesive layers are neglected. Fixed end at the left hand side, a fixed point loading equal to 4 N at the right hand side, length equal to 300 mm, depth of the beam equal to 9.6 mm, width of the beam equal to 40 mm, thickness of the actuators equal to 0.2 mm (one is on the upper side, the other on the lower side of the beam, symmetrically), as shown in Figure 3.

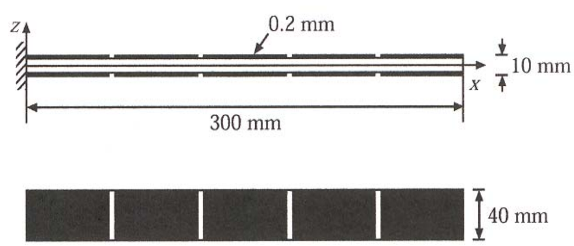

Figure.3. A cantilever beam with surface–bonded piezoelectric sensors and actuators

The beam is divided evenly into 30 finite elements. All the piezoceramics on the upper and lower surfaces of the beam are used as actuators. Equal-amplitude voltages with an opposite sign are applied to the upper and lower piezoelectric layers respectively to control the deformation of the beam subjected to the concentrated load. Due to the converse piezoelectric effect, the distributed piezoelectric actuators contract or expand depending on negative or positive active voltages. In general, for an upward displacement, the upper actuators need a negative voltage and the lower actuators need a positive one.
To investigate the effect of the number of actuator pairs on the deformation control, three sets of actuator pairs are

considered: all the five pairs of actuators, two pairs (the left and the middle ones) and one pair (the left one).

Let us activate one out of the five available pairs of actuators with a constant actuation equal to 200 Volts. The deflection curves due to the external loading and the several actuators, one after the other, are shown in the next Figure 4. One concludes that by using actuators far away from the fixed end of the beam, the shape control task becomes more difficult.

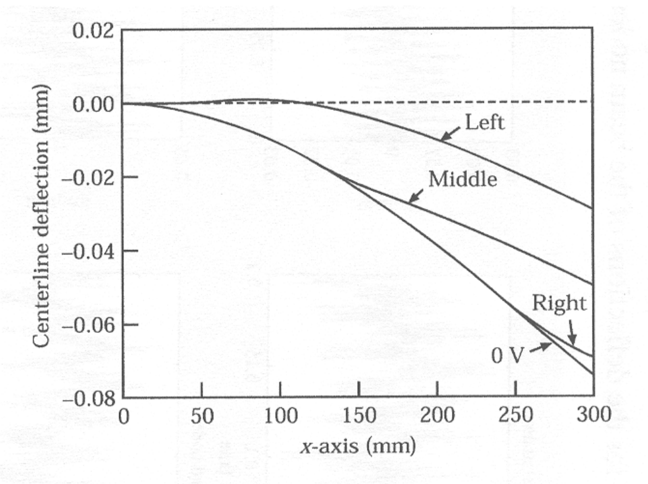

Figure 4.The centerline deflection of the cantilever beam with one pair of actuators at different position

For practical applications one would like to know the optimal location of the actuators and their actuation value with respect to a given shape control task. A first attempt has been done here by classical trial and error techniques.

After some numerical experiments the more satisfactory results are shown in the next Figure 5.

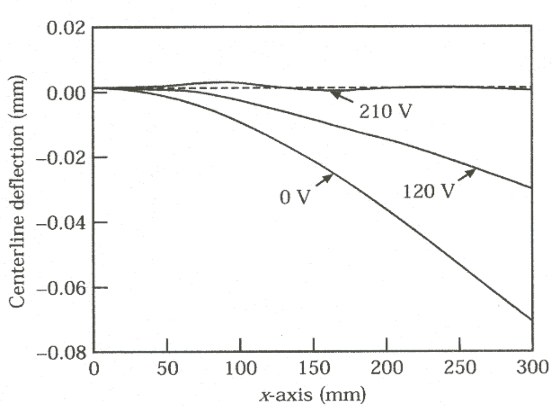

Figure 5. The centerline of the cantilever beam with two pairs of actuators located at the left end and middle span of the beam.

Here one pair of actuators next to the fixed end and a second one in the middle of the beam is used. The deflected shapes under the action of the external load and the previously mentioned actuator pair for different values of the actuation are shown in the Figure 5. The task has been the reduction of beam's deflections due to loading.

The previous results are directly comparable with the data published in paper [16].

IV. OPTIMAL SHAPE CONTROL USING GENETIC OPTIMIZATION

The most general problem of optimal shape control involves the definition of the number, position and actuation voltages of the actuators such that a given cost function is minimized. A good approximation of this general problem is considered here by means of a general optimization procedure, namely genetic optimization [17]. Every one actuator is considered as a separate design variable with it's voltage as the design parameter. The minimization of the beam's deflections in the least square sense consists the optimal design task. This cost function is depicted in the following picture (Figure 6) with respect to the first (near to the fixed end) and third pair of actuators, with all other actuators been considered as inactive (with zero voltages).

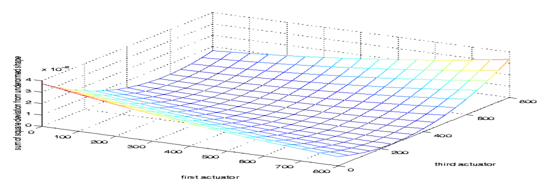

Figure 6. The minimization of the cost function in the case of two pairs of actuators.

An optimal design with five pairs of active actuators calculated by the genetic algorithm is shown in the next Figure.7.

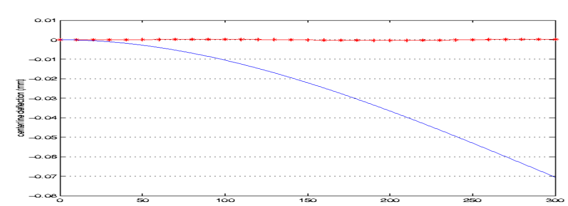

Figure 7. The centerline of the cantilever beam with five pairs of actuators with optimal values of actuation voltages.

The values of the optimal design are 348.108 Volts for the first pair of actuators (on the left side), 162.406 Volts for the second one, 271.654 Volts for the third one, 32.340

Volts for the fourth one and 421.164 Volts for the fifth one (on the right side).

For the time being all available actuators are used for the solution. This procedure will be extended in the near future to include 0-1 variables and the possibility of choosing a limited number of actuators among a large number of potential ones (optimal design of placement). This step is straightforward, since genetic algorithms can handle binary codes and discrete variables. Similar problems can be formulated and studied for dynamical problems. In that case the measures of the optimal control problem (like controllability measures in classical LQR problems) are used for the definition of the cost function. [18-20] Results in this direction will be presented elsewhere in the near future.

Acknowledgements

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