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PhD Dissertation
Cost-Benefit Analysis and Risk Assessment of Mining Activities in Terms of Circular Economy and its Environmental Impact

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Ευχαριστίες

Η παρούσα Διδακτορική Διατριβή επιλέχθηκε και εκπονήθηκε με απώτερο σκοπό να αναδείξει σε πρακτικό επίπεδο τη σημασία της αντικειμενικής αποτίμησης πάσης φύσεως τεχνικών έργων και επενδυτικών εγχειρημάτων, δίνοντας έμφαση στον τομέα της βιώσιμης ανάπτυξης στη μεταλλευτική δραστηριότητα και συνυπολογίζοντας όλες τις απαιτούμενες δεσμεύσεις που απορρέουν από τη σχετική νομοθεσία, με οικονομικούς όρους. Επιπροσθέτως, αναδείχθηκε σε βάθος η εφαρμογή της στοχαστικής μεθοδολογίας για την αντικειμενική αποτίμηση της επικινδυνότητας με μαθηματικούς όρους, εστιάζοντας στη μεταλλευτική δραστηριότητα με σκοπό τη βιώσιμη εξέλιξή της.

Ευχαριστώ ιδιαιτέρως τον επιβλέποντα καθηγητή μου κύριο Βαρουχάκη Εμμανουήλ για την εμπιστοσύνη που μου έδειξε καθ’ όλη τη διάρκεια εκπόνησης της παρούσας Διδακτορικής Διατριβής καθώς και για την επιστημονική του συνεισφορά στο εν λόγω έργο. Ακολούθως, ευχαριστώ θερμά τους συνεπιβλέποντες καθηγητές κύριο Ροζάκη Στυλιανό και κύριο Γαλετάκη Μιχαήλ για τη δική τους συνεισφορά προσφέροντας ο καθένας ξεχωριστά ανάλογα με το αντικείμενό του τις κατάλληλες συμβουλές και υποδείξεις ώστε να ολοκληρωθεί με επιτυχία η παρούσα διατριβή. Κατά ένα μεγάλο χρονικό περιθώριο, πέραν του αντικειμένου της παρούσας Διδακτορικής Διατριβής, συνεργάστηκα επαγγελματικά με τον επιβλέποντα καθηγητή μου κύριο Βαρουχάκη Εμμανουήλ, καθώς και με τον συνεπιβλέποντα καθηγητή μου κύριο Ροζακη Στυλιανό. Για το λόγο αυτό εκφράζω το σεβασμό και την ευγνωμοσύνη μου ειδικά προς το πρόσωπό τους.

Ιδιαιτέρως ευχαριστώ τον κύριο Αγιουτάντη Ζαχαρία για τη συνολική καθοδήγηση και συνεπίβλεψη της διατριβής μου, καθώς και για την παροχή των εξειδικευμένων συμβουλών του στο συνδυαστικό τομέα της περιβαλλοντικά βιώσιμης μεταλλευτικής. Επιπροσθέτως, παραθέτω τις ευχαριστίες μου προς τον Διδάκτορα, Ανώτερο Οικονομικό Αναλυτή και Υπεύθυνο Πολιτικής στην Ευρωπαϊκή Ένωση, κύριο Σφακιανάκη Εμμανουήλ για την επιστημονική του συνεισφορά και ακαδημαϊκή συνεργασία. Ακόμα, ευχαριστώ τους κυρίους καθηγητές Κομνίτσα Κωνσταντίνο και Άνθιμο Ξενίδη οι οποίοι και συνεισέφεραν στο παρόν επιστημονικό έργο με την ενεργό παρουσία τους, παρέχοντας επιστημονική αξιολόγηση.

Εκφράζω την ευγνωμοσύνη μου και τον απεριόριστο σεβασμό προς όλους τους δικούς μου ανθρώπους για την εμπιστοσύνη και την υποστήριξη που έδειξαν προς το πρόσωπό μου καθ’ όλη τη διάρκεια των Διδακτορικών σπουδών μου. Ιδιαίτερα προς τους γονείς μου, προσφέρω τις ευχαριστίες μου γιατί μου δίδαξαν τις έννοιες «αἰέν ἀριστεύειν», και «τοῖς γονεῦσιν ὀφείλω τὸ ζῆν, τῷ διδασκάλῳ ὀφείλω τὸ εὖ ζῆν», τις οποίες προσπαθώ να εφαρμόζω στη ζωή μου.

Abstract

The consecutive increase in demand for metals corresponds to the proportional increase in mining activities worldwide. Respecting the principles of the Circular Economy, as expressed through relevant legislation, the 4Rs action plan (reducing the production of mining waste, recovering and recycling the beneficial metals contained in mining tailings, and reusing them in separate industrial applications) should be implemented to transition existing or future mining activities into sustainable mining practices.

It is essential to note that secondary mining processes, particularly the recovery of metals from mining tailings, contribute positively to minimizing environmental and supply risks associated with metals while supporting the life cycle of mining companies. The utilization of metals contained in mining tailings, combined with an environmentally friendly action plan, offers additive financial benefits for each mining company that adopts it by selling the recovered metal mix for various industrial applications.

Supporting the life cycle of mining activities associated with innovative mining waste management practices in terms of Circular Economy requires the establishment of a closed system of industrial units enclosing the hazardous mining waste (tailings) to avoid any potential leachate risks as in the conventional waste management practices of tailings disposal to be reused in backfilling or stonewalling.

The precipitation of mining tailings containing metal ions, combined with the acidic character of these hazardous leachates, can occur when tailings disposal sites are overloaded or subjected to heavy rainfalls, and this can have a catastrophic impact on the environment and human health. Additionally, it refers to the loss of beneficial material from adsorption by the ground soil.

The proposed alternative technique of tailings management through a closed system of metal recovery minimizes the risks of environmental pollution and loss of metal ions due to precipitation in the downstream soil area, as opposed to conventional mining waste management practices. This PhD dissertation, considering the real industrial needs and risks of the mining sector, examines the level of sustainability for a proposed innovative technique for mining waste management, comparing it with the corresponding grade of sustainability associated with conventional mining waste management practices.

Hence, two comparable cases are examined and assessed. Case Scenario A(0) refers to the conventional methods of tailings disposal using geomembranes and geological fractions, accepting the potential risks of environmental pollution and the systematic risks of non-compliance with mandatory legal obligations that align with the policy of the Circular Economy. Case Scenario A(1) refers to the innovative method of tailings utilization to recover metals contained within them, which adopts a high degree of conformance with the mandatory legal obligations of the mining sector and implements the principles of the 4Rs.

The current scientific study employs an engineering risk analysis approach structured by the Cost-Benefit Analysis guidance and the Stochastic Risk Assessment methodology of Bayesian Analysis to determine the practical sustainability of each examined case based on the corresponding techno-economic approach. The implementation of the Cost-Benefit Analysis using applicable statistical simulation provides a forecast of the total financial risk related to each case over the next thirty working years. The application of Stochastic Risk Assessment using Bayesian Analysis assesses (a) the degree of financial riskescalation calculated through the implementation of Cost-Benefit Analysis for each examined case and (b) the cases that are likely to occur instead of pure theoretical conditions. Thus, the entire engineering risk analysis tool provides a comprehensive financial risk assessment that guides optimal business decision-making for each mining company before the commencement of mining activities.

Regarding the practical application of the current research, it is essential to note that combining Cost-Benefit Analysis with Stochastic Risk Assessment provides an objective method for assessing financial risk by converting all technical and legal parameters into monetary terms. Moreover, results show an extremely low deviation between the two methodological approaches for the cases that are likely to occur in actual industrial circumstances. At the same time, it is assessed that Case Scenario A(1) is more sustainable than the opposite Case Scenario A(0), showing a lower risk grade of approximately 4.8 times.

Considering the role of Financial Engineering risk analysis, supported by Cost-Benefit Analysis and Stochastic Risk Assessment using Bayesian Analysis tailored to industrial conditions, it is understood that this approach provides a comprehensive tool for evaluating the techno-economic sustainability of similar projects based on cost, risk, and potential benefits. This methodological framework of Financial Engineering risk analysis can be utilized by both financial institutions that fund sustainable mining action plans and practices, as well as mining company administrations aiming to develop their activities in terms of sustainable mining and the Circular Economy.

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1 Introduction

1.1. Background and Motivation

Mining activities aim to produce the demanded mass of metals to challenge the supply chain risks for separate industrial applications to overlap human needs. Unavoidable conditions of mining waste production derived from mining activities of primary extraction can cause serious environmental and human dangers due to their acidic character and contain metal trace elements that precipitate in the downstream aquifer zone (1) (2) (3). Unfortunately, the case of mining tailings leachates in the nearby ground soil layers and aquifer zone is a common catastrophic phenomenon well known as "acidic drainage" (1) (2) (4).

Regarding this, waste management solutions focus on the detection of alternative methodologies to manage the produced mining waste. In 1992, a global guidance emphasizing on the Sustainable Development Goals (SDGs), known as "Agenda 21", enabled the basic regulations to protect the environment from the potential damages of the industrial sector (5). According to this, several regions approved this primary green deal and the corresponding representative legal ministries ruled on that general policy of Sustainable Development. Different legislation/guidance in terms of local Agenda 21 (LA21) enacted alternative action plans focusing on waste management, especially for hazardous waste, such as the produced mining tailings, i.e., China (6), Australia (7), Europe (8) (9) (10).

This guidance development affects the Circular Economy policy that is commonly known around the world. The Circular Economy (CE) represents the entire guidance on waste management that supports the policy of 4Rs (reduce, reuse, recycle, and recover waste). Mining tailings are hazardous waste due to their acidic behavior. The trace elements that precipitate downstream have toxic behavior for human health, even when flowing into the aquifer zone or to the surface water deposits (1) (11) (12) (13). So, regarding mining waste management, any methodology that minimizes the hazard source for environmental pollution or serious causatives to human health, from the point of view of CE policy, is approved.

Despite any innovative methodology of tailings management being indicated as respecting the principles of CE expressed through the corresponding legal obligations, especially for the hazardous waste of the mining industry, there is the necessity for the determination of its financial sustainability (3) (14) (15). There are several cases of innovative tailings reprocessing through closed industrial plants of tailings management against the conventional methods of tailings disposal to ground soil or geological fraction mix sub-layers as geomembranes (16) (17) (18). The tailings disposal is the primary step to dry the tailings sludge for its consecutive reuse as raw material for backfilling, stonewalling, etc. Tailings disposal is a high-risk technique due to the high probability of tailings leachates as their pH is very low, so geological fractions or geomembranes cannot ensure the waterproofing of the geotextile layer (19) (20) (21) (22). Furthermore, trace elements of metals used to be geospatially distributed to the soil layers or in worst cases to the aquifer zone, when leachates occur (11) (21) (23). So, the beneficial material contained into tailings cannot easily be recovered.

Therefore, innovative methodologies of tailings reprocessing, supported by industrial thickeners using chemical reagents and implementing flotation methodology to filter the precipitated pulp, have already been developed, optimized and applied especially in Scandinavian regions showing recovery in a high level of efficiency (17) (18) (24). Unfortunately, these techniques and processing optimization is not entirely adopted by all the European or other regions in the world. One of the most inhibitory factors for this is the uncommon detection of the economic sustainability of each similar project. Therefore, the ignorance of techno-economic viability that characterizes any tailings reprocessing should be overlapped and compared with the corresponding evaluation of the conventional tailings management technique of free disposal.

Hence, the detection of techno-economic sustainability of the obliged innovative tailings management technique against the conventional methods is a crucial factor for the mining industry to adopt the policy of CE in mining activities globally. Furthermore, financial institutions that offer loans for the sustainable development of the mining sector should evaluate the submitted requests by the mining companies according to their activities in terms of CE regarding their tailings management action plan. The submitted action plans should be cost-evaluated, including safety factors, and meet the legal requirements of sustainable mining.

This PhD dissertation, by approving the mechanisms of metal recovery from mining tailings, the policy of 4Rs, and mandatory legal obligations for the mining sector, focuses on the financial risk assessment between the innovative and conventional tailings management processes. To achieve a high level of efficiency, an innovative action plan for tailings reprocessing, which is dimensional-costly costly, is financially assessed and compared with the corresponding financial risk assessment of the conventional tailings management process of free disposal. Emphasis is given to the financial sustainability for both examined cases and their relevant grades of efficiency. The implemented financial risk approaches aim to provide deterministic and stochastic risk results that could be valuable for business decision-making in the mining industry, and the sensitive estimation of the loans provided by financial institutions/funds to mining companies, to be developed.

1.2. Importance of Circular Economy in Mining Waste Management

Principles of the Circular Economy CE represent the adoption of a dynamic model focusing on a sustainable waste management system, even for the mining sector (3) (5) (9) (10) (25). The waste management plan is structured by the adoption of necessity for the reduction of produced waste, recovery/recycle of the valuable materials that are contained, and reuse them as raw material for industrial applications (17) (26).

On a global scale, the annual production of all mining waste is approximately 25 billion tons (27). At the same time, mining extractive waste annually produced and stored in the European Union are approximately 4.7 billion tonnes, and the tailings from mining activities refer to the second rank of the total produced waste in the European Union, while their current productivity ranges between 1.2-1.4 billion tonnes, annually (28) (29). Mining tailings are more hazardous waste than the rest of mining extractive waste due to their content of metal trace elements and acids in pulp (1) (29). So, without implementing the CE principles by applying practical methodologies, to efficiently manage tailings (minimizing their volume while removing, through recovery processing, the metal trace elements and purifying them to be reused in separate industrial applications), there is a high-risk probability for negative environmental and human impact from the consecutive operation of mining activities (14) (18) (27).

Regarding the supply chain risks of various metals, it is commonly known that according to the future consecutive increase of their demand in separate industrial applications by 2050, an unavoidable lack of their primary sources will occur (30). Hence, industrial activities that can provide secondary sources of metals to challenge the future supply chain risks for the entire industrial sector correspond to a significant amount of assistance in addressing the shortage of metal deposits. For instance, at the Talvivaara mine site in Finland, an applied industrial units plant was established to recover trace elements of metal mixture, including toxic (Lead-Plumbum, Chromium, Cadmium), critical (Nickel, Cobalt), and precious metals (Copper, Gold, etc), showing an efficient rate of recovery of approximately 50-80 % (31). Additionally, more optimized cases have shown rates of efficient recovery that range between 80-95% (18). In experimental-low scale, the corresponding recovery rates of efficiency fluctuate between (85-97%) (32) (33) (34).

Thus, from the practical point of view, applied cases of tailings management reprocessing, respecting the principles of CE, occur while demonstrating high efficiency for the removal of metal mixture from tailings capacity. To face the challenges of future metal supply chain risks while minimizing hazard sources for the environment and human health, the CE policy should be adopted worldwide. The role of techno-economics is crucial to distribute this philosophy and to be adopted by the mining companies on an industrial scale.

Focusing on the financial interest, which the legally obliged and innovative applied CE principles of tailings reprocessing on an industrial scale may have for the mining sector, the current study develops a financial risk assessment tool to prove the level of efficiency for the alternative practices of tailings management. The techno-economic evidence for the sustainability of tailings management processes is needed to ensure their industrial viability (4) (13).

1.3. Research Questions - Research Gap

Considering the importance of the Circular Economy on the mining sector and the environment, it is comprehended that despite methodologies for sustainable mining are already optimized and in several applied cases their efficiency is too high, still the research regarding their economic viability is limited. Cost-effective analysis that is tailored to each project's requirements is an indicative techno-economic assessment tool that practically ensures the corresponding applicability of the innovative tailings management processes on an industrial scale (4) (35).

In the managerial sector, the optimal business decision should meet the requirements of benefits' superiority over the costs (35) (36). Although the fact of applying CE principles, as a legal obligation in the mining sector, makes tailings management reprocessing a mandatory action plan, unfortunately, mining companies worldwide adopt business threats regarding the investment in secondary reprocessing to utilize their produced mining waste. As a result, instead of adopting CE principles, the adoption of the conventional tailings management processes associated with environmental risks is more probable to occur.

Despite that, several scientific research studies and investment guidance support the development of cost-effective analysis-assessment as an indicative tool to estimate the optimal business decision, even in mining waste management, there is a research gap on detecting the required techno-economic indicators that characterize the rate of economic sustainability between the alternative method of tailings reprocessing to the opposite conventional method of tailings disposal (37) (38) (39). The determination of this research gap is one of the most important incentives for the mining sector to adopt the policy of 4Rs (23) (40) (35).

The major research gap of the sustainability grade detection between conventional and innovative approaches of tailings management can indirectly cause the minimization of the mining sector's life cycle, especially in cases where environmental pollution or general non-conformance with the requirements of CE are detected, except for the direct hazards of the irreversible environmental damages (9) (10) (15).

Scientific research cannot easily approach the existing research gap due to the complexity of the project and the risk management sector, which cannot be well-modeled for all examined cases because of their separate nature of work in actual industrial conditions. Therefore, an expansion of the major research gap is the lack of a financial engineering risk analysis tool that could be adaptable to examine conditions and assess the optimal business decision considering the corresponding legal obligations, investment guidance, and technical parameters. Hence, the research gap does not focus purely on the detection of techno-economic sustainability indicators (of innovative approaches to the conventional approaches of tailings management), but on the required tool that decodes the targets and criteria to be determined and cost-evaluated (29) (35).

Moreover, except for the deterministic financial risk detection, even for mining companies or financial institutions that offer financial loans for business development in terms of green growth, to the first, there is a research gap on the future estimation of financial risk that corresponds to each examined case, ranging from innovative to conventional methods of tailings management (4) (17). Thus, a stochastic approach that detects the grade of the deterministic assessed financial risks is required to optimize the engineering risk analysis tool regarding optimal business-decision making and cross examine the results of deterministic financial risk analysis approach.

Considering the current described research gaps, it is comprehended that an engineering risk analysis tool structured by the combination of a deterministic and stochastic simulation, is needed to be applied in actual industrial conditions of mining waste management to assess the point of efficiency comparing the innovative-indicated method of tailings reprocessing (meeting the 4Rs policy's requirements and high rate of legal conformance) with the conventional method of tailings disposal (that meets a low legal conformance grade in terms of CE).

1.4. Research Objectives

Focusing on the existing research gaps, this PhD dissertation aims to provide an entire engineering risk analysis tool that demonstrates the determination of targets and their criteria that need to be analyzed and assessed to identify the point of efficiency of a proposed case for tailings reprocessing (Case Scenario A(1) and its real and theoretical sub-cases), meeting the corresponding requirements of 4Rs and a high rate of legal conformance, against the opposite case for tailings disposal that refers to the conventional tailings management cases (Case Scenario A(0)). Considering that two financial risk assessment tools are applied to evaluate the sustainability grade of each examined case.

The first appraisal tool corresponds to the Cost-Benefit Analysis (CBA), which is a deterministic approach indicated as the most appropriate cost estimator (35). By the application of CBA in actual conditions regarding tailings management, the corresponding techno-economic viability indicators are calculated by converting technical parameters to monetary terms for both cases. Emphasizing Case Scenario A(1), to ensure a high level of safety on the provided economic assessment, the proposed case of tailings management processing is over-dimensioned and costly-overrun. This leads to the financial risk assessment between the comparison of best-case conditions for Case Scenario A(0) and worst-case conditions for Case Scenario A(1).

Hence, the primary objective of this research focuses on the calculation of techno-economic sustainability indicators using the deterministic approach of CBA to estimate the fluctuation of financial risks that describe both assessed cases (Case Scenario A(0), Case Scenario A(1), and its sub-cases) (16) (23) (41). Based on the primary objectives assessed by the implementation of CBA, to provide additional objectives of the future financial risks and their grades describing the two cases, a secondary simulative tool, the stochastic financial risk assessment (SRA), is applied, too. The SRA methodology implies the terms and definitions of the Bayesian Analysis approach (23) (42).

So, noticing the prior case conditions that were determined by CBA's techno-economic indicators, based on the stochastic simulation provided through the Bayesian SRA approach, future financial risk grades, including monetary terms and risk probability that interpret the escalation of the deterministic financial risk assessment approach, are detected. The application of SRA in actual conditions regarding the financial sustainability comparison between the innovative and conventional methods of mining waste management, considering technical parameters, primary cost evaluation, and legal obligations for the mining sector, is a valuable tool that, except for its provisions, cross-examines the deterministic approach results of CBA.

The scope and innovation focus of this study is on the application of scientific methodologies, evaluating actual conditions of the mining industry, to provide a complete engineering risk analysis tool that calculates the indicators that characterize the economic sustainability of each examined case that a start-up mining company should evaluate before the beginning of its mining activities at site locations. Moreover, existing mining companies could adopt this engineering risk analysis tool to estimate their current or future sustainability grade, considering their corporate policy.

By applying the present engineering risk analysis tool, a deterministic risk matrix is calculated, showing the sustainability grades for all examined cases and sub-cases compared to each other. The structured risk matrix is a valuable evaluation tool for mining companies and financial institutions, supporting them. Finally, a cross-examination between the two applied approaches ensures their sensitivity level among CBA-SRA while verifying the accuracy of risk detection for the more probable cases that may occur in industrial conditions. It is essential to mention that this cross-validation detects the clear theoretical sub-cases that do not refer to actual circumstances.

1.5. Dissertation Structure

This PhD dissertation focuses on optimizing financial engineering risk analysis supported by cost-benefit analysis methodology and stochastic risk assessment, considering the Circular Economy's principles for the environmental impact of the mining sector regarding business decision-making for the mining waste management processes. Therefore, a primary analysis of the Circular Economy's principles, their expressed legal obligations, and their correlation with the environmental impact of mining activities is provided in the introduction section.

The second section focuses on the literature review for mining tailings management, the role of circular economy policy in this, and the conceptualization of the engineering risk analysis framework that needs to be applied to select the most appropriate decision-making for each mining company regarding the 4Rs policy. This section determines the severity of the financial engineering risk analysis tool application depending on the criteria and targets that should be identified and achieved, respectively.

The third section emphasizes the methodological frameworks for both approaches, which structure the entire financial engineering risk analysis tool to provide accurate business decision-making when selecting the most appropriate mining waste management process and evaluating its techno-economic parameters. It also mentions the importance of sustainable mining decisions regarding tailings management against conventional methods to the mining sector's life cycle.

The fourth section provides a detailed analysis of the terms and definitions of the circular economy policy in the mining sector. This section mentions the existing environmental pollution problems that can caused by conventional mining waste management activities and the principles of circular economy policy expressed through the corresponding legal obligations for the mining sector. Applied cases of innovative tailings management processing respecting the circular economy and its terms that show a high level of efficient metal recovery from mining tailings and increasing the life cycle of mining activities are mentioned.

The consecutive section describes the innovative, indicated methodology of tailings management that meets the requirements of the 4Rs. Emphasis is given to the mechanism of metal recovery, while some of the already applied cases for metal recovery from tailings are mentioned.

The role of the circular economy is to minimize potential environmental threats, increase the mining sector's life cycle, and offer financial gains to mining companies that conform with the corresponding legislation through cost savings and economic rate of return from the utilization of metals contained in tailings. The overall cost-benefit analysis is presented in the sixth section. The applied CBA framework considers the legislation that needs to be followed in terms of sustainable mining, calculating the financial risks that refer to conventional and innovative methods of tailings management. Statistical tools are applied to optimize the financial risk calculations that describe each examined case. This methodology provides a sufficient economic risk assessment between conventional and innovative mining waste management procedures. The cost evaluation permits accurate and objective business decision-making, especially for a start-up mining company that needs to select the optimal solution regarding the most economically sustainable waste management technique to treat its produced volume of tailings. The entire CBA analysis assesses the worst-case conditions for the innovative methodologies of tailings reprocessing by proposing an over-dimensional and cost-overdesigned work layout (Case Scenario A(1)) with the best-case conditions of the conventional tailings disposal technique, considering that there is no detected environmental pollution. So, the financial risk assessment provided through the deterministic approach of CBA ensures a superior level of safe estimation.

In the seventh section, accounting for the cost functions described by applying the CBA framework, the stochastic approach supported by the SRA-Bayesian Analysis methodology is applied to cross-examine the deterministic results from a purely scientific perspective. The scope of SRA is to estimate the generating financial risk grades that cause the deterministic results of CBA using the Bayesian Analysis involving the prior distributions of the examined risk probabilities that describe each case condition. The SRA methodology is applied to calculate the stochastic financial risk and stochastic grades of risk indicators.

The eighth section of the present study provides a detailed description of the results obtained using both CBA and SRA methodological frameworks. Techno-economic and risk indicators that describe the economic viability of each case condition are assessed and evaluated. Furthermore, a thorough, deep comparative evaluation is provided for the results of both CBA and SRA. This comparative evaluation offers a better comprehension of the role of the engineering risk analysis tool's practical application in the industrial sector. The final chapter of this section describes the physical meaning of the calculated results, comparing the assessed indicators that describe the examined cases with each other.

The final section summarizes the significant findings of this research. It also mentions the practical application of the developed methodological framework of a financial engineering risk analysis tool structured by CBA and SRA in the industrial and scientific sectors. The major findings that overlap the existing research gaps are validated by the detection of techno-economic sustainability indicators showing the superiority of innovative methodologies regarding tailings management to conventional methods. Moreover, suggestions for future research development and decisions in the mining sector in terms of green growth are provided.

2. Literature Review and Theoretical Framework

2.1. Review of Tailings Management in the Mining Industry

Mining activities are prolific worldwide due to the perpetual requirement for the production of metal materials for a variety of separate industrial applications. Mining activities include the working tasks of primary mining extraction, mineral processing for the separation and enrichment of the beneficial minerals, and environmental remediation. The environmental risks associated with mining activities primarily stem from the lack of a strategic business plan for managing the waste produced during mining processing, known as tailings. This occurs because chemical reagents used in processing are needed to separate beneficial from non-beneficial material of mining ores to be purified (1) (43) (44). Therefore, acid drainage has been observed in some abandoned mining sites globally. This leads to an increase in both environmental pollution and human health risks.

To face this challenge, the indicative solution does not refer to closing mining activities or minimizing their action plan. However, environmental legislation focusing on alternative and innovative mining waste management action plans has already been developed, mentioning the legal guidance/ obligations each mining company should conform to (9) (10). Until recent years, the tailings management involved disposing of the produced mining tailings mud to be dried. After that, the dry mixture product was used to make the stonewalling. According to scientific studies and several environmental monitoring systems, this methodology may cause high severity of risky phenomena, such as pollution of the aquifer zone or acid drainage in watersheds due to the rain falling (1) (45). To minimize potential environmental pollution hazards, the environmental protective legislation, considering the tasks and objectives of the Circular Economy, proposes that each mining company should manage its mining activities that produce tailings through closed industrial unit plants, in order to avoid risks of leachate spills, instead of their disposal as dried out mud (4) (9) (10) (18) (46).

The Circular Economy policy (as described in section 2) in mining activities refers to the reduction of the produced waste, recovery of beneficial materials before the waste's final disposal, recycling of the beneficial materials to be reusable, and reuse of the recovered materials before the final disposal of waste mass. According to the CE and general guidance of Sustainable Development, the environmental protective guidance worldwide, and especially in Europe, expressed through the corresponding European Directives (2006/21/EC, 2008/98/EC), emphasizes the need for alternative methodologies for tailings management.

Each European region conforming to the corresponding European guidance has established its national legislation that indirectly determines the mandatory legal obligations, focusing on waste management even for mining companies. The national legislation involves the environmental safety measures that should be implemented and an action plan in case of non-conformance, containing specific indicators for determining the relevant penalty costs. It is essential to mention that these indicators and the indicative action plan are generally proportional to the Gross Domestic Product (GDP) that characterizes each regional economy.

The main objectives of this PhD dissertation (as described in section 2) focus on a proposal for a tailings management action plan conforming with the legal obligations of the mining industry in terms of the Circular Economy of a start-up mining company that produces 150 ktonnes of mining tailings annually, the control of its techno-economic sustainability supported by a combined economic assessment tool, and a comparative financial risk assessment among the alternative tailings management plan with the outdated methodology of dry mud.

The proposed case for tailings management (as described in section 6.3.1) refers to an indicative action plan to utilize mining tailings and recover/reuse the contained metals. This processing considers existing experimental results from scientific studies and the indicative action plan emphasizing the tailings management processes supported by a closed system of industrial units, as already implemented most commonly in Scandinavian regions (20) (33) (34) (47) (48).

The suggested processing of tailings management refers to an additive process of tailings purification in metal units. As scientific experimental studies have showcased (18) (33) (49), the efficient recovery of metals from mining tailings is structured by two classes of chemical reagents. The first class of reagents refers to acids, and their occurrence aims to separate metal ions from non-metallic chemical compounds. The second reagent class is catalysts that rapidly react to the chemical reaction of metal ions' de-binding and precipitation (18) (33) (34). The entire process occurs inside closed unit plants (buffer tanks) structured explicitly for tailings management processing. It is essential to mention that experimental studies resulted in over 95 % metal recovery efficiency through one major step (due to laboratory conditions) (32) (33). Based on this, the proposed case of the tailings management system is structured by three steps, each containing two stages of acid input and one stage of catalysis to ensure a high-efficiency rate. This also leads to an overdesigned cost assessment of the proposed project.

Continuously, two techno-economic analysis assessments (as they are demonstrated in section 6) for a start-up mining company that is estimated to produce 150 ktonnes of mining tailings are separately presented, one for the case of non-conformance with the innovative legal obligations that support CE policy (Case Scenario A(0)), and one for the case of fully legal conformance with the updated environmental legal obligations of sustainable mining.

Based on the primary economic assessment that characterizes each examined case, the Cost-Benefit Analysis Methodology is applied to assess the efficiency point of sustainability for the proposed Case Scenario A(1) against the conventional Case A(0) (as demonstrated in section 6). Finally, the Stochastic Risk Assessment methodology (analyzed in section 7) is implemented to assess the financial risk grade regarding each examined case's economic sustainability.

The overall financial risk assessment results, structured by both methodologies of CBA-SRA, are clearly demonstrated and discussed in section 8. In conclusion, the major key findings are presented, suggested future research tasks are recommended, and the valuable applicability of the presented combined financial risk assessment tool using the deterministic financial risk assessment by CBA and stochastic risk assessment by SRA, in the business development of sustainable mining or the research and development policy of financial institutions.

2.2. Engineering Risk Analysis Perspectives in Mining Waste Management

Humans' high need for metal materials necessitates the development of mining operations, especially in places highly enriched in metal ions. After separating beneficial and non-beneficial materials, industrial enrichment mechanisms increase metal output.

These mechanisms, known as metallurgical procedures, produce a vast volume of mining/metallurgical hazardous waste (MMW) at final disposal sites. MMW’s composition usually includes metal filings in low-pH site conditions. Thus, the environmental pollution hazard is high unless sustainable methods are implemented to reduce the concentration of heavy and toxic metals in MMW at every disposal site (1) (41).

Engineering risk analysis is applied to assess the optimal business decision in mining activities, considering their environmental impact and the requirements of the Circular Economy. Therefore, engineering risk analysis focuses on detecting the most economically sustainable mining waste management process for a start-up mining company to minimize potential environmental pollution hazards. Figure 1 shows the targets and criteria that are considered by the engineering risk analysis tool (24) (25) (46).

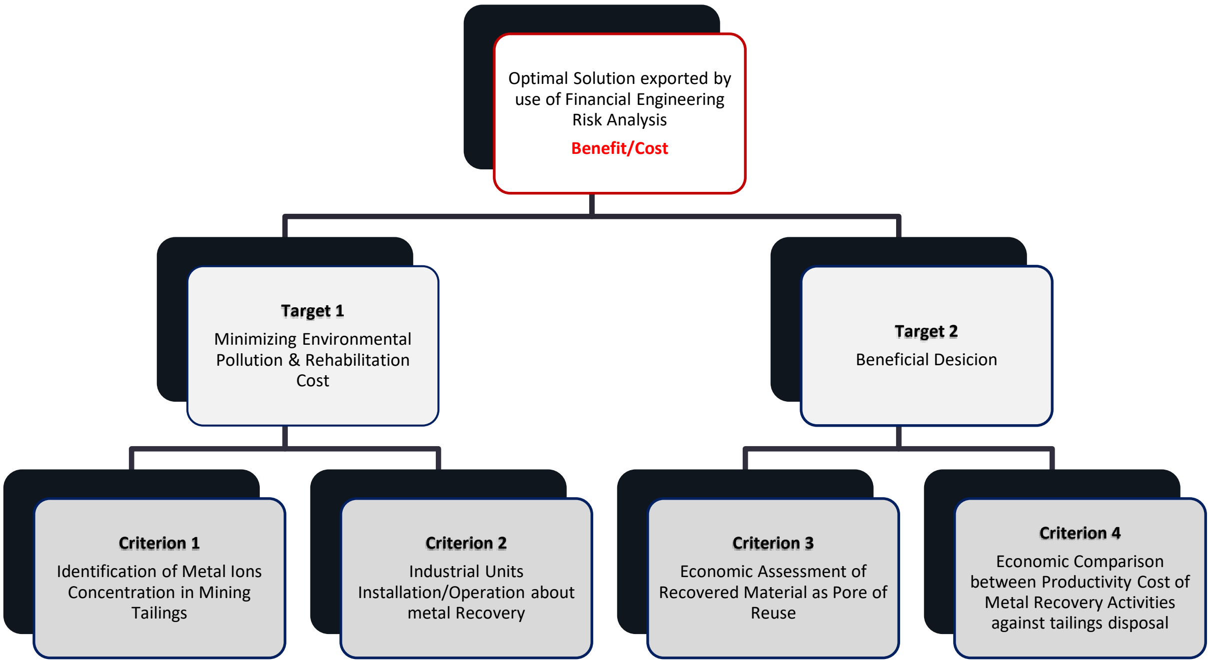


Figure 1 Hierarchy of Targets & Criteria considered by Engineering risk analysis methodology to export the Optimal Business Decision according to the Benefit to Cost ratio in Mining Waste Management

The exported business decision is based on the total beneficial scenario, which, apart from environmental pollution and its rehabilitation cost, evaluates the possible case of reusing the recovered metal product as raw material for the global financial marketplace. According to this, a model of a scientific database in which all the required parameters are converted to monetary units needs to be implemented.

This database consists of the criteria presented in Figure 1. Evaluating these criteria ensures the validity of each operational strategy. Each criterion's elements are described below.

Criterion 1. Identification of Metal Ions Concentration in Mining Tailings.

Identifying metal ion concentration in mining tailings is crucial for evaluating environmental rehabilitation costs. Scientific study sample results should be considered when determining the chemical composition of mineral ores in metal ions. This metal ion concentration process is calculated in mg/kg, ppm, or %wt. The annual production volume of each mine industry's waste and their chemical composition analysis could reasonably estimate the total metal product content.

Criterion 2. Recover Industrial Units Installation/Operation

Considering the scientific evidence of the total estimated metal product content, the necessary installation and operation capacity of the industrial units needed for implementing recovery procedures could be calculated. The recovery capacity/technical specs of the industrial units are proportional to their installation and annual productivity costs.

Criterion 3. Economic Assessment of Recovery for Metal Material into tailings as Reusable Material

After criterion 2, an economic assessment of the sustainability monitoring for metal recovery processes is needed. This data assessment evaluates the potential economic gains from utilizing the total volume of recovered metal materials. The economic assessment of metal recovery processes is the most significant step of engineering risk analysis tools due to its ability to offer an objective estimation of the actual sustainability of such a project.

Criterion 4. Economic Comparison of Productivity Cost for Metal Recovery Activities against Conventional methods of tailings management. An economic comparison between the productivity cost of metal recovery activities against free disposal is required to verify the sustainability superiority of the former with respect to the conventional tailings disposal methods. This assessment considers the environmental rehabilitation costs in cases of tailings disposal and provides consultancy to the existing mining companies that do not conform to the innovative legal obligations that support the policy of Circular Economy.

The underlying engineering risk analysis project plan for facing mining waste management challenges is based on four criteria. Criterion 1 refers to the chemical process of mining tailings gained by scientific case studies. Criterion 2 refers to the cost of primary installation and maintenance of industrial equipment to waste-treat mining tailings while metal recovery procedures occur. Criterion 3 refers to the approach of benefit that a start-up company aims to receive by the economic utilization of a recovered metal mixture and cost savings due to legal conformance with the Circular Economy's policy. Criterion 4 refers to the financial comparison between the innovative business proposal of tailings management and the conventional methods of disposal, with no implementation of recovery processes.

Regarding mining waste management, due to the severity of the produced hazardous waste that may cause high environmental risks, the selection of the optimal business decision should be highly accurate and meet the requirements of actual industrial conditions. The methodology of cost-effective analysis, known as "Cost-Benefit Analysis", is an indicative risk analysis tool that evaluates the sustainability of the tailings management project by converting all technical parameters into monetary terms (50) (51). To verify the business decision-making by CBA, a strong statistical analysis-assessment tool supported by the Stochastic Risk Assessment through Bayesian Analysis is required (4) (52) (53).

This study determines how Cost-Benefit analysis and Stochastic Risk Assessment contribute positively to (a) the environmental effect of MMW reduction, (b) decreasing the environmental rehabilitation cost, and (c) research into economically sustainable methods of recovering metal from MMW.

Financial Risk Assessment through Cost-Benefit Analysis for Mining Waste Management enhances the development of an innovative engineering risk analysis methodology that could be implemented on an industrial scale. This methodology adopts the structural materials of CBA and applies them to assess the financial risk for each business decision that the mining company's administration should make. The current study examines two case scenarios for a start-up precious metals mining company that aims to produce a huge volume of mining tailings annually.

Engineering risk analysis structured by CBA and SRA is applied to a) investigate if the innovative methodology of metal recovery from mining tailings that refers to Case Scenario A(1) is technically and economically sustainable, and b) assess its point of efficiency against the conventional method of tailings disposal, which corresponds to Case Scenario A(0).

First, Case Scenario A(0) refers to the presentation of mining activities without the 4R's policy and non-implementation of environmental protection measures due to the tailings disposal with non-adoption of recovery of beneficial materials contained in them. This business decision corresponds to the full approval of penalty cost for non-compliance with the environmental protection policy in case of environmental inspection.

Second scenario A(1) refers to the presentation of mining activities with full implementation of environmental protection requirements. This business decision corresponds to the investment in a closed system of industrial units for metal recovery and avoiding the free disposal of tailings onto soil areas.

Considering a) each project's nature of work, b) legislative requirements for environmental protection, and c) escalation of penalty cost for non-compliance with the corresponding legislation, the total cost for each case scenario is extracted. Cost-benefit analysis evaluates the sustainability of each case scenario by its Financial Risk. The major scope of this study is to ensure the adaptability of the CBA appraisal tool to each similar subject of study, in which the lowest Financial Risk indices characterize optimal business decisions. CBA's evaluation involves converting the parameters of each case scenario into monetary terms.

Further research focuses on detecting each case's techno-economic indicators, which demonstrate its efficiency level. The indicators that are analyzed and assessed are the total CBA (benefit-to-cost ratio), LTP (Long-Term Perspective), and ERR (economic rate of Return) for each case (35) (46) (50).

Moreover, the present study's innovative research is enriched with the practical role of the Stochastic Risk Assessment by converting nominal law requirements to mathematical-statistical parameters for the two examined scenarios to extract the appropriate business decision. The SRA of Bayesian Analysis aligns the results of financial risk assessment calculated through the CBA methodology for further forecast accuracy.

The SRA calculates the Risk Index (R) for both scenarios A(0): acceptance of penalty costs due to non-conformance with environmental protection requirements and A(1): investment in the metal recovery by mining tailings to be fully implemented with the European and global guidance regarding the Circular Economy. Thus, Stochastic Risk results provide high sensitivity to the CBA's indices, permitting the consecutive monitoring of the potential risk indices for both examined cases. Thus, the sustainability of each case is characterized by the minimum values of LTP and R, and maximum values of ERR and CBA indicators (23) (54).

2.3. Overview of Cost-Benefit Analysis and Stochastic Risk Assessment

The scope of CBA is to analyze, assess, and identify both the operational project cost and its benefits. As it is comprehended, CBA evaluates the sustainability or non-sustainability of each business decision. The CBA's framework, a crucial tool in this evaluation, comprises seven main steps and five major study subjects (50). These are presented below.

<u>Main steps of CBA</u>

1. **Description of the Context**

The first step in the CBA process is the 'Description of the Context '. This step is crucial as it provides a comprehensive understanding of the legal, economic, and political background of the local country where the project will be implemented. It is essential to consider factors such as the socio-economic situation, the existing economic policy, the development of operational activities, and the flexibility of legislation policy, as these can significantly impact the success of the project.

2. **Definition of Objectives**

The “Description of the Context” identifies the effects of the project on the local society, economy, and environment that need to be analyzed. Furthermore, evidence of the project’s benefits to local society is mandatory. These benefits have to meet the requirements of current legislation.

3. **Project Identification**

Identification of the project consists of a) physical equipment (human resources, machinery, etc) that is going to be used and b) the organization that will be responsible for the project’s quality control. These factors ought to ensure the operation’s efficiency in an environmentally friendly mode.

4. **Technical and Environmental Sustainability**

This step includes a) a strategic analysis in which are mentioned the reasons of each business decision approval, b) required job positions about the project’s phases implementation, c) human resources and responsibilities, d) environmental protection plan during work activities, e) total project management chrono diagram (considering milestones, major tasks, critical pathways etc), and f) whole cost estimation based on the previous schedule.

5. **Financial Analysis**

Financial analysis includes a) the project’s profitability to its owner and administration, b) financial sustainability according to positive economic balance maintenance, considering productivity and procurement costs (eg., the cost of used equipment, job salaries, the cost of possible mishaps, etc.).

6. **Economic Analysis**

The results from financial analysis are evaluated, and more sustainable mechanisms are implemented to reduce money loss. In this step, based on the previous financial assessment, indirect taxes and general financial burdens are reduced by alternative operational procedures.

7. **Risk Assessment**

Risk assessment considers a combination of the project’s probabilistic analysis, quality control analysis, and hazard analysis. This contributes positively to identifying the key tasks that may hurt the project’s development and formulating the risk prevention plan (RPP) accordingly.

Major CBA’s Subjects of study

1. **Cost of Opportunities**

Opportunity cost (CO) is the loss of potential gain from other alternative solutions when one is characterized as the ideal to be implemented. Often, approved business decisions chosen according to the financial growth rationale may negatively impact the whole business plan because of other parameters that have not been considered. The ideal project solution must adopt the Q-C-T (Quality, Cost, Time) pattern.

2. **Long-Term Perspective**

The long-term perspective (LTP) consists of the cost of the project's work activities over 10-30 years. In this task of CBA, the value of future costs and benefits is estimated, considering all the possible hazard effects on the project's life. So, the identification of hazards is mandatory. Thus, CBA evaluates the hazards as approved or unapproved, and by this evaluation, the project's critical pathway is extracted.

3. **Economic Performance Calculation**

Project objectives have monetary value (positive for benefits and negative for costs). CBA is based on these values, assessing each objective's effectiveness. Total project performance is characterized as beneficial or not. This assessment includes both the Economic Net Present Value (ENPV) and Economic Rate of Return (ERR) indicators.

4. **Microeconomic Estimation**

Each project, except for its environmental or financial impact, has a social impact, too. As a pre-operation microeconomic study, CBA must determine and calculate economic performance factors on this path. The ENPV considers the direct environmental and financial effects. Otherwise, indirect effects, such as social effects (eg., operation approval or disapproval by the whole area population, problems in human resources, etc.) have to be reduced to the lowest amount. By the elimination of indirect effects during the ongoing future time, better-modeled analysis is achieved.

5. **Incremental Assessment**

Incremental assessment (IA) compares two possible scenarios based on the project's work activities. The first scenario is described by risk approval and its possible penalty cost. The second scenario includes mechanisms, procedures, or implementation of environmental safety measures (e.g., equipment to be used, job positions, etc.) according to the legislation and their cost/benefits. Thus, each scenario is assessed by ERR and total CBA. This comparison uses mathematical models to determine each scenario's efficiency. By IA, it is easy to output a cash-flow analysis of each possible applicable scenario, especially when perpetual mechanisms (Recovery/Recycling mechanisms in terms of Circular Economy) show low levels of ERR indicators.

The scope of SRA in mining waste management is to assess the risk grade for each business decision regarding conventional tailings disposal and innovative metal recovery from mining tailings through a closed system of unit plants. The SRA methodology is applied after the primary statistical techno-economic method of CBA, which converts all of the required technical parameters into monetary terms.

By implementing SRA, CBA's financial risk assessment, which evaluates the sustainability of each examined case, is calibrated to verify its incremental risk grade. Two significant variables structure the SRA methodology. The primary key variable refers to the probability that determines the economic feasibility of each examined case (42) (55). The secondary variable refers to the total cost function by CBA and the money loss that characterizes each case scenario, respectively (4) (56).

The SRA is a pure stochastic mathematical-statistical analysis-assessment tool that is applied to a variety of scientific sectors (finance, engineering, mathematics, medical, etc.) to provide accurate estimations considering the examined current conditions and terms that determine the potential or expected risks (57) (58). Therefore, SRA is characterized as a clear scientific methodology for the detection of risk based on unlimited actual conditions.

For the stochastic financial risk detection, both the total risk probability and the corresponding cost function are required. Hence, the SRA, considering these two factors, calculates the financial risk grade. It is essential to mention that the stochastic financial risk grade refers to the rate of financial risk fluctuation (59) (60).

The stochastic financial risk assessment variables setting depends on the examined case terms and conditions. For instance, in cases of theoretical estimations of examined cases, the setting of variables is achieved following the typical methodologies that determine the most suitable distributions to be used. In contrast, in actual cases that may refer to existing conditions (considering legal obligations, technical limitations, or other physical parameters, etc.) the SRA is applied using a mixture of deterministic with stochastic variables (55).

In the present study, both methodologies, CBA and SRA, are applied to assess the point of efficiency between two examined scenarios that refer to actual conditions of the conventional and innovative methods for tailings management, respectively, while offering a combined financial risk analysis-assessment engineering tool. Both methodologies are needed for an accurate financial risk assessment. The deterministic method of CBA calculates the total cost function of each case, including statistical analysis. The stochastic approach of SRA is applied to verify the results of CBA or assess any potential deviation.

2.4. Role of Circular Economy in Tailings Management

Following the terms and definitions declared in Agenda 21 which corresponds to a memorandum action plan for sustainable development, the policy of Circular Economy is established to provide an indicative perpetual cycle of industrial operations respecting the environment and human health (5). According to waste management, especially in the case of mining activities that produce huge volumes of hazardous waste, the philosophy of Circular Economy, which refers to the policy of 4Rs should be adopted to avoid any risks, even for the environment or human health (1) (18) (61).

The strategic plan of CE aims to minimize the environmental, health, and safety risks while creating chances for business development in terms of green growth by the utilization of waste (62) (63). The Policy of CE is supported globally by the general guidance of sustainable development (5). Respecting this, in the European Union, specific guidance is enacted to provide specific obligations for the waste management plan that each European region should conform to. Focusing on the mining waste management plan, the corresponding directives 2006/21/EC and 2008/98/EC (9) (10) indicate the mandatory procedures that mining companies should follow to manage their produced tailings.

This guidance describes the innovative and alternative methodology of tailings management that is already implemented in northern Europe (Finland, Sweden) (18) (25), through a closed system of industrial units (by the controlled addition of acids or catalysts) to avoid any potential risk of tailings leachate to the environment. This tailings management plan refers to the innovative and environmentally safe method of utilizing mining waste while offering financial gains by reusing recovered/recycled metal materials. Moreover, this methodology, due to its high level of efficiency in metal recovery, especially in critical metals contained in tailings mass, faces the future challenges of metal supply risk that is estimated to occur in the new manufacturing era by 2050 (30).

In the present study, conceptual terms and definitions of CE in an industrial scale are considered to assess the techno-economic sustainability of the innovative compared to the conventional tailings management processes. To assess this, the actual case conditions of tailings management and the regional legislation terms are considered (64). Each European region's legislation conforms with the general guidance of the corresponding European Directives, and its terms/ legal obligations that express the CE's requirements are proportional to the region's Gross Domestic Product. Therefore, the current study focuses on the financial risk assessment considering the environmental impact and CE terms expressed by the Greek legislation, which conforms with the European and Global guidance of Circular Economy and Sustainable Development.

2.5. Conceptualization of Risk Analysis, Engineering Perspectives, and Circular Economy on Tailings Management

Engineering risk analysis evaluates a business decision or operational activity before its implementation. Its main advantage is that it provides the required scientific evidence about each decision's correctness. One of CBA's major advantages is the well-modeled assessment of project objectives, describing them with monetary values (50). Thus, the separation of beneficial from costly decisions is easy to accomplish. On the other hand, it seems that this modeled system would not be suitable for all project types because of their varied nature of work. However, this could already be reckoned in microeconomic estimation ME and incremental assessment IA supported by the SRA (23) (40) (55).

According to the waste management practices of mining activities, CBA is used to compare possible scenarios' negative costs and potential benefits (in terms of CE) and environmental rehabilitation (40) (65) (66). The lack of investment in innovative mining waste management describes the first Case Scenario A(0) that refers to conventional methods of tailings disposal. In contrast, the negative cost of investment in a closed system of industrial units (following the guidance of CE) to waste treat mining tailings, including potential benefits by the utilization of recovered metal materials (rare earth elements (REEs), strategic metals (SM), or other precious metals (PM)) refers to the second Case Scenario of innovative tailings management. The negative cost of each case scenario is calculated by the negative goal function (GF), which considers each negative cost expressed through the loss function (LF) calibrated by the relevant Probability Density Function (PDF), which describes the fluctuation for the estimation of negative probability.

CBA is suitable for both scenarios, including applicable mathematical models, to complete all required tasks. The main disadvantages of each business decision are the higher penalty cost of non-compliance in the first scenario and the cost of investment, including any probable financial gains in the second scenario. Thus, CBA tasks could evaluate the sustainability of each scenario based on LTP, ERR, and total CBA indices.

There are no relevant references/studies in which the legislative obligations are transitioned to countable techno-economic indices or mathematical indicators, especially in challenging cases of the mining industry. So, there is no sufficient scientific evaluation for the appropriate business decision in industrial mode, between the conventional and innovative methodologies of tailings management.

The two options for the administration of each mining company are a) non-compliance with the obligations and guidance of environmental legislation and full acceptance of potential penalty costs (Case Scenario A(0)), and b) investment in actions for full conformance with the guidance of environmental legislation and gain of potential economic benefits by the reuse of recovered materials (Case Scenario A(1)).

The cost-benefit analysis on investment projects objectively evaluates all the legal and technical parameters converted into monetary terms. The scientific role of SRA is to align the total GF calculated by CBA to prove a more sensitive future approach to grading the sustainability of any similar project. The grading of sustainability is expressed through the Risk Indicator. So, the most sustainable Case Scenario is characterized by the minimum LTP and R indicator while adopting the maximum ERR and CBA indicator. Furthermore, as a general observation for the efficiency of Case Scenario A(1) against Case Scenario A(0), the Internal Rate of Return (IRR) is calculated after the overall CBA. The IRR indicator is not an accurate decision-making rule because the Net Present Value (NPV) changes within the forecast period. By the end of the CBA, IRR shows the stochastic grade of efficiency among the comparable scenarios, A(1) to A(0) (50).

Considering scientific studies on the technical-economic analyses according to the CBA methodology, data analysis is provided, including partial parameters (40) to evaluate the project's total sustainability (65). This subject of science could be tailored to each project's requirements. Therefore, applying CBA examines specific factors that determine the total cost of investments and the potential benefit. For instance, regarding the metal recovery procedures that should be implemented in the case of electronic equipment waste management, there is a necessity for the estimation of the machinery's installation cost, the charges of transportation, the operational cost, and the potential benefit (67). Another indicative application of the CBA to assess the sustainability of metal recovery by photovoltaic waste focuses again on the total investment cost configured by the primary installation cost for the required equipment, operational cost, and income benefit (37).

The proposed Case Scenario A(1), which refers to mining waste management in terms of 4Rs for mining tailings, is evaluated through the obligations of CBA. So, its evaluation is proportional to the numerical values of Long Term Perspective and Economic Rate of Return against the relevant indices of the opposite Case Scenario A(0). The optimal business decision between these two Case Scenarios refers to the minimum numerical value of LTP and R and the maximum numerical value of ERR and CBA indices.

Thus, the sustainability of each case scenario is evaluated through the final CBA index configured by the ratio of benefit to cost B/C and R index configured through SRA. To compare the grading of stochastic risk assessment, the ratio of the Risk index multiplied by the cost functions is applied between the two examined cases. If the CBA ratio exceeds the numerical value 1, then the examined case is optimal. Otherwise, regarding the ratio of R indices multiplied by the cost functions for each case, if its numerical value exceeds 1, the examined case is characterized as risky (68). This case does not refer to the optimal business decision.

Considering this axiom, to accurately estimate the sustainability of each case scenario examined, CBA's output (Goal Functions for both cases) needs to be calibrated through Bayesian Analysis. The provided SRA aims to calibrate the output of CBA's Goal Functions through Risk Integrals that assess the degree of financial risk (69) (70).

2.6. Conceptual Framework for Integrating CBA & SRA in Mining Operations

This study's research target is to identify the optimal business decision that the administration of a start-up mining company should follow. The provision of the optimal business decision is supported by the financial risk assessment for two examined cases A(0) and A(1), considering the terms of the Circular Economy for environmental protection. Due to the combination of mandatory legal obligations that express the Circular Economy's requirements and technical parameters that each mining company should apply to satisfy the purposes of the 4Rs (reduce, recover, reuse waste), the CBA is implemented to convert all these factors into monetary terms while providing a detailed financial risk assessment in an objective mode. Further research focuses on detecting each case's techno-economic indicators, demonstrating their efficiency level. Indicators that are analyzed and assessed are the total CBA, LTP, and ERR for each case.

Moreover, the present study's innovative research is enriched with the practical role of the Stochastic Risk Assessment by converting nominal law requirements to mathematical parameters for the two examined scenarios to extract the appropriate business decision. The SRA of Bayesian Analysis aligns the results of financial risk assessment calculated through the CBA methodology for further forecast accuracy.

The SRA calculates the Risk Index (R) for both scenarios A(0): acceptance of penalty costs due to non-conformance with environmental protection requirements and A(1): investment in the metal recovery by mining tailings to be fully implemented with the European and global guidance regarding the Circular Economy. Thus, Stochastic Risk results provide high sensitivity to the CBA's indices, permitting the consecutive monitoring of the potential risk indices for both examined cases. Thus, the sustainability of each case is characterized by the minimum values of LTP and R, and maximum values of ERR and CBA indicators.

The CBA methodology considers legal obligations expressed through economic parameters for each current project. This study investigates how alternative mining waste management systems could be sustainable by recovering metals from mining tailings. Thus, CBA is applied to these projects' circumstances.

CBA tasks vary for every technical project, but the methodology tasks stay standard. So, CBA should fulfill the seven steps mentioned in section 1.2. The CBA process chain consists of the following targets, which should be determined regarding the pre-operation of tailings management (50) (71).

CBA targets to determine - Strategic Management (72)

1. Objective Definition

2. Scope Definition

3. Project Impacts/Monetary Evaluation

4. Identification and Responsibilities/Work Sharing of involved stakeholders

5. Financial risk assessment based on project impacts evaluation

SRA tasks accounting for the CBA results to provide a more accurate financial risk assessment approach. The targets that should be determined before the actual operation of tailings management and detecting the optimal business decision are the following.

SRA targets to determine - Strategic Management (4) (23) (54)

1. Accounting of Cost Functions calculated by CBA

2. Selecting the most appropriate statistical distribution that describes the actual conditions of the examined cases

3. Identification of Stochastic Financial Risk Assessment

4. Identification of the stochastic financial risk grade

5. Approval of the most optimal business decision

By this methodology, mining waste management infrastructure activities are socio-economically and environmentally feasible (71) (73). Infrastructures consist of recycling mechanisms, used equipment, human work teams, required plant installation costs, chemical reagents, etc. Mathematical formulas determine all these factors and describe their impact on the total project's cost economically.

These formulas are specific to each CBA target. The total chain of CBA provides the ability to identify optimal solutions by considering each target's equation results. These equations refer to the total goal functions for both examined cases. So, they are aligned with the Beta distribution through SRA to calculate the risk indicator for each Case Scenario. Further information regarding determining mathematical variables for both CBA and SRA is described in the Methodology Section.

During audit in Case Scenario A(0), the non-conformance might be detected or might not be. Therefore, in this case, the binomial distribution is the most suitable mathematical tool to describe the probability of non-compliance.

On the opposite side, Case Scenario A(1) has a high grade of conformance with legal obligations. Despite this, there is a risk of environmental pollution due to any potential leachate of hazardous waste in specific areas inside the industrial field. Therefore, the Binomial and Erlang distributions are applied to describe two major partial cases that refer to the real challenges that need to be assessed, respectively.

The binomial distribution is applied to prove the Probability Density Function when environmental pollution might or might not be detected during an audit control. The Erlang distribution is applied in the condition of standard non-conformance per audit control, so the potential penalty increase grows linearly (42) (74).

In Case Scenario A(1), each distribution refers to two conditions. The first condition refers to potential environmental pollution detection in different locations inside the industrial field, so the negative targeting adopts a stable numerical value. The second condition refers to potential environmental pollution detection in a specific location inside the industrial field, so the negative targeting increases annually. At the same time, there is a high degree of compliance with mandatory legal obligations.

Specific Probability Density Functions express all the probable cases of Scenario A(1). These functions are applied to extract the relevant negative goal functions that approach the total financial risks of Case Scenario A(1). Comparative evaluation of these goal functions guides the range of financial risk that refers to Case Scenario A(1) through CBA. Moreover, these goal functions are aligned through SRA to calculate the total risk indicators that describe each Case Scenario's conditions.

3. Research Methodology

3.1. Research Design and Approach of the Financial Engineering Risk Analysis Model

This PhD dissertation focuses on applying engineering risk analysis perspectives to assess the optimal business decision for a start-up mining company regarding tailings management, considering the legal obligations that express Circular Economy policies and are mandatory for the mining industry. The philosophy of the applied engineering risk analysis tool accounts for the results from similar scientific studies that support the innovative and alternative tailings management processes and environmental risks of mining waste (23) (41) (75).

It is logical to assume that a financial engineering risk analysis model for business decision-making should be applied before the start of mining operations to minimize the source of a huge volume of hazardous waste production without an action plan that meets the Circular Economy policy's requirements. Engineering risk analysis in business decision-making for mining activities allows a comparison between conventional and alternative tailings management processes based on technical and economic criteria (4) (15) (33) (76).

The conventional method of tailings management, known as dry-sludge, refers to the utilization of mining tailings as binding material in separate industrial applications such as the production of concrete mix, backfilling, stonewalling, etc., after the moisture removal. This conventional method involves tailings disposal until moisture removal is achieved (21) (77) . This corresponds to high environmental hazard sources and risks. There are occurred cases in which environmental damage cases caused by tailings disposal dam collapse is irreversible due to the acidic nature of the waste, and the placement of geomembranes is not an efficient method to avoid widespread environmental pollution (78) (79) (80) (81).

Hence, an indicative engineering solution to address the environmental risks caused by tailings disposal focuses on establishing closed industrial unit plants to treat the produced mining waste through metal recovery processes (18) (24) (82). This innovative methodology has already been implemented in Scandinavian regions and conforms with the legal obligations for sustainable mining declared in European Directives 2006/21/EC and 2008/98/EC. Application of this technique for tailings management, except for the ergonomic plan that depends on the quality of equipment to be used, requires a financial assessment to estimate the grade of sustainability.

A financial risk assessment is needed to prove that this innovative methodology (Case Scenario A(1)), which fully conforms to environmental protection legal obligations, is more economically effective than the conventional tailings disposal (dried-sludge) methodology. Considering the axiom that potential environmental damage from hazardous waste (such as mining activities' tailings) is catastrophic and irreversible, it is comprehended that a model of engineering risk analysis needs to be implemented before the actual mining operations occur. So, both methodologies, CBA-SRA, that structure the engineering risk analysis tool, have significant importance for the mining sector due to their ability to assess the optimal business decision for tailings management.

The financial risk assessment provided by the CBA offers an accurate approach to the total investment cost needed in Case Scenario A(1) compared to Case Scenario A(0). This financial risk assessment accounts for the probability of non-conformance that refers to each examined case, revenues from the utilization of recovered metals, and cost savings due to environmental legal conformance for the Case

Scenario A(1) to the opposite conventional tailings management methodology in Case Scenario A(0). Based on the financial risk assessment through CBA, specific economic indicators of sustainability that characterize the grade of technical and economic efficiency in terms of Circular Economy and corresponding legislation for each examined case are objectively calculated.

The current study, especially for Case Scenario A(1), examines five sub-cases that may refer to actual conditions in mining activities during environmental inspections. The financial risk assessment through CBA is provided for all these sub-cases, even though some may refer to purely theoretical approaches.

Considering the scientific methodology SRA of Bayesian Analysis, a transparent stochastic methodology is applied to verify the results of CBA. Considering statistical tools, the SRA methodology offers a stochastic financial risk assessment for detecting the sustainability ranking of each examined case. Moreover, the advantage of SRA is that it detects the degree of financial risk escalation identified by CBA.

3.2. CBA and SRA Case Studies Selection

Regarding optimal business decision-making about tailings management in the mining sector, a financial engineering risk analysis research approach is required to assess the sustainability grade among the two examined cases of conventional to innovative tailings management processes. Each methodological tool provides separate offers for the total objective evaluation. Corporation management systems of the mining sector should consider the overall risk analysis (including technical parameters and economic assessment) that focuses on their tailings management plan to minimize their negative environmental impact, as guided by the legislation of environmental protection that expresses the Circular Economy's principles before their actual mining activities.

The general guidance of investment projects indicates that CBA is the most appropriate economic appraisal tool (50). Thus, CBA methodology plays a significant role in the financial risk assessment of the examined cases of tailings management. The economic evaluation of each business decision should be considered before a start-up mining company requests a financial loan from funds or financial institutions.

The CBA methodology is indicated as a typical and valuable financial assessment tool in sustainable tailings management systems, which invest in closed unit plant systems that separate beneficial from waste material through recovery processing (3) (83). CBA could be applied to assess the grade of economic sustainability between conventional and innovative methodologies of tailings management, or between different processing approaches of metal recovery supported by closed industrial unit plants (16) (84) (85). Despite guidance on implementing CBA to evaluate each business decision in tailings management economically, its application in actual cases where CBA provided the corresponding sustainability indicators is limited to an extremely low range.

SRA methodology, through Bayesian Analysis in business decision-making, is an indicative method for detecting potential or expected risks expressed in monetary terms (4) (39) (54). As a statistical approach tool, Bayesian analysis estimates the potential or expected degree of risk that describes each examined condition. The applicability of Bayesian analysis to calculate stochastic risk grade depends on the natural parameters and their transition into numerical values for each assessed problem (42) (55). Therefore, in business decision-making through engineering risk analysis, due to the complexity of the combined sector (finance and engineering), a mixture of statistical distributions that refer to the natural parameters is indicated to provide an accurate approach (42) (56) (86).

The most helpful key finding from the application of SRA is the detection of the sustainability grade that describes the financial risk escalation calculated from CBA and refers to each case scenario. Accounting for this and the cost functions calculated by CBA, the verification of efficient points between the two examined cases is accomplished. Moreover, SRA results identify which of the sub-cases that refer to actual conditions are more probable to occur despite their assessed sustainable cost calculated by CBA. So, the SRA tool provides a higher level of sensitivity in the strategic plan of sustainable mining.

3.3. Methodological Framework for Cost-Benefit Analysis

The current study focuses on applying CBA to evaluate which of the two probable case scenarios (conventional or innovative tailings management methodology) is more appropriate for a start-up mining company to follow regarding tailings management, considering the principles of CE and the legal obligations that describe them. The engineering risk analysis tool structured by CBA takes into account the cost of the selected option that corresponds to the most sustainable decision in the total needed business development financial loan. Therefore, the applied CBA methodology refers to a deterministic financial risk assessment that includes the probable cost savings, benefits, and statistical approach of future economic efficiency that characterizes each case.

Therefore, CBA should imply the techno-economic findings needed as declared in the corresponding guidance for investment projects (50). These indicators, tailored to the project of tailings management needs, are the total benefit-to-cost ratio (B/C or CBA index), the long-term perspective, and the economic rate of return.

The B/C ratio corresponds to each examined case condition's total benefit and cost ratio. The benefit of each case refers to the difference between the cost of the opposite scenario and the outflow cost of the existing one. The benefit function includes costs, cost savings, and financial gains that describe each case condition. It is essential that the B/C ratio has a significant effect on the discount rate that a financial institution provides loans and funding to each mining company. A high B/C ratio corresponds to high or scalable discount rates. The discount rate of funding mining companies for sustainable mining growth depends on each financial institution's policy. Funding institutions' representatives evaluate the B/C ratio before funding, which is estimated (from the administration of a mining company) to be achieved. However, the B/C ratio is not a unique financial risk indicator that is considered to provide a financial loan. There are additional parameters, for instance, the preferred period of investment or repayment, the unexpected conditions of probable crisis situations in the market, and the unexpected political circumstances that may cause serious economic instability (30) (40) (50).

The LTP indicator refers to the cost function that represents the fluctuation of financial risk depending on the grade of compliance with the environmental protective requirements and the corresponding level of environmental risks. LTP indicator expresses the ratio between the cost function of the existing scenario and the opposite one.

The ERR indicator corresponds to the economic rate of return, accounting for cost savings and financial gains from utilizing recovered metals from mining tailings. Therefore, the ERR indicator corresponds to the difference between each case's full percentage of efficiency and the LTP indicator.

The LTP indicator is weighted by an important weighting factor that describes the mining company's corporate policy according to waste management and CE principles. However, the LTP indicator always has a more significant impact than ERR. The current study calculates this weighting factor with the minimum numerical percentage value to achieve a higher safety approach.

To assess all these CBA indicators that structure the financial risk assessment of technical parameters converted to monetary terms, the CBA methodology uses a deterministic statistical approach that describes each case's actual conditions and is applied to calculate the total cost. The CBA financial risk assessment involves the cost of penalties due to non-fully legal conformance in the sustainability evaluation of the first Case Scenario A(0), which refers to the conventional methods of tailings disposal and total investment cost to implement an innovative methodology of tailings management as in Case Scenario A(1), which refers to the alternative methodology of tailings management adopting a higher rate of legal conformance. The total cost functions for each case consider the potential benefits and cost savings, respectively.

3.4. Methodological Framework for Stochastic Financial Risk Assessment Models

Considering the cost functions calculated from CBA that describe each examined condition, the current study provides an innovative approach regarding the deterministic financial risk escalation rate supported by the Bayesian analysis of SRA. Despite SRA being a theoretical approach, it is an indicative tool to estimate the degree of risk fluctuations in a variety of scientific applications, especially in financial management and engineering (23) (42).

The philosophy of the SRA methodology is based on the axiom that if knowledge of a current condition and its deterministic terms exists, then it can easily be used to approximate a derived future condition stochastically (23) (55) (69). So, the methodological structure of SRA to be applied in business-decision making for the selection of the most appropriate and sustainable case focusing on tailings management requires two significant variables: the cost functions from CBA and the stochastic distribution that includes the risk probability of non-conformance and the corresponding working period of each condition, respectively.

Considering the CBA financial risk assessment cost functions, SRA identifies the increase or decrease in grades. Hence, SRA is applied to assess the economic sustainability ranking that corresponds to each case. To achieve this, it is necessary to assess the producing function of the corresponding cost functions multiplied by the suitable probability distribution and use the at-risk probability that refers to each case as a differential in the risk integral (56) (68) (69). Hence, the milestone for the application of SRA is the setting of the Risk Integral function, considering the physical parameters of each case to be examined.

Considering that during an environmental inspection, each non-conformance with the principles of CE might or might not be detected, the indicative statistical approach of the risky condition is through the binomial Probability Density Function and its prior, the Beta distribution, due to the occurrence of false-trial methodology (68) (69) (70). This condition happens by default in cases A(0) and A(1).

Especially in scenario A(1), to provide a greater safety level of approach, despite the non-conformance with CE principles that may or may not be detected during environmental inspection, in specific sub-cases, it is considered that the potential non-conformance is expected to occur during the working period, even though there is a high level of legal conformance. Therefore, in these sub-cases, the statistical approach of the cost function is based on the stochastic Erlang Probability Density Function. The prior distribution of the Erlang Probability Density Function, in normal conditions of expected risk within a period, is the Gamma distribution. However, due to the potential risk occurrence, applying a mixture of Erlang with Beta distributions is indicated to achieve a higher stochastic risk approach (42) (55) (56) (87).

Examining the two case scenarios A(0) and A(1), which refer to conventional and innovative methodologies of tailings management, respectively, the risk grade is calculated through the corresponding defined stochastic risk integrals considering the stochastic factors of the applied Probability Density Functions and their prior distributions. Multiplying the calculated stochastic risk grade by the relevant cost functions that describe each case, the total stochastic financial risk grade, including monetary terms, is finally calculated.

3.5. Life Cycle Assessment in Mining

Life Cycle Assessment (LCA) in mining involves five significant steps, from the primary detection of purified ore in metal materials to the final step of closure for mining activities. Hence, the first step of a mining life cycle refers to the geological research. The second step refers to the design of an action plan for mining activities. The third step refers to the construction of the exploitation sites. The fourth step refers to the actual productivity period of the mine site. The last fifth step refers to the mining closure actions (88) (89).

The LCA in the mining sector is a crucial factor for the entire business development in terms of "urban mining". Consecutive job offers and management opportunities focusing on marketing metal materials extracted from mining activities increase the regional GDP. Therefore, the necessity of alternative and innovative techniques to positively affect the mining sector's LCA should be adopted. Mining LCA is increasing while mining companies conform to the legal obligations of sustainable mining and follow the principles of CE (90) (91) (92).

Considering the five significant steps of the mining life cycle, financial gains for a mining company are realized only during the actual productivity of mining activities. The mining company loses money in all the other steps due to investment in geological research, designing and sloping the mining site area, and closing the active sites after exploitation.

Optimizing the life cycle of mining activities indicates investment in sustainable mining processes, such as reprocessing mining tailings (12). Moreover, reprocessing tailings to recover and reuse the metal materials implies the general guidance that principles the philosophy of CE through implementing the 4Rs policy Without sustainable tailings management practices, environmental inspections could dramatically reduce a mining company's life cycle due to the penalty costs for non-compliance with the corresponding legal obligations (9) (10) (54) (93).

Considering that tailings management processing should occur during the actual productivity period of mining activities, it is essential to mention that optimizing these procedures to maximize the LCA while minimizing the cost of mining closure is the most challenging part (2) (94). Hence, LCA in the mining sector needs the costs of sustainable mining practices to estimate each mining company's actual life cycle and how it corresponds to it.

Therefore, the financial engineering risk analysis methodology structured by CBA-SRA, considering the principles of CE in sustainable mining, has a significant role in both 'urban mining' and the development of LCA in the mining sector.

LCA methodology generally proves that innovative methodologies of metal recovery through reprocessing of mining tailings contribute positively to the risk minimization of acidic leachates as in conventional tailings disposal cases (24) (93). However, to accurately estimate the incremental rate of LCA in the sustainable mining sector, the operating cost of tailings management through metal recovery techniques should be calculated depending on actual conditions (tailings chemical process, tailings capacity, and concentration of metals into the tailings deposit). The LCA approach continuously compares the sustainable mining action plan with the conventional one.

3.6. Limitations of Research

Considering the significant role of mining activities in the industrial sector for satisfying human needs and respecting the terms and conditions of sustainable development in the mining sector, this PhD dissertation aims to prove that alternative and innovative methodology for tailings management is economically sustainable. Based on existing scientific research, there is a high environmental pollution risk where mining waste are freely disposed of to be dried and utilized as reusable material in separate industrial applications (3) (18) (23) (47) (95). Therefore, methodologies of reprocessing mining tailings are indicated as an appropriate business decision (16) (18).

To assess the sustainability of these indicated methods, a risk assessment tool that calculates their effectiveness and viability to be adopted by the mining sector. Hence, an engineering risk analysis tool that converts technical parameters, legal obligations, and existing scientific evidence into numerical values and monetary terms is valuable in front of the challenges for the new manufactured era of green growth.

The current PhD dissertation, based on the existing studies regarding the processing of metal recovery from mining tailings, presents an innovative upscaled case scenario (Case Scenario A(1)) that adopts the guidance of reprocessing tailings and is economically assessed to identify its total financial cost. Emphasizing the existing research gap about the practical cost of an actual tailings capacity reprocessing on an industrial scale by providing specific techno-economic sustainability indicators, the CBA is applied to assess the required sustainability indicators' numerical values while providing an entire techno-economic analysis estimation.

The techno-economic analysis assessment aims to prove the cost approach of such an innovative tailings management reprocessing by overrunning the examined case work layout to offer a high level of safe estimation. The overdesign of working activities ensures a high level of efficient recovery, described by higher investment costs. Moreover, the techno-economic analysis assessment supported by CBA evaluates the at-risk penalty costs and techno-economic indicators of sustainability due to environmental non-conformance with the mining sector's corresponding legal obligations for the conventional tailings disposal case (Case Scenario A(0)).

By applying CBA to the entire cost evaluation of these two Case Scenarios A(0) and A(1), the overall deterministic financial risk analysis is accomplished, providing comparative scientific evidence for detecting the point of efficiency between the two comparable conditions. It is essential to mention that in Case Scenario A(1), except for the practical sub-cases that may occur, theoretical sub-cases are examined, focusing on their financial risk estimation, too. To verify the deterministic approach provided by CBA, a stochastic simulative approach supported by SRA is applied, considering the cost configuration of CBA.

The SRA methodology describes the grade of the financial risk escalation demonstrated by the CBA. It emphasizes stochastic statistical variables (tools) to determine how each mining company’s corporation policy affects its financial risks, considering the principles of CE. Furthermore, the SRA methodology, as a purely scientific method, identifies which of the theoretical sub-cases are less probable to occur in actual conditions on an industrial scale.

Through SRA methodology, stochastic risk indicators that characterize each examined case are calculated, including monetary terms or purely involving the total risk probability of non-conformance. The stochastic risk results contribute positively to the comparative assessment for the grade of sustainability that describes each case condition.

The overall financial engineering risk analysis tool structured by the CBA-SRA configures the deterministic risk matrix, comparing stochastic financial risk indicators that refer to the corresponding at-risk costs of each assessed condition. The entire risk matrix provides evident data regarding the complex cross-examination of financial risk assessment between each examined case and sub-case.

4. Circular Economy adopted by Financial Engineering risk analysis for Tailings Management

4.1. Definition and Principles of Circular Economy in Mining

The philosophy of the circular economy supports the policy of the 4Rs and aims to utilize waste produced in every sector of human life. The final scope of the CE is for everyone to adopt the importance of minimizing the volume of produced waste and supporting the recovery and recycling processes. The last target is to reuse the recovered and recycled materials from waste (96) (97) (98) (99).

Based on that and following the guidance of sustainable development (5), especially in mining activities, legal obligations that indicate mandatory procedures that focus on mining waste management have been enacted by the European Union (9) (10), and as an expansion by all the European countries with strict severity proportional to the regional GDP, such as in Greece (64) (100) (101).

The principal targets of those obligations that express the CE policy are minimizing environmental pollution risks by enacting high penalties and environmental rehabilitation costs for the mining companies that do not conform to the policy of the 4Rs. Thus, the need for minimization of environmental pollution risks is very important in the life cycle of mining industries (89) (90). The cost of negative environmental impact and waste treatment is proportional to the concentration of metal ions in mining tailings (102).

Minimizing negative environmental impact consists of both heavy metal removal and acidic character reduction in the final waste disposal site (1) (43) (44).Heavy metals' ions, such as Cd, Cr, Pb, As, Ni, Cu, Zn etc., exist in mining tailings (103). These ions are dissolved in the final disposal site's aqueous environment as shown in Figure 2. The water of this place comes from mineral enrichment waste. As a result, already contains a low amount of chemical acids. By the addition of metal ions, chemical redox processes begin. So, the whole pH becomes lower. In addition to heavy metals, other metal types, such as precious, strategic metals, rare earths, etc., exist in mining tailings too (48) (97) (104) (105) (106) (107).

Figure 2 Pictographic view of the flow of leachates from the tailing dam of an iron ore mine to nearby surface water bodies (97)

Considering the nature of mining activities, CE's policy, as shown in Figure 3, focuses on minimizing unavoidable risk sources. The legal obligations of sustainable mining respecting the principles of circularity are an incentive for the mining industry to proceed with the adoption of alternative methods of managing and utilizing their produced tailings.

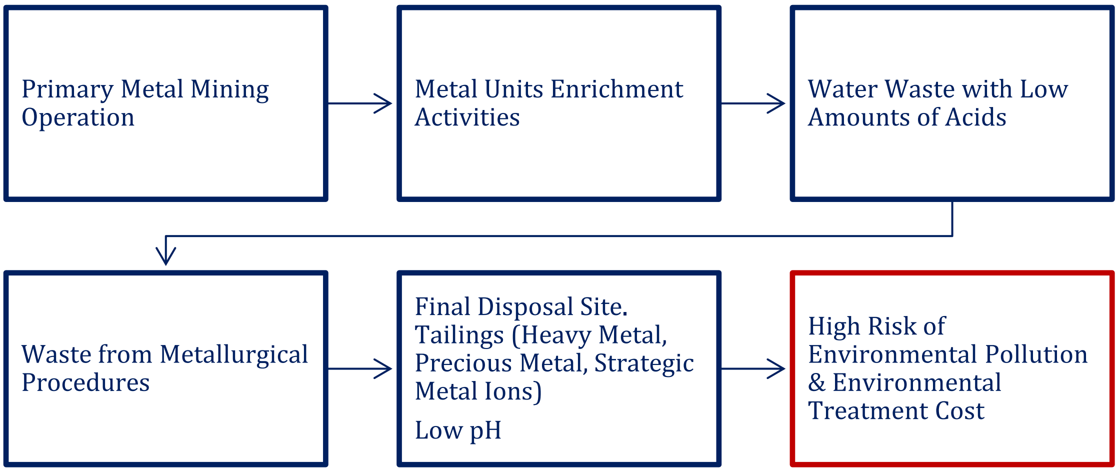


Figure 3 Origin of Environmental Risks in Mining Activities: implementing the conventional Tailings Management Procedures

Removal processes of the metal ions contained in tailings are required to minimize hazardous sources of environmental pollution (1) (41) (48) (62) (82) (108). CE policy, supported by the corresponding legal terms, obliges the mining sector to adopt environmental legislative requirements to avoid high financial risks due to the penalties of non-conformance or minimization of the life cycle of mining activities.

Non-compliance with environmental protection requirements refers not only to the detected environmental pollution during audit control by the authorized representatives of national environmental organizations. Non-conformance is characterized by four criterion probabilities that each of them corresponds to a specific legal obligation for a mining company. The four legal obligations that each mining company is required to implement are mentioned as they are declared by European Directives 2006/21 and 2008/98 (9) (10) and Greek Legislation (64).

1. Avoiding Uncontrolled and Systematic Disposal of Hazardous Waste on Ground Soil. In case of non-compliance with this mandatory legal obligation, the risky negative probability refers to probability $\boldsymbol{\theta}_{(1)}$.
2. Adoption of an Environmental Protection Policy according to the Implementation of Environmental Protection Mechanisms (Reduce, Recover, Reuse Waste) in terms of a Circular Economy by Investing in Mining Waste Management and Metal Recovery. In case of non-compliance with this mandatory legal obligation, the risky negative probability refers to probability $\boldsymbol{\theta}_{(2)}$.
3. Eliminate the hazard of the Environmental Pollution source. In case of non-compliance with this mandatory legal obligation, the risky negative probability refers to probability $\boldsymbol{\theta}_{(3)}$.
4. Avoiding a Huge Volume of Hazardous Waste to the Final Disposal Site. In case of non-compliance with this mandatory legal obligation, the risky negative probability refers to probability $\boldsymbol{\theta}_{(4)}$.

Weighting of the partial negative probabilities $\boldsymbol{\theta}_{(1)}$ $\boldsymbol{\theta}_{(2)}$ $\boldsymbol{\theta}_{(3)}$ $\boldsymbol{\theta}_{(4)}$, considering their impact to the total non-conformance as the law obligations declare it, extracts the total negative probability $\boldsymbol{\theta}_{(0)}$. The average of partial and potential negative probabilities calculates the total probability. This happens because the partial probabilities that are considered seem to have the same negative impact on the total non-conformance, so they have the same weighting factor equal to 0.25 (9) (10) (5).

According to the theoretical estimation, the total probability of non-compliance in cases of conventional tailings management, as in Case Scenario A(0), ranges between 0 and 100%. However, considering minimum risky conditions due to the lack of achievement for the 1st, 2nd, and 4th legal obligations expressed through $\boldsymbol{\theta_{(1)}}$ $\boldsymbol{\theta_{(2)}}$ $\boldsymbol{\theta_{(4)}}$ probabilities respectively, the range of the total probability of non-compliance is transitioned to 75-100%.

Based on the exact theoretical estimation in the cases of adopting an innovative indicated tailings management option, as in Case Scenario A(1), the total probability for non-compliance is configured considering maximum risky conditions due to the lack of achievement only for the third legal obligation expressed through probability $\boldsymbol{\theta_{(3)}}$. So, the maximum range of the total probability for non-compliance in Case Scenario A(1) is transitioned to 0-25%.

4.2. Principles of Circular Economy in Mining through the Financial Engineering Risk Analysis Tool of Business Decision-making

The financial engineering tool is supported by the deterministic approach of CBA and the stochastic approach of SRA, considering the CE policy and the legal obligations that express it. Both methodological approaches focus on detecting the sustainability grade of each business decision for a start-up mining company regarding the selection of tailings management among conventional and innovative processing.

CBA is based on the Benefit/Productivity Cost ratio, B/C. CBA's criteria are extracted from each project's main working activities (38) (40) (50) (98). This technical economic tool, which could be used to determine operational strategy, considers the real needs of each project. It is mentioned that the criteria for decision analysis/assessment are customized. As a result, the total exported guideline would apply to specific industrial working activities (109).

Therefore, the CBA method is selected over conventional methods, such as Multi-Criteria Decision Analysis (MCDA), Analytic Hierarchy Process (AHP), etc., as the most effective. The use of CBA in terms of circular economy, especially in mining waste management, is the most appropriate tool of assessment (50) (110).

Considering the axioms of CBA, mining waste management activities must be evaluated based on their techno-economic sustainability, considering their corresponding impact on the environment. This requires the hierarchy of operating procedures. All the operating tasks of CBA refer to the mandatory legal obligations with which each mining company should comply.

Regarding the selection of the most appropriate business decision for tailings management approach among the conventional one of free disposal to the innovative one, respecting the CE's requirements of reprocessing tailings, by the side of a mining company, CBA provides a comparative economic assessment of two Case Scenarios, A(0) and A(1) respectively, considering their configured costs. The total cost of Case Scenario A(0) refers to the penalty cost of non-compliance with the legal obligations regarding the policy of environmental protection and Circular Economy. The cost of Case Scenario A(1) refers to the total cost of operation, primary installation, and maintenance for a closed system of industrial units that waste treats mining tailings (11) (111).

The total productivity cost of Case Scenario A(1) consists of installation and operational activities regarding the metal ions removal (14) (43) (112) (113). Economic assessment for this Case Scenario contains a) cost for chemical reagents, b) cost for energy, c) cost for primary installation, d) maintenance cost for the whole equipment to be used, and e) benefit from the utilization of recovered metal materials (63) while the environmental rehabilitation cost is minimized (40) (50) (114) (115). By this assessment, the total investment cost is determined.

The CBA methodology provides a comparative financial risk assessment for the negative cost of each examined scenario A(0) and A(1). To determine the negative cost in each case, there is a necessity of the partial factors that configure it. For instance, in actual conditions for both cases, the total non-conformance might or might not be detected during the audit . This happens because, in site conditions, the environmental inspection could be accomplished in random locations inside the industrial field. So, in this case, the indicated statistical variable, which is the most suitable predictive tool for describing the negative goal achievement to estimate the density of risk probability's occurrence, is the binomial distribution expressing the trial-and-error methodology.

Despite this, especially for Case Scenario A(1) and its additive sub-cases, an alternative statistical tool that theoretically describes a worst-case condition for the expected risk probability density of non-conformance, even investing in environmentally friendly tailings management reprocessing, has been considered. This theoretical approach ensures the sensitivity of CBA results while offering the chance for further comparative analysis between the actual and theoretical sub-cases of the innovative Case Scenario A(1). So, in these sub-cases the most appropriate Probability Density Function is the Erlang distribution. Considering these circumstances, the separate sub-cases' financial assessments for the additive at-risk cost of non-conformance are based on the legislative configured penalty cost and the cost based on the total investment cost that characterizes the quality of recovery, respectively.

Hence, to provide a highly sensitive approach to potential negative goal achievement, both Erlang and Binomial distributions are considered and defined separately. So, the total negative cost is cross-examined based on the natural meaning of the parameters that configure it.

Even though tailings reprocessing for metal recovery procedures eliminates the hazard sources of non-conformance in Case Scenario A(1), there is still a probability of total non-conformance (with a lower value than in Case Scenario A(0)) due to only the probability $\boldsymbol{\theta}_{(3)}$.

The financial risk assessment, using CBA methodology, evaluates which scenario is the most sustainable, comparing total negative costs for Cases A(0) and A(1). The stochastic approach of SRA is implemented to cross-examine the CBA's deterministic financial risk assessment.

SRA detects the degree of financial riskescalation for each examined condition, considering the legislative requirements of CE. Its main scope is to verify the CBA's results, providing a high level of sensitivity. Calculated risk indicators from SRA provide the estimated grade of financial at-risk costs and the rate of their fluctuation during the working period.

To calculate the risk indicators, considering the cost functions of CBA, the prior distribution is needed to calibrate the multiplication between the stochastic Probability Density Functions and their prior distributions. The current study is based on the application of SRA in the mining sector, considering that the Sarmanov family distributions (BetE) are the most appropriate to define the risk grade for each condition (42) (55).

Due to the potential of risk detection during the environmental inspection in a mining company, the mixture of Beta and Erlang distributions, configuring their parameters with numerical variables that refer to industrial circumstances, is the most optimal stochastic statistical tool (42) (56) (87). Therefore, the prior distribution of risk probability's occurrence in all examined cases is the Beta distribution.

Implementing the SRA, which is a straightforward scientific upgrade risk assessment tool, verifies the deterministic results from CBA. Moreover, a comparative evaluation between these two approaches, CBA-SRA, shows their deviation. Based on this, detecting the less probable cases that may occur in actual site conditions is easy.

The SRA focuses on detecting the rate of financial risk that describes each examined case. At the same time, CBA emphasizes the cost-benefit evaluation that characterizes the most appropriate business decision, which implies the requirements of CE and the legal obligations that are mandatory for the mining sector. The engineering risk analysis tool structured from CBA-SRA provides a complete methodology for selecting and evaluating objectively the sustainability that characterizes the two main examined cases of conventional to innovative tailings management processes.

4.3. Strategic Plan of Circular Economy expressed through Legal Policy for the Implementation of Tailings Reprocessing in the Mining Sector

Figure 4 presents a strategic plan for engineering risk analysis tools based on the regulations and mandatory legal obligations expressing the CE policy. The engineering risk analysis model, focusing on the optimal business decision-making for a mining company regarding its produced tailings management, is structured by four separate criteria that determine the two significant targets of sustainable mining due to the deepening thoroughness of the CE's principles (9) (10) (41) (3) (2).

The two targets that need to be achieved are minimizing environmental risks, such as environmental pollution or costs due to penalties for non-conformance and rehabilitation costs, and maximizing the recovery process to expect potential financial gains by utilizing beneficial materials in tailings. This dynamic strategic plan ensures the technical and economic sustainability of the selected business decision by covering all the requirements of green growth.

Further analysis regarding the definitions of the partial aims and scopes describing each criterion is conducted separately. It is essential to mention that despite both methodologies of CBA and SRA serving in detecting separate indicators, their synthesis provides the unique scope of detecting the sustainability grade among the examined cases in actual industrial conditions. Considering the deterministic results of the risk matrix, which are configured by the deterministic and stochastic approaches, a complete appraisal tool for the optimal business decision selection in the mining sector is provided. This financial engineering risk analysis tool can be utilized even by mining companies or financial institutions that offer financial loans for green development to increase their life cycle.

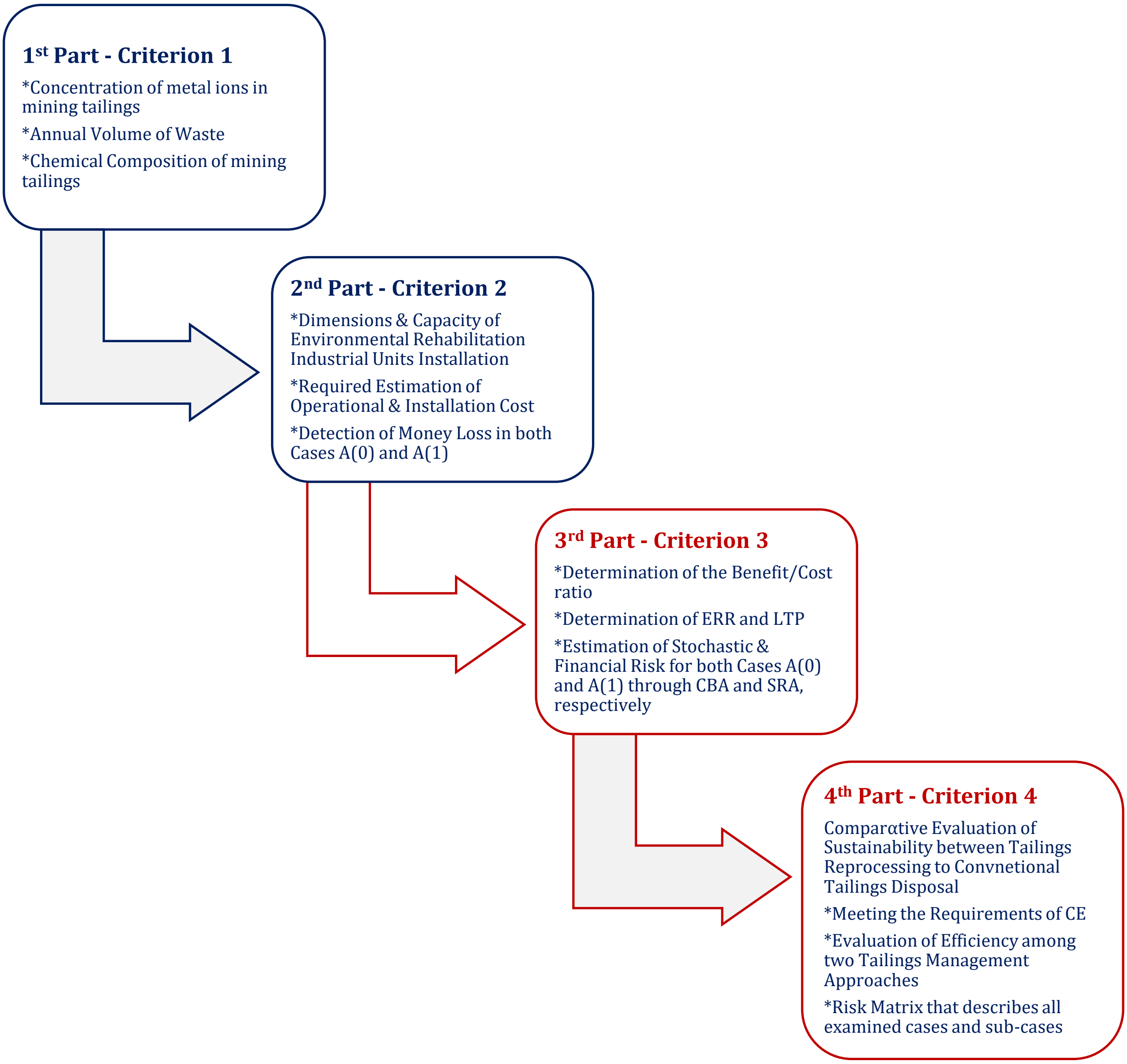


Figure 4 Hierarchy of CBA & SRA Structural Criteria that support the Entire Financial Risk Assessment among Conventional to Alternative Methodologies regarding the Tailings Management

4.3.1. Defining the 1st Criterion and Description of the Problem

Criterion 1 focuses on detecting the total mass of acid mine drainage (AMD) that needs to be treated. For both examined cases, the annual volume of mining waste, their chemical composition, and metal ions' concentration are estimated according to indicative experimental scientific studies (1) (116) (117). The dissolved Base Metal Ions (BMI) in Acid Mine Tailings (AMT) are Fe^{2+}, Fe^{3+}, Al^{2+}, Al^{3+}, Mn^{2+}, Cu^{+}, Ni^{+}, Ni^{2+}. Dissolved Precious Metal Ions (PMI) in AMT are Ag^{-2-+3}, Au^{-3-+5}, Pt^{-3-+6} (96).

Major Strategic Metal Ions (SMI) in AMT are Li+, Co^{-3-+5}, Ta^{-3-+5}, Pd^{0-+4}, Nb^{-3-+5} etc. SMI consists of Rare Earth Elements (REEs) such as (La, Ce, Pr, Nd, Pm, Sm, Eu, Gd, Tb, Dy, Ho, Er, Th, Yb, Lu, Y, Sc), too. Most of them present as chemical oxides in downstream of AMT (55). Plenty of BMI and PMI are existing near acid mine drainage (AMD), as native diluted ions. Lower concentration of them, as chemical salts, is existing in downstream place, as shown in Figure 5 (1).

Metal ions' different range of concentration depends on their specific weight and the ability to form chemical bonds with non-metallic elements. The presence of low pH levels in tailings, has a catalytic impact on ionic disintegration procedures. The positive metal ions, which exist in high concentrations, are used to make chemical bonds with acidic roots such as SO_3^{2-}, SO_4^{2-}, CO_3^{2-}, NO_3. Thus, produced chemical salts are used to precipitate in the downstream area of the final deposit place.

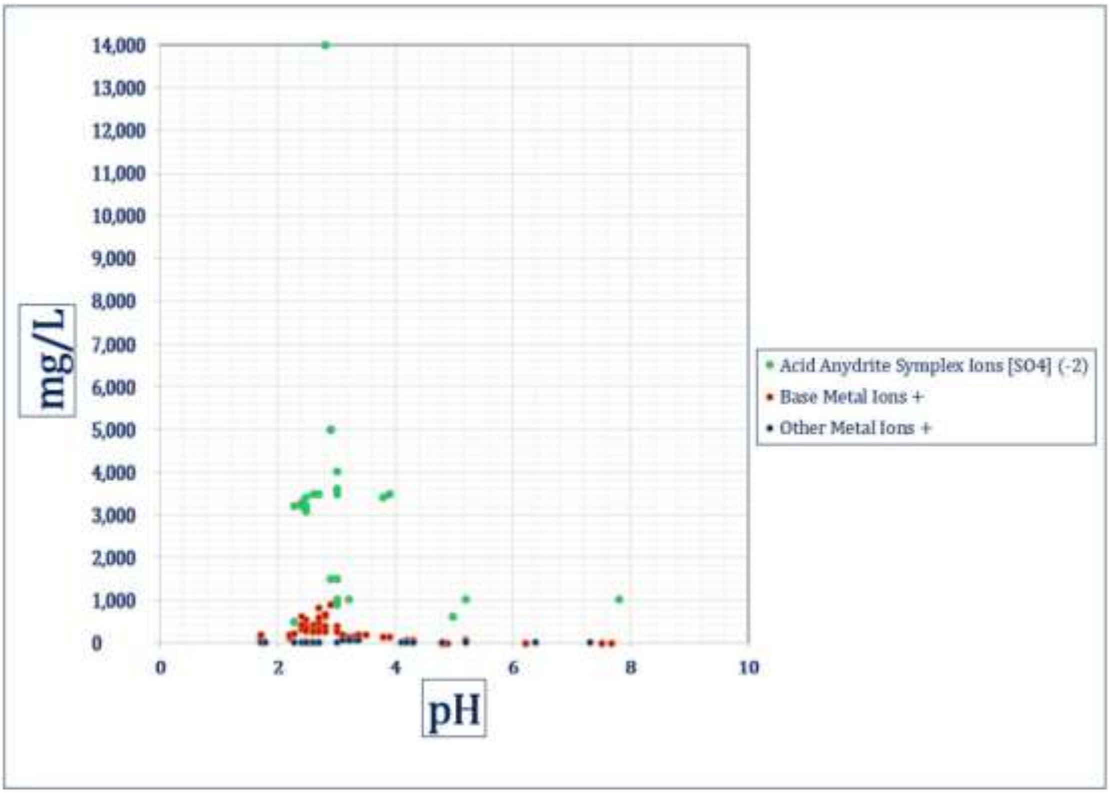


Figure 5 Distribution of metal ions as a function of pH of water samples collected near mining sites and from downstream rivers (based on (1))

4.3.2. Defining the 2nd Criterion

The second criterion involves all the required parameters regarding the cost estimation of installation and operation for a closed system of industrial units that supports metal recovery from mining tailings, such as in the proposed Case A(1). In contrast, the approved financial risk by each start-up mining company that does not conform to the legal obligations is assessed, as in Case A(0). Regarding these, to provide an accurate estimation of the potential money loss in both examined cases, the configuration of the Loss Function (LF) is needed. LF aims to determine the monetary values of all techno-economic parameters of both cases. The significant parameter of Case Scenario A(0) refers to the configuration of penalty cost for non-conformance. In the opposite Case Scenario A(1), the most significant parameters are the total cost of investment and total benefit.

The LF that refers to Case Scenario A(0) in the current study is configured according to the guidance of the relevant Greek Legislation (64) that meets the requirements of the European and Global Environmental Protection Policy expressed through the corresponding Directives (9) (10) (8). Due to the acidic and toxic behavior of mining tailings, the economic penalty is proportional to a) the Category of Risk classification for mining companies, b) the severity of hazardous solid & liquid waste, and c) the total compliance of the company in terms of Circular Economy and Environmental Protection Mechanisms.

The LF that refers to Case Scenario A(1) is configured according to the partial costs, configuring the total investment cost. These costs are a) the cost of primary installation for the equipment to be used, b) the cost of maintenance, c) the cost of operation that contains the cost of chemical reagents and the cost of energy, and d) the benefit gained by the utilization of the recovered metal mixture.

4.3.3. Defining the 3rd Criterion

Regarding the CBA methodology, two techno-economic indicators significantly affect total sustainability. These indicators are the LTP and ERR. Criterion 3 focuses on detecting these indicators for both examined cases. The weighting factor for each one of them is proportional to the expectations of each mining company's administration. The current study considers that these indicators have the same severity on total sustainability. The LTP indicator refers to the grade of minimization for the financial risk of both cases. So, the indicator ERR is expressed by the difference between the numerical value 100% and LTP in each case.

Moreover, the Benefit-to-Cost ratio detection calculates the final CBA indicator for Cases A(0) and A(1). To identify this ratio, the configuration of the negative goal achievement function should be expressed through the Goal Function (GF) in each case. GF is configured by the LF and Probability Density Function (PDF) (118). The role of PDF is to assess the total negative probability of achievement for each Case Scenario. So, by determining the financial risk through GF, the total financial risk is estimated in both cases (119).

As a result, CBA evaluation by the 1st, 2nd, and 3rd criteria provides an accurate approach to economic assessment of the total cost of a) metal recovery operation from mining tailings, as in Case A(1), and b) compliance and non-compliance with required legal environmental safety measures. CBA's evaluation could provide an accurate financial risk assessment of total economic benefits from a) the reuse of recovered base and precious metal raw material, b) the reduction of environmental pollution by the implementation of recovery mechanisms, and c) REE's recovery and reuse in terms of industrial utilization while offering economic and social gains to each cooperative agency.

The SRA methodology is applied to align the GF of financial risk assessment for Case Scenarios A(0) and A(1). Therefore, after defining the natural meaning of the GF's parameters with mathematical variables for each case, the final integral that demonstrates the risk grade for both cases is configured (53) (95) (86).

4.3.4. Defining the 4th Criterion

The techno-economic risk indicators are calculated separately for each Case Scenario using SRA. These indicators refer to stochastic financial risk and purely stochastic risk assessment, which characterizes the sustainability grade for Case Scenario A(0) and Case Scenario A(1) (including its sub-cases), respectively.

The SRA approach detects the degree of financial riskescalation. Therefore, the comparative evaluation of these results detects the sustainability rate of each case compared to the others. Hence, the risk matrix configured by the stochastic approach is a valuable tool for verifying the financial risk assessment of all examined cases.

Moreover, the exported risk matrix is an applicable managerial tool that demonstrates the comparative risk grade ratios considering economic parameters that express legal obligations and technical factors in terms of the 4Rs policy.

4.4. Cases of Circular Economy in Mining Waste Management

Considering the negative impact that free disposal of mining tailings may cause on the environment and human health, the principles of CE have been enacted through the corresponding legislation that is mandatory to be implemented by the mining sector (3) (9) (10) (23) (25) (41). Emphasizing these indicated procedures of alternative tailings management, scientific research has been developed to optimize processes of metal recovery from mining waste by reprocessing (2) (14) (103).

Mining tailings reprocessing has already been developed and adopted in Scandinavian regions, especially Finland (18) (24). Considering the severity of metals in human life and the industrial life cycle, alternative methodologies for the utilization of metal materials contained in mining tailings have been applied on an industrial scale, too (18) (25).

In the face of climate change and green transition, the necessity of specific metals for specific industrial applications and their supply risk makes them economically important. As a result, these metals, due to their criticality, are classified as critical raw materials (CRMs) (30). The CE policy supports the guidance for recovery and utilization of all flows of waste, even though mining waste, to be utilized and reprocessed to extract the beneficial materials they contain (24) (62) (83).

More specific practical cases have shown that recovery efficiency in lead and zinc has been limited to approximately 90% from mining tailings (17). Additional case studies conforming to the CE policy have resulted in the efficiency of precious metals recovery above 90% (19) (120). Case studies regarding REEs recovery from mining tailings have shown that the efficiency of recovery could be over 93% using the optimal process of acids and catalysts (82) (121).

Several mining companies in Finland, adopting the CE policy, have achieved the following rates of recovery efficiency (18):

- 75% of Chrome concentrates at the Kemi mine site, implementing gravity separation
- 92-95% of Gold at the Kittilä mine site, implementing flotation and oxidization by the addition of acids and catalysts
- 95-96% Copper and 92-93% Zinc at the Pyhäsalmi mine site, implementing a flotation technique involving chemical reagents and catalysts
- 80% Nickel and Zinc at the Talvivaara mine site, implementing bioleaching and chemical reagents
- 80-85% Gold at the Vammala mine site, implementing gravity separation and flotation
- 58% Nickel at the Lahnaslampi mine site, implementing gravity separation with no chemical reagents or catalysts

Considering these applicable techniques by mining companies that adopt the philosophy of Circular Economy, it is comprehended that recovery efficiency may differ between separate mine sites due to their chemical contents and the reprocessing method of tailings management. So, there is practical sustainability of the CE adoption by the mining sector by implementing innovative methodologies of tailings management as in Case Scenario A(1) which are evaluated by this .

5. Method of Metal Recovery from Mining Tailings

5.1. Stereochemistry of Rare Earth Elements and other metals. Its Role in Recovery from Mining Tailings

Metal recovery is based on the stereochemical engineering mechanism of chemical compounding of chemical salts between metal ions and complex anions of acidic anhydrite (33) (34). The electron structure of metals has significant importance on their stereochemistry and their ability to form chemical bonds with other chemical substances, such as acids and reagents. The hierarchy of electron shells and sublayers occupation provides the exact number of outer shell electrons. As a result, principal quantum numbers are extracted, so it is easier to comprehend the layout of hybrid orbitals. The hierarchy of both electron shells and sublayers' occupation is shown in Figure 6. The growth of orbitals is shown in Figure 7.

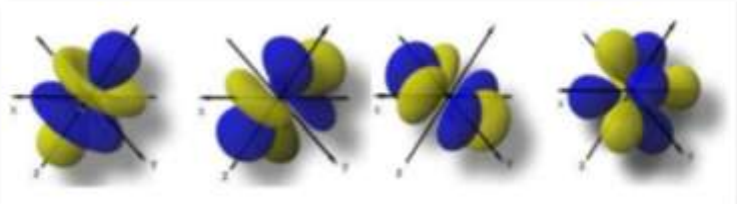


Figure 6 Growing ''f'' Orbitals in Spatial 3d System a) f_{z^3}, b) f_{xz^2}, c) $f_{x(x^2-3y^2)}$, d) f_{xyz} (122)

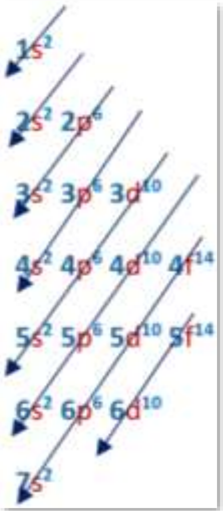


Figure 7 Occupation of electron shells and sublayers

The Electron shells **K-Q** are equal to numbers 1-7, and the sublayers of each one are symbolized with letters s, p, d, and **f**. The quantum number of major electron shells is "n." The quantum number of each sublayer is mentioned with the symbol "**l**". The magnetic quantum number "ml" defines the orbital's spatial orientation in terms of a 3d system. The quantum number "ms" defines the spin direction of a single electron in each electron pair. The maximum number of electrons for each shell's sublayer is a) 2 electrons about sublayer ''s'', b) 6 electrons about sublayer ''p'', c) 10 electrons about sublayer ''d'', and d) 14 electrons about sublayer ''f''.

The number "**ml**" is proportional to the number "**l**". Sublayer "**s**" equals to **l**=0, sublayer "**p**" equals to **l**=1, sublayer "**d**" equals to **l**=2, and sublayer "**f**" equals to **l**=4. The range value of **ml** takes integer numbers from negative to positive "**l**". So, REEs which have both "d"and "f" sublayers have a range of quantum numbers "**ml**" [(-2)-(+2)] and [(-3)-(+3)] respectively. When the "**l**" number is characterized as "**d**", the limit down and limit up of the number "**ml**" are -2 and +2, respectively. When the ''**l**" number is characterized as ''**f**", the limit down and limit up of the number "**ml**" are -3 and +3, respectively.

The electron structure of REEs is expressed by hybrid orbitals of "d & f" sublayers. Thus, their position in the Periodic Table is the ''d & f' section (3^d.secondary group and 4^{th} & 5^{th} period). All REEs from La – Lu (lanthanides) and from Ac – Lr (actinides) have 0 – 14 free electrons in their outer sublayer, respectively. According to the REEs electron structure the number of their possible "**d** & **f**" orbitals in a 3d system can be estimated (122). The stereochemical orbitals' system tends to be more stable due to its homogeneous growth on all 3 axes dimensions x,y,z. Thus, the abundance of single external electrons of chemical compounds, which are able to make chemical bonds with rare earth elements, is proportional to the stereochemical stability of the produced orbital system.

REEs have similar physicochemical behavior, especially in forming chemical bonds with negative complex ions of acidic roots. REEs' single external electrons can make electron pairs with other bonding electrons. This electrochemical phenomenon is exploited to recover REEs in the aqueous environment by adding chemical acids.

All metals tend to reject their single external electrons and make chemical bonds with nonmetal elements, which receive these electrons, too. For example, metal elements form a chemical bond with phosphoric root $(PO4)^{-3}$ complex ion in a chemical reaction between phosphoric acid and a mixture of REEs. The final product is [Metal-(PO_4)] chemical salt.

Example $3M^x + xH_3PO_4 \longrightarrow M_3(PO_4)_x + 3x/2H_2$

Considering this chemical mechanism, it is comprehended that the use of an acidic aqueous environment has a catalytic impact on the production of metal-chemical salts. So, in metal recovery procedures, the main factors that need to be combined/ calibrated are the optimal amount of acid in a specific temperature environment (for each metal) with the ideal mature time (33). In an acidic environment, the mobilization of metals' external electrons is higher due to their tendency to make orbital chemical bonds with complex ions of non-metallic acidic roots. Thus, the required Ionization Energy ($\mathbf{E_i}$) in a low pH environment is lower than in a higher pH environment.

As a result, the most catalytic chemical mechanism for recovering all types of metal from mining tailings is the addition of organic and inorganic acids. This chain mode of recovery should take place inside industrial units, which must be monitored, and the whole operation should be under control.

Each metal element's recovery efficiency varies according to the type of acid used, the acidic range concentration, and the interaction time. It has been reported that the most efficient acid for REEs' recovery is Di-Ethylhexyl phosphoric acid (**D2EHPA**), which can enrich the concentration of scandium and other REEs (*Sc is the most expensive metal of all REEs (123), and separating it from other metal materials (33) (124) (125). Separation and enrichment of REE concentration is due to the formation of the corresponding chemical salts (125) (126).

For example, the chemical bond between Nd with the phosphoric acid root $(PO4)^{-3}$ is $Nd_3(PO_4)_4$. The electron structure of Nd is $[Xe]4f^4\,6s^2$. In the chemical compound of $Nd_3(PO_4)_4$, $4f_{x^3}$,$4fy^3$,$4fz^3$,$4f_{xyz}$ orbitals of Nd tend to grow their dimensions in a 3d spatial system by forming lateral bonds with external electrons of $(PO_4)^{-3}$ complex ion. Table 1 presents the explanation of quantum numbers and orbital characteristics.

Table 1 Explanation of Quantum Numbers and Orbitals' Symbolism

Quantum Numbers			Orbitals		
Electron Shell n	**Sublayer l**	**Magnetic Quantum number ml**	**Character**	**Number**	**Orbital's Symbol**
1	0	0	s	1	1s
2	0	0	s	1	2s
	1	-1,0,+1	P	3	$2p_x$,$2p_y$,$2p_z$
3	0	0	s	1	3s
	1	-1,0,+1	p	3	$3p_x$,$3p_y$,$3p_z$
	2	-2,-1,0,+1,+2	d	5	$3d_{yz}$,$3d_{xz}$,$3d_{xy}$,$3d_{x^2-y^2}$,$3d_{z^2}$
4	0	0	s	1	4s
	1	-1,0,+1	p	3	$4p_x$,$4p_y$,$4p_z$
	2	-2,-1,0,+1,+2	d	5	$4d_{yz}$,$4d_{xz}$,$4d_{xy}$,$4d_{x^2-y^2}$,$4d_{z^2}$
	3	-3,-2,-1,0,+1,+2,+3	f	7	$4f_{x^3}$,$4fy^3$,$4fz^3$,$4f_{xyz}$, $4f_x(z^2-y^2)$,$4f_y(z^2-x^2)$,$4f_z(x^2-y^2)$
5	0	0	s	1	5s
	1	-1,0,+1	p	3	$5p_x$,$5p_y$,$5p_z$
	2	-2,-1,0,+1,+2	d	5	$5d_{yz}$,$5d_{xz}$,$5d_{xy}$,$5d_{x^2-y^2}$,$5d_{z^2}$
	3	-3,-2,-1,0,+1,+2,+3	f	7	$5f_{x^3}$,$5fy^3$,$5fz^3$,$5f_{xyz}$, $5f_x(z^2-y^2)$,$5f_y(z^2-x^2)$,$54f_z(x^2-y^2)$
6	0	0	s	1	6s
	1	-1,0,+1	p	3	$6p_x$,$6p_y$,$6p_z$
	2	-2,-1,0,+1,+2	d	5	$6d_{yz}$,$6d_{xz}$,$6d_{xy}$,$6d_{x^2-y^2}$,$6d_{z^2}$
7	0	0	s	1	7s
	1	-1,0,+1	P	3	$7p_x$,$7p_y$,$7p_z$
8	0	0	s	1	8s

Figures 8, 9 demonstrate the orbital types s, p, d, and f in the 3d system. Figure 10 shows the steps of metal recovery that were followed in the most optimized existing scientific experimental study (33).

In dissolution pH of 0.35 and D2EHP with an acidic range 1-11 M, 98% of Sc was extracted, whereas iron(Fe), yttrium(Y), and lanthanides(La) were left in the tank solution (33). The final extraction order was $Sc^{3+} > Fe^{3+} > Lu^{3+} > Yb^{3+} > Er^{3+} > Y^{3+} > Ho^{3+}$.

After the first separation, P_2O_4 was dissolved in a mix of hydrochloric and nitric aqueous acid (HCL & HNO3)aq, with an acidic range of 1-5 M, to extract and separate iron and other metal elements. The second separation achieved the extraction order $Sc^{3+} > Th^{3+} > Ce^{3+} > Lu^{3+}$ (33).

After the second separation, P_2O_4 was dissolved in sulfuric aqueous acid $(H_2SO_4)_{aq}$, with an acidic range 1-5 M, to extract and separate amounts of cerium, thorium and lanthanum from other metal elements. The third separation achieved the extraction order Sc> Ce> Th > La (33).

This is a typical recovery procedure that uses a variety of acids in a perpetual mode of 2,3, or 4 steps to achieve the highest ratio of recovered material (32) (33) (34) (124).

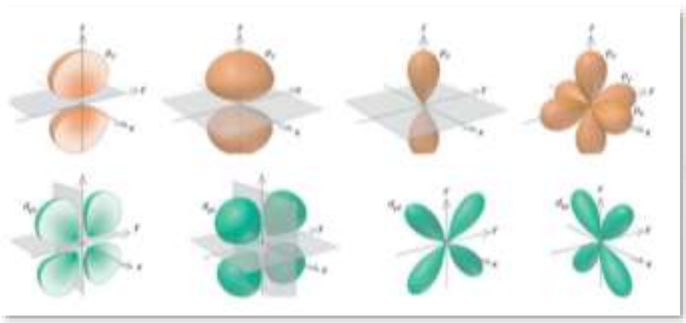


Figure 8 Isomeric of orbitals type f in 3d spatial system (122)

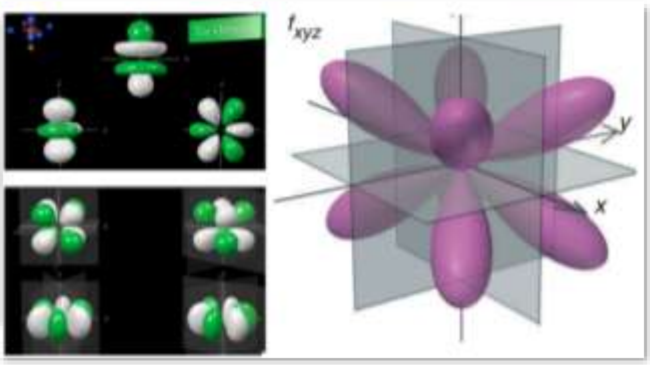


Figure 9 Orbitals s, p, d in spatial 3d system (122)

Figure 10 Separation and Purification Procedures of REEs (33)

Repetitive procedures of acid addition and filtration achieve the purification/Enrichment stage. The precipitated chemical salt of each step is the REE/other metals-enriched cake. The floating product of each phase of enrichment is input into the next purification procedure (22).

The whole philosophy of metal recovery is based on a) their electron structure b) ionization of metals' external electrons and its impact on forming chemical bonds with non-metal elements, b) the orientation of orbitals while they are tending to get a more stable 3d structure, along all directions of 3d system, by making lateral bonds with non-metal complex ions.

5.2. Mechanism of Metal Recovery from Mining Tailings

Recovery is a crucial factor in Mining Waste Management. The ability to ensure high amounts of recovered metals from abandoned mining tailings contributes positively to environmental protection and the reuse of waste in terms of the Circular Economy, meeting the requirements of relevant International and European legislation.

Operation cost is one of the most inhibiting reasons for the implementation of Recovery activities. This enhances the scientific importance of the exploration of engineering techniques to increase the efficiency of the whole procedure (127) (128). As a result, metal recovery could be more sustainable on an industrial scale.

Metal Recovery's Efficiency is determined by the pH, Temperature, Concentration in acids (metal's ability to form stronger chemical bonds by producing complex chemical salts), and the gravitational force of metals (due to their specific weight while precipitating in the downstream area) (1) (116) (129).

According to scientific studies, the recovery efficiency is proportional to the addition of acids, catalytic organic-inorganic compounds, and microbes (22) (32) (49) (123) (130) (131). Aside from the presence of chemical reagents and microbes, an important role is the optimal residence time in which plenty of metals create complex ionic structures and precipitate downstream.

Figure 11 presents the efficiency of REEs and iron recovery (22). The mechanism involves four steps of recovery. The floated liquid material from the first step is input into the next one. This flow sheet is repeated three more times. In the first step, aqueous dissolution is used with deionized water and chemical acids (H_2SO_4 and HNO_3 1% w/w with C=1-2M).

In the second, third, and fourth steps, an aqueous dissolution by deionized water and chemical acids (H_3PO_4 and HCL 1% w/w with C=1-2M) is used. The ratio of aqueous dissolution's mass and tailing's volume is 1gr/Lt in all recovery steps. Catalytic compounds such as Cyanex 572, Cyanex 923, and D2EHPA accelerate the downstream of chemical salt production (34). The ratio of acids could vary from 1-5M and is proportional to the catalytic compounds, preference time of recovery, etc. (33) (132). Figure 11 shows the mechanism of filtration of REEs using NF membranes, and the corresponding ratio recovery efficiency using three different catalysts.

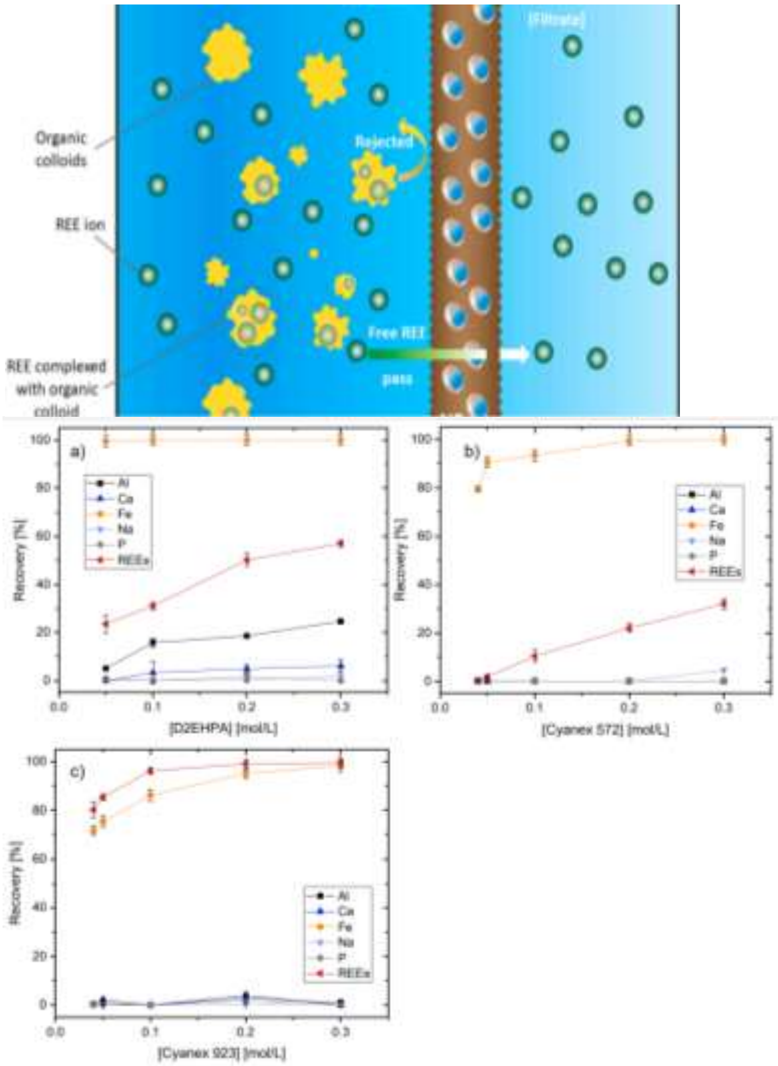


Figure 11 Efficiency of Metal Recovery using catalyst concentrations from 0,04 to 0,3 moles/Lt a) D2EHPA, b) Cyanex 572, c) Cyanex 923 (33) (34)

REE recovery efficiency up to 95 % is achieved by the 4 stages of oxidation, using H_2SO_4 & HCL (in the 1st step), HCL & H_3PO_4 (in the 2nd , 3rd, and 4th steps). The entire procedure is conducted in a temperature range of 18-22 ^oC . The catalysts' role could be replaced by the action of microbes/bacteria (99) or other organic compounds (72) (77). The presence of microbial bacteria reduces, in high amounts, the cost of chemical mass flow (83).

In other experimental studies, it is mentioned that the efficiency of REEs and other metals recovery up to 80-95% is achieved by the combined use of chemical (organic compounds, catalysts, acids) and biological reagents mix and nanomembranes NF (33) (32). NF used in recovery procedures is to filter clear metal ions or acidic roots and separate them by the chemical salts' complex ions. In the following step, the extra use of chemical and biological reagents increases the ionic separation of metals by non-metals, making the potential use of a nanomembrane more efficient. Table 2 shows the recovery efficiency of Germanium and other REEs at specific pH conditions using other catalysts.

Table 2 Critical Raw Materials-REEs extraction by stainless steel microfiltration membrane (32)

Membrane	Extractant	pH	Rejection Efficiency	Rejected REEs
Stainless steel with a pore size of 5 μm	2-Ethylhexyl-phosphoric acid-2-ethylhexylester (EHEHPA)	3,33	94	La
		3,14-3,63	99	Ge
		>3,5		Pr
		4	>90, (91-99)	Nd, Eu, Er, Y, Sm, Gd, Dy

According to the relevant scientific experimental studies (33) (49) (32) (122), the achievement of high recovery effectiveness requires the optimal combination of:

a) Concentration of chemical reagents (M) and content (%w/w, or %w/v) in an aqueous environment
b) presence of microbes
c) temperature range
d) time of residence
e) RPM number of the blender when a stirred tank reactor is used
f) COD, chemical oxygen demand

The optimal combination mechanism of the parameters (chemical & biological reagents, temperature, time of residence, etc.), except for the recovery's efficiency increment, provides lower energy consumption. Based on the scientific results (32) (133) (134) (135), the sustainability of the whole recovery operation assumes optimization of:

a) of chemical and microbe/bacterial reagents mass-flow ratio
b) of energy consumption and required equipment

On an industrial scale, recovery mechanisms are implemented in three or four stages. Each stage involves pH reduction, downstream of chemical salts, filtration, and drying of sunk material. The floated material from stage one is input to the second enrichment stage. The next chapter focuses on the recovery mechanism description in the proposed Case Scenario A(1).

5.3. Comparative Analysis of Metal Recovery Techniques

The innovative approach of reprocessing mining tailings involves separate techniques for the final metal recovery procedure. The following are the most common and efficient techniques of metal recovery from mining tailings (3) (33) (34) (82). (109).

- Gravitational separation
- Floatation separation
- Sink/Float separation using chemical reagents
- Sink/float separation using catalysts (cyanide or phosphates)

The tailings deposit ore involves plenty of metals as trace elements, so the separation between each other is the most challenging part of the tailings reprocessing (1) (2) (117) (17). Hence, from a practical point of view on an industrial scale, a mining company that utilizes its produced tailings should apply the most optimal chemical engineering mechanism to ensure the higher efficiency of metal trace elements removal.

The chemical reaction that is required to separate metal trace elements from the tailings involves the occurrence of chemical salts among the anhydrite of acids and the metal cations (33) (34).

The produced chemical salts are used to precipitate downstream. The metal mixture of trace elements is then purified and can efficiently be utilized as a selling material for the global metal marketplace. The presence of catalysts, despite not being characterized as an environmentally friendly technique due to the hazard caused by the occurrence of cyanide, rapids the creation of chemical salt compounds (3) (14).

From the point of view of a cost-effective solution, the occurrence of catalysts is an indicative method. However, considering the hazard source in an emergency where environmental risks are incredibly high, mining companies that adopt the CE policy to increase their life cycle implement a mix of acidic addition and phosphates to decrease the proportion of required cyanic catalysts. Moreover, the whole processing is indicated to be applied in an entire system of closed unit plants, with no disposal to ground soil layers armed with geological barrier sub-layers or geomembranes (16) (17) (18) (78), as obliged by the corresponding legislation that supports the policy of the Circular Economy (9) (10).

Reprocessing of tailings could occur in conventional cases of free disposal of tailings in ground soil layers protected with geomembranes or geological sub-layers with extremely low porosity. In these cases, reprocessing is achieved by the gravitational separation method using an underground pipeline system. Through it, the precipitated tailing dissolution flows to the filtration stage. However, due to the acidic behavior of the tailings capacity and unexpected weather conditions, such as heavy rainfalls, there is a high-hazard source of leachate detection (78) (3) (2). At the same time, there is no legal conformance due to the prohibition of disposing of tailings over geological structures (9) (10) (64).

Hence, the innovative approach of implementing tailings reprocessing through closed industrial unit plants using chemical reagents and catalysts is an indicative method of sustainable mining that follows the principles of CE and the mandatory legal obligations. The challenging part of the optimization for chemical engineering processing of tailings management is proportional to the quality and chemical contents of each mining site's tailings (34) (111) (125) (127).

It is essential to mention that on an industrial scale, for a mining company, eliminating hazardous sources of environmental risks while optimizing the recovery processing to utilize the removed metal mixture from the tailings rapidly is valuable, respecting the guidance provided through the project and risk management schedule plan (46) (54) (36). Therefore, the standardization of chemical engineering procedures to reprocess tailings depends on the content of metal ions. The reprocessing cases that have already shown a high level of recovery efficiency involve the addition of both acids (such as HCL, H_2SO_4) and catalysts (P_2O_5 or Cyanex) implementing the sink/floatation method where the metal chemical salts precipitate and are filtered in thickeners (16) (18) (33) (34).

6. Cost-Benefit Analysis and Techno-Economic Assessment

6.1. Overview of CBA in Mining Waste Management

Implementing CBA to assess the economic sustainability of each examined case between Case Scenario A(0) and Case Scenario A(1) involves combining a primary techno-economic assessment that detects the negative cost flows for each case with a statistical approach for detecting the escalation of risk probability that affects total cost fluctuation.

CBA aims to assess the total cost and potential benefit, including cost savings and financial gains, that characterize each of the two probable decisions of tailings management: conventional (Case Scenario A(0)) and innovative (Case Scenario A(1)). Therefore, the first step of CBA focuses on the primary techno-economic evaluation of the two examined cases, considering the mining industry's legal obligations that express the principles of Circular Economy and the 4Rs policy.

Emphasizing this primary assessment, the loss function that estimates the total money loss is calculated and defined for each of the examined cases. To optimize the primary economic estimation and provide a future financial risk assessment, the factor of detecting the risk probability of non-conformance occurrence, which is separated for each case, is needed. So, the Probability Density Function is configured following the terms and definitions of the corresponding legislation for environmental protection. Multiplying the loss function (based on the primary economic assessment) with the Probability Density Function, the goal function is calculated, which refers to the final negative goal achievement converted to monetary terms and characterizes the cost-effectiveness of each examined case separately.

6.2. Case Scenario A(0)

6.2.1 Economic Analysis -Assessment – Risks of Non-Conformance with Circular Economy Legal Obligations in Conventional Mining Waste Management Methods

Economic Analysis-Assessment of the Case Scenario A(0) considers the legal obligations that configure the total grade of non-conformance. These criteria refer to the partial probabilities of negative goals that guide the total negative probability of non-conformance, as mentioned in section 4.1.

Considering that each country's Gross Domestic Product (GDP) affects the severity of techno-economic indicators that configure the penalty cost of non-compliance with indicative environmental protection policy, specific calculation data are available for companies in each region as declared by the corresponding legislation. Therefore, the provided economic calculation data for the penalty cost of non-conformance by the Greek Legislation (64) expresses the legal obligations of Directives 2006/21/EC and 2008/98/EC, while its indicators are proportional to the relevant national GDP.

Considering that Case Scenario A(0) is described by the Trial-Error methodology, it is comprehended that the Binomial distribution will calculate its financial risk. This calculation is based on the configuration for the Penalty Cost of non-compliance according to the requirements of Greek legislation. So, implementing the Binomial distribution on the project’s needs is also tailored to the Greek legislation’s requirements to be evaluated in industrial mode. The following equation shows the calculation mechanism for the penalty cost of non-compliance, while its partial coefficients are shown in Table 3.

E.P.=f(SP, Σ_{EAE}, ΣB, Σεεπi) , **EP**=SP* Σ_{EAE} * ΣB* Σεεπi (1)

E.P. refers to Economic Penalty which is calculated.

S.P. refers to the Standard Penalty Cost based on the Environmental Risk Category of each Company.

Σ_{EAE} refers to the factor of the annual gross revenue of each operation activity.

Σ=3 for non-financial activities.

ΣB is the weighting factor, proportional to the risk of– non-compliance and its impact on the environment and human health by environmental pollution.

$\Sigma\varepsilon\varepsilon\pi_i$ refers to the multiply of individual coefficients (mitigating or aggravating).

Table 3 Greek Legislation Requirements for the Configuration of Penalty Cost about Non-Compliance with Environmental Protection Policy (64)

Standard Penalty Cost		
Risk Category B	**Risk Category A2**	**Risk Category A1**[1]
6,000 €	9,000 €	12,000 €

Factor ΣB					
Severity of non-Compliance	Liquid or Solid Waste/ Compounds			Administrative Responsibility of Non-Compliance	Sound Effect or Other Non-Compliances
	Hazardous Waste	Non-Hazardous Waste	Gas Pollutants		
Low	0.4	0.2	0.3	0.2	0.1
Medium	2.5	1.4	2.0	1.0	1.0
High	6.5	5.5	6.0	5.5	5.0
Extremely High	16.0[1]	13.0	14.0	10.0[1]	10.0

Mitigating/Aggravating Factors			**$\Sigma\varepsilon\varepsilon\pi_i$**
A/A	**Description**	**Escalation**	**Coefficients**
1	History of Compliance (tactical inspections, period time of 10 years)	Positive / Medium Negative (2-3 Non-Compliances) Negative (>4 Non-Compliances)	0.8/1.0 1.2 3.0
2	Behavior/Cooperation during Audit Control	Excellent/Good/Negative	0.9/1.0/1.1
3	Compliance after Audit Control	Full/Medium/None	0.6/0.8/1.0
4	Intention for gaining high financial gains (directly or indirectly)	No/Yes	1.0/1.1
5	Total level of Compliance with the environmental requirements after Inspection	High/Medium/Low	0.9/1.0/1.1
6	Public Profitability	Yes/No	0.9/1.0
7	Social Profitability	Yes/No	0.9/1.0

Furthermore, Table 4 shows the configuration for the probability of non-compliance $\theta_{(0)}$, considering the minimum risky conditions of scenario A(0) considering only the non-legal conformance, with environmental protection requirements according to the corresponding legislations and guidance (9) (10) (5) (100) (101) .

[1]All the mining companies are classified in Risk Category A1, due to the production of huge volume of hazardous waste as declared in the national legislation (Greek Government Gazette, 2011). So, in case A(0) during Audit Control, PC=12,000 €, ΣB=26, Σ_{EAE}=3, and Σεεπ is extracted by the multiply of $\Sigma\varepsilon\varepsilon\pi_1$-$\Sigma\varepsilon\varepsilon\pi_7$ as shown in Table A1

Table 4 Configuration for the minimum risky Probability of Non-legal Compliance with Environmental Protection Policy in Terms of Circular Economy and 3R's

Years	Number of Individual Probabilities of Non-Conformance with Environmental Protection Requirements	Uncontrolled and Systematic Disposal of Hazardous Waste on Ground Soil	Business Decision according to the Implementation of Environmental Protection Mechanisms (Reduce, Recover, Reuse Waste)	Identified Environmental Pollution Source	Huge Volume of Hazardous Waste to the Final Disposal Site	Total Probability of Environmental Non-Compliance
i	M(i)	$θ_{(1)}$	$θ_{(2)}$	$θ_{(3)}$	$θ_{(4)}$	$θ_{(0)}$
1	4	1.00	1.00	0	1.00	0.75
2	4	1.00	1.00	0	1.00	0.75
…n	4	1.00	1.00	0	1.00	0.75

Considering Equation (1), the configuration for the penalty cost of non-compliance is output by multiplying between the factors of SP $Σ_{EAE}$, ΣΒ, $Σεεπ_i$.

The factor SP equals 12,000 euros for a mining company that belongs to the Risk Category A1. The factor $Σ_{EAE}$ equals 3 for a start-up company (64). The factor ΣΒ is output by the addition of its partial indicators due to the presence of hazardous waste and the responsibility of each company's administration for the lack of environmental policy. Finally, Σεεπi is output through the multiplication of partial Σεεπ negative indicators during the time of operation. So, Table A1 (in the Appendix) shows the configuration of the penalty cost for non-compliance with the environmental protection requirements and its escalation during the first five consecutive years of operation.

It is considered that there is only one non-conformance per annual audit control. In addition, it shows the configuration of the annual $Y_{(i)}$ factor due to the negative targeting by the nationally authorized environmental inspection authorities.

In actual conditions, the total negative cost of non-conformance may be configured due to partial non-compliance in different audit control sites inside the industrial field. So, the Probability Density Function is needed to calibrate the potential negative goal, which refers to the one audit control, considering the additional inspections. Therefore, the total goal function refers to the product of the Probability Density Function with the loss function. It is essential to mention that even if there is no identified environmental pollution, the total probability for non-compliance in Case Scenario A(0) is at least 75%. The high numerical value for probability $θ_{(0)}$ is due to the lack of environmental protection policy in Case Scenario A(0). Thus, it is extracted that the implementation of environmental protection measures is more critical to the configuration for the estimation of the total probability for non-conformance than the identified environmental pollution.

All these parameters configure the escalation of Financial Risk in Case Scenario A(0) for the first five consecutive non-conformances, considering that there is only one non-compliance annually. The combination of partial parameters S.P., ΣΕΑΕ, ΣΒ, and Σεεπi achieves the configuration of total penalty cost. Coefficient Σεεπi is determined by partial coefficients $Σεεπ_1$, $Σεεπ_2$, $Σεεπ_3$, $Σεεπ_4$, $Σεεπ_5$, $Σεεπ_6$, and $Σεεπ_7$, as they are described on Table 2 (64). Σεεπi coefficient could not have a numerical value lower than 0.4 and higher than 1.5 for the first four consecutive years of non-conformance (Greek Government Gazette, 2022). In addition, after the fourth year, the $Σεεπ_i$ coefficient adopts a stable numerical value. Σεεπi coefficient, except for the major variable for the determination of total annual penalty cost, contains the probability of non-compliance indirectly.

6.3. Case Scenario A(1)-Circular Economy-Oriented Approach

6.3.1. Work Plan Layout

This chapter provides a detailed description of the proposed scenario A(1), for a start-up mining company that is estimated to produce 150 ktonnes of mining tailings annually, and refers to the investment in metal recovery from mining tailings to face the challenges of the opposite scenario A(0). The examined tailings mass corresponds to actual conditions because a) it is half of the annual tailings mass of 325.8 ktonnes mentioned in a relevant already applied case study of metal recovery in Portugal (136), and b) it is approximately equal to an average annual produced tailings mass of 135-165 ktonnes (137).

To identify the total investment cost for tailings management, each mining company should estimate the tailings mass it aims to produce. The total investment cost is configured according to the estimated tailings mass (assessed by the equipment's primary installation/maintenance costs, chemical reagents, and energy consumption).

The two central parameters that affect sustainability efficiency need to be determined to evaluate the economic sustainability of this operation, which are the cost of a) chemical compounds and b) energy mass flow. Additive technical parameters that could increase the cost of investment are the cost of maintenance and the potential revenue (as shown in Appendix Tables A3-A5) by utilizing the recovered metal mixture. Considering that there are 300 working days per year, the daily average mass of waste that needs to be treated, in terms of recovery procedures, is easily extracted at 500,000 kg/day.

The designed flow scheme to treat 150 ktonnes of waste annually involves three major steps. Each step contains three stages, as shown in Figure 12. Reactors, which are used per step, must be able to extract metal materials by adding chemical compounds (reagents) and catalysts. At the same time, a blender machine provides the required rpm to accelerate the precipitation of the produced chemical salts (138). After each step, sunk material is filtered, and the floated material is input into the next stage of recovery.

The work nature of such a project involves a circular system process, which requires an excellent, organized schedule plan. As a result, the use of autonomous machinery equipment is preferable due to its ability to provide well-modeled process activities.

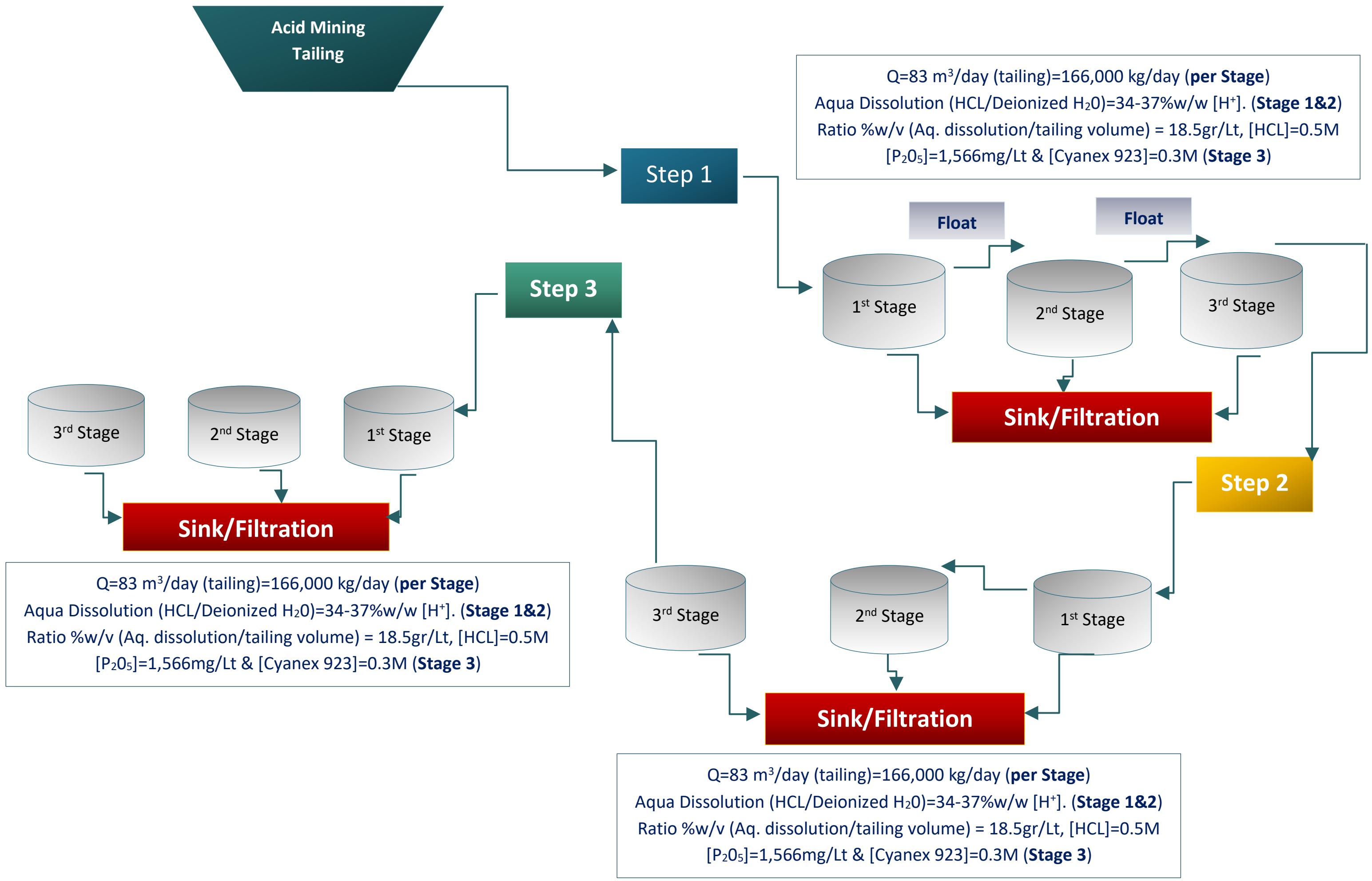


Figure 12 Flow-Scheme of the Case Scenario A(1) on Metal Recovery from Mining Tailings

The input volume of waste per stage is shown in the flow scheme above, estimated at 83 m^3. The required residence time inside each stage is mentioned to be 2 hours. So, after the 2 hours of residence time for the first volume part of tailings on the first stage and its filtration, the second volume part of tailings is input to the reactor of stage 1 while the floated material of the treated tailing part is input to the reactor on the second stage.

The total duration of beneficial metals' recovery for each part of mining tailings is estimated at 18 hours. Thus, total metal recovery by the whole daily volume of mining tailings is achieved after 20 working hours through fully autonomous equipment. Table 5 presents a daily schedule plan for this project type to consider the nature of the work.

Table 5 Daily Metal Recovery Chrono – Diagram, of the proposed Case Scenario A(1)

Daily Project Schedule of Metal Recovery (250 m^3 Gold Mining Tailings)																		
Codification Color / Part of Waste	1st Part	2nd Part	3d Part															
Steps of Recovery	1st Step						2nd Step						3d Step					
Stages of Recovery	Stage 1		Stage 2		Stage 3		Stage 1		Stage 2		Stage 3		Stage 1		Stage 2		Stage 3	
Residence Time of Recovery (Hours)	1st	2nd	3d	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th	17th	18th
1	1st Part																	
2	2nd Part	1st Part																
3	3d Part	2nd Part	1st Part															
4		3d Part	2nd Part	1st Part														
5			3d Part	2nd Part	1st Part													
6				3d Part	2nd Part	1st Part												
7					3d Part	2nd Part	1st Part											
8						3d Part	2nd Part	1st Part										
9							3d Part	2nd Part	1st Part									
10								3d Part	2nd Part	1st Part								
11									3d Part	2nd Part	1st Part							

Daily Project Schedule of Metal Recovery (250 m³ Gold Mining Tailings)																		
Codification Color / Part of Waste	1st Part	2nd Part	3d Part															
Steps of Recovery	1st Step						2nd Step						3d Step					
Stages of Recovery	Stage 1		Stage 2		Stage 3		Stage 1		Stage 2		Stage 3		Stage 1		Stage 2		Stage 3	
Residence Time of Recovery (Hours)	1st	2nd	3d	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	15th	16th	17th	18th
12										3d Part	2nd Part	1st Part						
13											3d Part	2nd Part	1st Part					
14												3d Part	2nd Part	1st Part				
15													3d Part	2nd Part	1st Part			
16														3d Part	2nd Part	1st Part		
17															3d Part	2nd Part	1st Part	
18																3d Part	2nd Part	1st Part
19																	3d Part	2nd Part
20																		3d Part
Total Hours of Metal Recovery Operation (20hrs/day) - 250 m³ Volume of Gold Mining Tailings																		

6.3.2. Estimated Techno-Economic Analysis of the Case Scenario A(1)

The demonstrated work layout, as shown in the Flow-scheme in section 6.3.1, adopts the ratio of chemical acids and catalysts mentioned in the experimental scientific study (32) to achieve the expected efficiency of metal extraction >95%. The reported hydrochloric acid [HCL] concentration ranges between 0.5 and 1M. In the case scenario A(1), the concentration of HCL is equal to 0.5 M for both stages 1 and 2.

The total mass of Hydrochloric acid in each stage of recovery is estimated at 568 kg. According to the acid marketplace (139) (140), the total cost of chemical reagents is estimated at 20,448 €/day.

The next stage of recovery is achieved by the catalysts' action in low pH conditions. The used catalyst is D2EHPA (extraction of base metals) or Cyanex 923 (extraction of REEs), due to its efficiency on metal recovery. The ratio P_2O_5 / $(\text{Dissolution})_{aq}$ is 1,566 mg/Lt and for Cyanex 923 is about 0.3 M. According to the catalyst marketplace (141) (142), the total cost estimation of P_2O_5 is 6,162 €/day and for Cyanex 923 is estimated at 2,601 €/day.

Finally, the total exported recovery cost per day is approximately 29,211 €. Thus, based on this flow chart, the annual cost of chemicals and catalysts for metal recovery by gold mining tailings of 150,000,000 kg is estimated at 8,763,300 €. The economic analysis of the mass flow of chemicals and catalysts is presented in Table 6.

Table 6 Cost Estimation of Reagents & Catalysts on REEs and other metals Recovery

Steps	Stages	Tailing Mass (kg/d)	Total Qt (m^3/d)	Qt (m^3/stage)	Chemical Reagents	Chemical Reagents Concentration (M), or mg/Lt	Reagents Mass (kg)	Costs (€/kg)	Daily Costs (€/Stage)	Annual Costs (€/Stage)
Step 1	Stage 1	500,000	250	83	HCL_{aq} (34-37%) 1%w/w	0.5	568	6	3,408	1,022,400
	Stage 2			83	HCL_{aq} (34-37%) 1%w/w	0.5	568	6	3,408	1,022,400
	Stage 3			83	P_2O_5 & Cyanex 923	1,566 mg/Lt & 0.3 M respectively	130 & 1,220 respectively	15.8 (P_2O_5) & 0.71(Cyanex 923)	2,054 (P_2O_5) & 867 (Cyanex)	876,300
Step 2	Stage 1	500,000	250	83	HCL_{aq} (34-37%) 1%w/w	0.5	568	6	3,408	1,022,400
	Stage 2			83	HCL_{aq} (34-37%) 1%w/w	0.5	568	6	3,408	1,022,400
	Stage 3			83	P_2O_5 & Cyanex 923	1,566 mg/Lt & 0.3 M respectively	130 & 1,220 respectively	15.8 (P_2O_5) & 0.71(Cyanex 923)	2,054 (P_2O_5) & 867 (Cyanex)	876,300
Step 3	Stage 1	500,000	250	83	HCL_{aq} (34-37%) 1%w/w	0.5	568	6	3,408	1,022,400
	Stage 2			83	HCL_{aq} (34-37%) 1%w/w	0.5	568	6	3,408	1,022,400
	Stage 3			83	P_2O_5 & Cyanex 923	1,566 mg/Lt & 0.3 M respectively	130 & 1,220 respectively	15.8 (P_2O_5) & 0.71(Cyanex 923)	2,054 (P_2O_5) & 867 (Cyanex)	876,300
Annual Total Cost of Chemical & Catalysts (€)										**8,763,300**

The estimation of the operation cost of metal recovery from mining tailings also includes the additional energy cost. Operation cost varies with each type of electrical and mechanical equipment used. Electricity and natural gas are the most common energy flows, providing the required power for all of these steps in industrial mode (75).

Due to the higher cost of electricity than natural gas, energy multipliers are used to transform energy from natural gas into electricity (143). Furthermore, using chemical acids and catalysts reduces the required energy throughout the recovery procedure (75) (144).

According to the European Commission (25), the energy consumption for waste management of gold mining tailings in Orivesi is reported to be 1kWh/t , and the total energy consumption at the site per tonne of ore processed is 53.5 kWh/t. At Ovacik, the total energy consumption is reported to be 60kWh/t. As the European Commission mentions, energy consumption is proportional to the chemical composition of each tailing type.

In Case Scenario A(1), energy consumption estimation concerns precious metals mining tailings. As a result, the configuration of the required energy will be similar to that of Orivesi's and Ovacik's waste management cases.

The annual volume of mining tailings in Orivesi is reported to be 780,000 m^3. The density of mining tailings varies from 1,580 to 2,220kg/m^3 (145). In Case Scenario A(1), the density of mining tailings is rounded to 2,000 kg/m^3. The relevant average annual volume of tailings is estimated at 75,000 m^3.

The average electricity price per MWh in Europe is reported to be 135 €/MWh (146) and the respective price of energy by natural gas is 22.71 €/MWh (147). The energy crisis 2022 affects the energy price, so the electricity cost is configured by the reported price of 240 €/MWh (148), while natural gas costs 72.9 €/MWh (149).

Considering the required energy flow for precious metals mining waste management, it is estimated that the required power of energy for recovery mechanisms ranges between 53.5 – 60 kWh/t. As a safety factor, the highest energy value per tonne is extracted from the economic estimation of the required annual energy cost. The estimation of total energy cost is 1,329,660 €/ year. The comparative estimation of energy cost analysis for Case-Scenario A(1) with other gold mining companies is presented in Table 7.

Table 7 Estimation of Energy Cost Analysis on the Case-Scenario A(1)

Mining Tailing Site	Annual Volume of Tailings (m^3)	Annual Mass of Tailings (kg)	Energy Consumption MWh/Ore tonne	Average Electricity Price Europe 2022 (€/MWh)	Average TTF Natural Gas Price Europe 2022 (€/MWh)	Electricity 40% (MWh/Ore Tonne)	TTF 60% (MWh/Ore Tonne)	Total Estimated Energy Cost (€)
Orivesi	780,000	1,560,000,000	0.0535	260	72.9	0.0214	0.0321	12,330,380
Ovacik	150,000	360,000,000	0.06	260	72.9	0.024	0.036	2,659,320
Case Scenario A(1)	75,000	150,000,000	0.06	260	72.9	0.024	0.036	1,329,660

6.3.3. Cost Analysis of Metal Recovery and Waste Processing - Cost of Installation and Maintenance

The Total Operation Cost, by the clear side of processing in Case Scenario A(1), includes both the energy and chemical reagents' consumption costs (127) (76). This Techno-economic analysis refers only to the cost of recovery activities, assuming the existence of the fully autonomous industrial installation.

The cost of installation varies with each piece of electromechanical and mechanical equipment that will be used. Reactors should provide a stable flow of raw material and chemical reagents in an aqueous environment, an NF membrane on the inner surface, and stable spin movement of the blender rotor. The residence time is estimated at 2 hours for each stage (33). So, the daily volume of waste to be treated should be 250 m^3.

The required capacity of each cylinder reactor, which is used in each stage, is estimated at 90 m^3 (radius 2,4 m and its height 5 m). The productivity cost of each reactor is calculated by the techno-economic analysis & assessment of its technical parameters. The major parameters of this calculation are a) Metal Injection Molding (MIM), b) Binder Jet (BJ), and c) Laser Powder Bed Fusion (LPBF) (150).

Table 8 presents the Productivity Cost Analysis of chemical reactor manufacturing. In addition, the total productivity cost of the required chemical reactor for each stage of this project is assessed.

Table 8 Productivity Cost of Chemical Reactors with Capacity 90 m³ based on Scientific Estimation (150)

Activity	Technical Parameters & Costs Description	Average Input	Unit Cost (Whole Productivity) ($)	Unit Cost per Reactor (Capacity 90 m³) ($)	Total Unit Cost per Reactor (Capacity 90 m³) ($)
Metal Injection Molding (MIM)	Sintering cycle time (hrs) per batch	20	101	101	3.792
	Cost of manufacturing space ($/m²)	1000	101	3282,5	
	Process step yield (%)	90	105	105	
	Operator wages ($/yr)	50000	101	101	
	Annual maintenance as a fraction of capital cost (%)	5	101	101	
	Electricity cost ($/kWh)	0,0729	101	101	
Binder Jet (BJ)	Printing layer thickness (mm)	0,1	144	144	5.699
	Printing time (sec) per layer	60	144	144	
	Depowdering cycle time (min) per part	30	144	144	
	Cost of manufacturing space ($/m²)	1000	144	4680	
	Process step yield (%)	90	155	155	
	Annual operator wages ($/yr)	50000	144	144	
	Annual maintenance as a fraction of capital cost (%)	5	144	144	
	Electricity cost ($/kWh)	0,0729	144	144	
Laser powder bed fusion (LPBF)	Printing layer thickness (mm)	0,04	541	541	21.504
	Printing scan speed (cm/sec)	700	541	541	
	Powder removal cycle time (hrs) per part	1,5	541	541	
	Cost of manufacturing space ($/m²)	1000	541	17582,5	
	Process step yield (%)	90	675	675	
	Annual operator wages ($/yr)	50000	541	541	
	Annual maintenance as a fraction of capital cost (%)	5	541	541	
	Electricity cost ($/kWh)	0,0729	541	541	
	Total Production Cost of each Reactor				**30.994 $** **30.388 €**

By this assessment, the total reactor's production cost is estimated at 30.388 €. However, the installation cost of each cylinder chemical reactor is expected to be greater than this price. The cost of installation involves a) the cost of pump connection, b) the cost of other electronic equipment to be used for placement, c) the cost of transportation fees, and d) the cost of stakeholders (workers' salaries, economic charge by the equipment to be used, trucks, cranes, etc.).

According to the Aspen Capital Cost Estimator (ACCE) (151), indicative installation costs are extracted proportional to each reactor's capacity and technical specifications. For example, in the case of a diameter/height=0.5:1 ratio (which is more suitable to the hypothesis scenario's dimensions), the installation cost of each reactor is estimated at 170,000 €.

The entire dynamic project is divided into three main steps, each comprising three stages of recovery. Thus, the total installation cost of 9 reactors is estimated at 1,530,000 €. The fully autonomous equipment contributes positively to the efficient recovery of beneficial metals during daily work. The annual maintenance cost is approximately estimated at 20% of the cost of primary installation (150) (151). So, there is an additional cost of 306,000 € due to the service of the applied equipment.

The primary installation and maintenance costs, energy consumption, and chemical reagents configure the total negative cost in Case Scenario A(1). This total cost refers to the total money loss from investment in sustainable waste management processing.

6.4. Financial Risk Assessment and Decision-Making Framework supported by Cost-Benefit Analysis Methodology

This scientific study aims to evaluate the criteria for case scenarios A(0) and A(1) while providing their direct or indirect impact on the corresponding implemented actions (35). Taking into consideration that the industrial need for metal materials will increase in the future (152) with the proportional increase of their supply risk (112), it is necessary to consider the utilization of mining waste (153) through metal recovery procedures (154). Therefore, case scenario A(1) is compared with the opposite case Scenario A(0).

This comparison is based on a partial techno-economic analysis for both scenarios, considering that in the case of scenario A(0), there is no policy of conformance by the side of a start-up mining company with the legislative environmental protection measures. This guides to the full acceptance of penalty costs for non-compliance.

In contrast, in the case scenario A(1), there is a high rate of compliance with the policy of green growth by the side of a mining start-up company in terms of the circular economy (24) and the development model of 4Rs (118). This comparison for the economic assessment of each case scenario is based on existing experimental results according to the chemical mass flow (32), procedures of metal recovery (133), and the degree of efficiency (15).

After the primary economic assessments' comparison, the CBA appraisal tool is used to identify ERR (155) and LTP (156) for each case scenario. The methodology for exporting these partial indices for both examined cases requires estimating the total negative cost and the relevant potential benefit (63).

In the case scenario A(0), the total negative cost refers to the potential cost of non-compliance, and the benefit equals the avoidance of money loss due to non-investment in establishing a closed industrial mining waste management system.

In case scenario A(1), the total negative cost refers to the total investment cost, while the benefit equals the potential passive income by utilizing the recovered metal mix. CBA evaluates the whole of conditions, an industrial scale based on a) the proposed work layout (63)and the implementation of guidance for investment in terms of green growth (35), in the case scenario A(1). However, CBA evaluates the total negative cost due to non-compliance with the Circular Economy and the terms of the environmental protection policy (9) (10) (64) (101)in case scenario A(0).

So, the CBA methodology assesses the sustainability for cases A(0) and A(1) by converting all their parameters into monetary terms. The optimal applicable mathematical function required to describe each case scenario's cost estimation is a milestone for the efficiency and accuracy of CBA's output indices.

For the negative cost determination in each case, partial factors are necessary to configure it. For instance, in case A(0), the total non-conformance might or might not be detected during the audit . This happens because, in actual conditions, environmental inspections can be accomplished in random locations inside the industrial field. So, in this case, the trial-and-error methodology is the most suitable for describing the negative goal achievement.

In the opposite scenario, at least one non-conformance might be detected annually while investing a standard cost for waste treatment through a closed system of industrial units. Therefore, the Erlang distribution is the most suitable methodology for detecting the probability of a negative cost (119) (157).

Thus, considering that case scenario A(0) is described by the false-trial methodology (118), while the opposite case scenario A(1) is described by stable productivity and revenue cost per year (119), the Binomial and Erlang distributions are selected, respectively (158).

According to the Bayesian Analysis, the scientific approach for calibrating CBA's results is continuously enhanced through ''Financial'' SRA (95). So, two different integrals referring to each case scenario's industrial conditions are established (13). The integrals' limits vary from each other due to the specific examined conditions (86).

CBA is a techno-economic tool that examines the sustainability of each business decision through applicable mathematical models that convert all partial parameters into monetary terms. These models focus on the Financial Risk Assessment, tailored to each project's needs and the relevant legislation. This study emphasizes two case scenarios. Scenario A(0) refers to the consecutive mining operations without implementing the legislative requirements regarding the circular economy (4Rs). In this case scenario, avoiding the corresponding penalty cost for non-conformance with environmental protection guidance is impossible. On the other hand, scenario A(1) refers to the mining operations that adopt an environmentally friendly manner of work through the investment in a closed system of industrial units to minimize the huge volume of produced waste and to recover the metal materials contained in the tailings. The optimal business decision, based on CBA, is determined by the minimum LTP and maximum ERR (50).

Figure 13 shows the steps for the entire Financial Risk Analysis-Assessment, expressed by the Goal Functions for both case scenarios, considering their partial primary economic assessments. In addition, the CBA's partial indicators LTP and ERR for each case scenario are extracted. The configuration of Stochastic Risk Functions is also needed for both case scenarios. Finally, it provides the CBA Indices by the ratio between the numerical values of the two Stochastic Risk Indices.

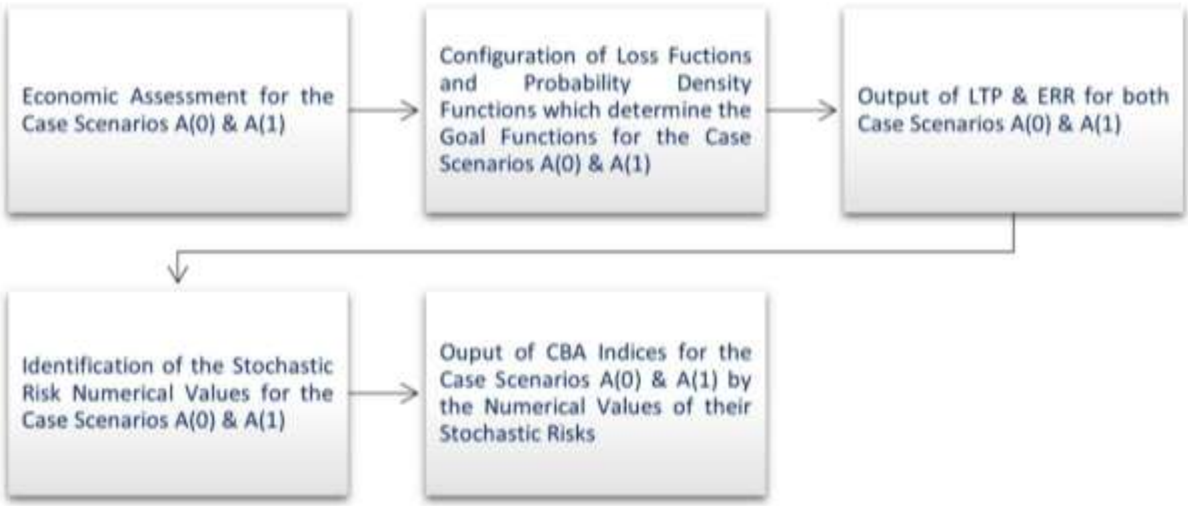


Figure 13 Steps of the Financial Risk Analysis-Assessment

For the implementation of CBA is required the determination of a) loss function L[A(0)_A(1),Y], b) goal function G[A(0)_A(1),Y], c) Probability Density Function [PDF] for each case scenario A(0), A(1). Except for CBA's partial functions, the corresponding determination of total FR is necessary through Bayesian Analysis (BA). FR is provided by the combination between G[A(0)_A(1),Y], [PDF], and Beta distribution Beta[t,r,θ(0)] (t=300 refers to the annual working days, r refers to the annual audit control sites, and θ(0) refers to the probability for non-compliance with environmental protection requirements). Case Scenario

A(0) refers to non-compliance with environmental requirements; as a result, its financial risk is calculated by the escalation of the corresponding penalty cost for non-conformance with Global, European, and Greek legislation (9) (10) (64) (101).

Generally, according to (79), a total Audit Control is mandatory for all mining industries annually due to their high environmental risk level. The complete audit control involves specific audit control sites in the nearby waste disposal area. It is taken into consideration that four specific locations are needed to describe the level of environmental pollution.

Thus, audit control sites per year equal 4 (as per the horizon directions, r=4). In addition, it has to be mentioned that non-compliance with environmental protection requirements does not only refer to the identified environmental pollution. Non-conformance is characterized by four criterion probabilities (64).

The first criterion refers to the “Uncontrolled and Systematic Disposal of Hazardous Waste on Ground Soil” (probability $\theta_{(1)}$). The second criterion refers to the “Industrial Policy according to the Implementation of Environmental Protection Mechanisms (Reduce, Recover, Reuse Waste)” (probability $\theta_{(2)}$). The third criterion refers to the “Identified Environmental Pollution Source” (probability $\theta_{(3)}$). The fourth criterion refers to the “Huge Volume of Hazardous Waste to the Final Disposal Site” (probability $\theta_{(4)}$). So, the total probability of non-compliance with environmental requirements θ(0) is calculated by the average of $\theta_{(1)}$, $\theta_{(2)}$, $\theta_{(3)}$, $\theta_{(4)}$.

According to the theoretical estimation, the probability for non-compliance in Case Scenario A(0) ranges between 0-100%; however, due to the lack of achievement for the criteria mentioned above 1,2,4, this probability range is transitioned to 75-100%. The binomial distribution is considered to determine the goal function in this Case Scenario (118). During the audit control, the Total Conformance or Total Non-Conformance with environmental protection requirements is examined, so the entire procedure is based on the trial–and–error methodology (158).

Case Scenario A(1) refers to full conformance with the environmental protection policy. Its financial risk is calculated by the total investment cost for the installation of the compact industrial unit to a) eliminate the risk probability for free tailings disposal, b) recover metals from tailings, and c) reuse the recovered metal materials by disposing of them in the global metal market industry. Economic assessment for this Case Scenario contains a) cost for chemical reagents, b) cost for energy, c) cost for primary installation, d) maintenance cost for the whole equipment to be used, and e) benefit from the utilization of recovered metal materials (63). By this assessment, the total investment cost is determined.

Based on the theoretical estimation (119), the probability for non-compliance in Case Scenario A(1) ranges between 0-100%; however, due to the lack of achievement only for criterion 3, this probability range is transitioned to 0-25%. Thus, to determine the goal function in this case scenario, the Erlang distribution is considered due to the examination of the current conditions (159). During the audit, the grade of non-conformance with environmental protection requirements is examined (119).

6.4.1. Cost-Benefit Analysis in Case Scenario A(0)

6.4.1.1. Loss Function

Taking into consideration that Σεεπi is the primary factor that determines the fluctuation of the total penalty costs K(i), the loss function for Case Scenario A(0) is established by the combination of varied Y(i) coefficients and the primary total penalty cost K(1). Equation 2 shows the Loss Function mechanism during the working period. The extracted functions refer to one non-compliance annually.

$$L(A(0), Y(i)) = K1 * [Y(i)]^2 \begin{cases} Y(i) = 1 & 0 \le i \le 1 \text{ Years} \\ Y(i) = 3.27 & 1 < i \le 4 \text{ Years} \\ Y(i) = 8.17 & 4 < i \text{ Years} \end{cases} \quad (2)$$

6.4.1.2. Probability Density Function

The Probability Density Function describes the total probability of the annual negative cost described through the Loss Function, which is estimated to increase for the consecutive years after the first non-compliance in Case Scenario A(0). N refers to the audit control sites per year, estimated to be at least 4. Audit control sites for the consecutive years are increasing in an arithmetic progression (in Case Scenario A(0) due to the non-approval of environmental protection measures). In contrast, the stable factor equals the numerical value 4. Thus, audit control sites for the second year are at least 8, audit control sites for the third year are at least 12, etc.

The natural meaning of PDF is to determine the incremental additive of growth for negative cost (160). For this reason, the PDF considers the estimated future negative cost in terms of the constant corresponding negative cost in the case of 4 audit control positions per year. In the following equation, the N parameter, located in the numerator of the f(Y) function, refers to the additive control sites year by year after the first non-compliance. The N parameter, placed in the denominator function of the f(Y), refers to the stable numerical value of audit control sites in the arithmetical progression.

E[Y] refers to the estimated numerical value for Y(i). E[Y] is exported by multiplying N and probability $\theta_{(0)}$. Parameter N refers to the additive control sites year by year after the first non-compliance. Standard deviation σ^2 refers to the combination of the N factor (additive control sites year by year after the first non-compliance) and the total probability for non-conformance, as shown in the following equations (3-5).

$$f(Y) = N! * [\theta(0)]^{\wedge}Y * [[(1 - \theta(0)]^{N-Y}] * [Y! * (N - Y)!]^{-1} \quad (3)$$

$$E[Y] = N * \theta(0), \quad (4) \qquad \sigma^2 = N * \theta(0) * [1 - \theta(0)] \quad (5)$$

6.4.1.3. Goal Function

The Goal function describes the total negative cost for non-compliance with environmental requirements, which is exported by the calibration of exported negative costs of the Loss Function through the Probability Density Function. The physical meaning of this estimation concerns the axiom a) that numerical values exported by the loss function refer to one non-compliance annually, while b) f(Y) numerical values exported by the PDF refer to the incremental additive of growth for the negative costs by LF.

Equation 6 shows the configuration of the Goal Function (GF) during the whole period of environmental monitoring control.

$$G(A(0), Y(i)) = K1 * [Y(i)]^2 * f\big(Y(i)\big) \begin{cases} Y(i) = 1 & 0 \le i \le 1 \text{ Years} \\ Y(i) = 3.27 & 1 < i \le 4 \text{ Years} \\ Y(i) = 8.17 & 4 < i \text{ Years} \end{cases} \quad (6)$$

According to the configuration of the goal function, the total negative cost for non-conformance with environmental protection requirements is estimated. There is a classification mode of three different classes for the goal function. First class G(A(0),Y(1)) refers to the first year of non-compliance. Second class G(A(0),Y(2,3,4)) refers to the three consecutive years of non-compliance, 2^{nd}, 3^{rd}, and 4^{th} after the first one. Third class G(A(0),Y(5)) refers to the consecutive years of non-compliance over the 5^{th} one. This classification mode is described by the Greek Legislation (64).

The physical meaning of the configured goal functions is to show the negative impact on the incremental total penalty cost due to the corresponding increment of the total probability for non-conformance. Through the structure of the goal functions, the total penalty cost per year is demonstrated more realistically. Additionally, the PDF assumes a stable numerical value when the factor Y(i) also assumes a stable numerical value. Thus, the annual goal function will remain stable for each year after the 4th one, when environmental protection requirements are not implemented.

The Goal Function (shown in Equation 6) determines the total economic assessment extracted through binomial distribution theory for Case Scenario A(0). The presented economic assessment is exported considering that $\theta_{(3)}=0$ to provide financial risk results for the best-case situation where there is no identified environmental pollution by audit controls. So, the exported financial risks are calculated only through the mining industry's axiom of non-environmentally friendly behavior. Further information is demonstrated according to the additive financial risk for non-compliance with environmental specs over 30 years in Table A2.

The sum of these financial risks is compared with the corresponding sum by the relevant assessment for Case Scenario A(1). Through CBA, this comparison provides the optimal engineering solution for the economic sustainability of mining waste management. As mentioned, this comparison is a combined appraisal tool that converts all the parameters for both scenarios into monetary terms. Its output results are based on the theoretical manner of applied mathematics.

6.4.2. Cost-Benefit Analysis in Case Scenario A(1)

The Loss Function shows the total negative cost due to the investment cost for the case project in which the mining tailings are waste treated through a closed system of industrial units (as shown in the Flow-scheme in chapter 6.3.1) (63) (161). Therefore, there is a need for the installation of an industrial connection between buffer tanks using pipelines. This system should be fully autonomous and work for 20 hours per day. The proposed case scenario A(1) is upscaled, considering the existing scientific experimental studies on metal recovery (34) (99). The combination of the technical parameters (162) that support this operation is tailored to treat 150 ktonnes of mining tailings annually (63). Considering that the annual working days are 300 and the fact that the density of mining tailings varies from 1,600-2,220 kg/m^3 it is anticipated that the average density is 2000 kg/m^3 (111). So, the daily mass of the produced tailings is approximately 500,000 kg, and their proportional volume corresponds to 250 m^3.

The work layout of the case scenario A(1) involves three major steps. Each step contains three different stages. In the first two stages, the acidic aqua dissolution HCL(aq) (34-37%) with 1%w/w is added (with a concentration of 0.5 M) to de-bind the existing chemical salts (between the metal cations and complex anions of the acidic anhydrides) that are contained into the tailings (34). The third step involves the addition of catalysts (P_2O_5 in a ratio of 1566 mg/L and Cyanex 923 with a concentration of 0.3 M) to accelerate the metal recovery procedure. It has adopted the methodology of sunk/filtration. So, each material precipitating in each step contains high amounts of metal ions (99). As a result, a mixture of different metals (base, precious, and REEs) is recovered daily. At the same time, the aqueous dissolution, including catalysts and acids, could be disposed of or transferred to authorized acid treatment companies. The cost analysis for chemical consumption and its cost is presented in Table 5.

Besides, the energy consumption for the mining waste management of tailings through recovery procedures is considered 0.06 MWh/t (25). Due to the higher cost of electrical energy than the energy of natural gas, it is suggested that the whole operation be supported by a combined energy system between electrical generators (at 40%) and natural gas multipliers (at 60%) (75). Therefore, the energy consumption and cost are estimated based on the most recent energy price values. Table 7 in case scenario A(1) (148) (163). It is essential to mention that due to the crisis in Ukraine in 2022, the price value for energy showed an extremely high amount. Considering that this price value has been lowered two years later, the energy cost calculation was accomplished under the worst-case conditions (price value shown in 2022) as a safety factor.

An additional negative cost considered through the Loss Function is the primary installation and maintenance of the whole equipment. For the configuration of this cost, the capacity of each reactor needed to treat 83 m^3 of tailings daily is considered. So, the productivity cost for each reactor is estimated at 30,388 € (150), while the total installation cost for each reactor, including the pipeline connection and partially needed equipment, is calibrated by the ACCE (Aspen Capital Cost Estimator) index. The installation cost is estimated at 170,000 € (5.6 times more) per reactor (150) (151) (151). So, the total cost of installing nine reactors, including their partial equipment, is estimated at 1,530,000 €. The Cost analysis for the configuration of the productivity cost per reactor is modified to the needed capacity (90 m^3) as shown in Table 8.

There are various correlations between the maintenance cost and the primary installation cost ratio. This ratio ranges from 17% to 23%. The estimated maintenance cost would be approximately 20% of the installation cost (43). Therefore, the annual maintenance cost for the consecutive years of operation after the first year is estimated at approximately 306,000 €.

The benefit of the reuse of recovered materials is estimated according to their contamination into the tailings. Moreover, based on the chemical process of the mining tailings, the estimated total metal mass that could be recovered is assessed (117). Considering the fluctuation of the price values for the three major metal categories (base metals, precious metals, and rare earth elements), the annual positive income is estimated (152). So, the tailings mass of 150 ktonnes annually is estimated to recover approximately 2,152 kg of metals (including base metals, precious metals, and rare earth elements). The annual estimated benefit is approximately 5,438,333 €. Further description is provided in Tables A3, A4, and A5, which are presented in the Appendix section and show the configuration of the potential benefit by utilizing the recovered metal mixture, considering the annual increase of metal price values.

Based on this Techno-Economic Analysis, from the clear side of engineering perspective, it is comprehended that the crucial factors of efficiency, due to their effect on the total investment cost, are the costs of chemical engineering mechanism, energy consumption, primary installation and maintenance of the equipment, and potential revenue by the utilization of metal recovered mixture (Tables 6, 7, 8, A3-A5). The selected chemical engineering mechanism supports the work layout of recovery, containing the addition of acids or catalysts (18) (11). Moreover, the whole system should be supported by the required energy capacity. The entire procedure minimizes electrical energy consumption by adopting a combined energy support system that utilizes multipliers to convert natural gas energy into electrical power. Considering that the costs for chemical reagents and energy consumption significantly impact the total investment cost annually, the optimization of waste management is proportional to their combination in actual circumstances.

6.4.2.1. Loss Functions and Goal Functions in Sub-Cases of the Case Scenario A(1)

The Loss Function refers to the annual stable negative cost that describes Case Scenario A(1). This function is configured by the partial costs of operation (Cost of Primary Installation or Maintenance per year (CIM), Cost of required Energy (EC), Cost of Chemical Reagents (CCR), and Benefit Value (BV) by the reuse of recovered metal materials) and is described by the real Goal Function too. The total investment cost K(i) expresses the total stable money loss for investment and operation of the metal recovery project through a closed system of industrial units, including chemical reagents and catalysts. Loss Function for each operation year in Case Scenario A(1) is established by the sum of these partial costs (as shown in Table A6). Safety Factor +20% ensures the Loss Function's accuracy in case of economic assessment deviation from the actual techno-economic conditions. Factor Y(i) in the actual Loss Function refers to the total cost ratio between the present year (1,2,3,…n) and the first one Y(i)=K(i)/K(1). Equation (7) shows the Loss Function mechanism during the working period. The extracted functions refer to the total investment cost in terms of full compliance with environmental protection requirements through the implementation of 3R's mode annually. The results for the stable annual investment cost are shown in Table A7. The at-risk Loss Function for all sub-cases of the Case Scenario A(1) is calculated as shown in equations 7a, 7b, and 7c, respectively.

The theoretical $Y(i)_t$ indicator of negative targeting (configured by the Σεεπi ratio as in Case Scenario A(0)) refers to the potential negative targeting due to legal non-conformance, considering unexpected conditions despite investing in the Circular Economy principles. The $Y(i)_a$ indicator relates to the negative targeting of money loss due to the investment in sustainable tailings management processing, and is configured by the total investment cost ratio of the primary year to the consecutive years of operation.

$$\mathbf{L(A(0), Y(i)) = K(i) * Y(i)_t}^2 \begin{cases} \mathbf{Y(i) = 1,} & \mathbf{K(i) = K(1) = 374,400€} \quad \mathbf{0 \leq i \leq 1 \;\; Years} \\ \mathbf{Y(i) = 3.27} & \mathbf{K(i) = K(1) = 374,400\,€} \quad \mathbf{1 < i \;\leq 4\, Years} \\ \mathbf{Y(i) = 8.17} & \mathbf{K(i) = K(1) = 374,400\,€} \quad \mathbf{4 < i\; Years} \end{cases}, \; sub-cases\; A(1)_{1,2} \quad (7a)$$

$$\mathbf{L(A(0), Y(i)) = K(i) * Y(i)_t} \begin{cases} \mathbf{Y(i) = 1,} & \mathbf{K(i) = K(1) = 374,400€} \quad \mathbf{0 \leq i \leq 1 \;\; Years} \\ \mathbf{Y(i) = 3.27} & \mathbf{K(i) = K(1) = 374,400\,€} \quad \mathbf{1 < i \;\leq 4\, Years} \\ \mathbf{Y(i) = 8.17} & \mathbf{K(i) = K(1) = 374,400\,€} \quad \mathbf{4 < i\; Years} \end{cases}, \; sub-cases\; A(1)_{3,4} \quad (7b)$$

$$\mathbf{L(A(1)\_SF, Y(i)) = K(i) *} \\ \mathbf{Y(i)}_{a-t} \begin{cases} \mathbf{Y(i) = 1,} & \mathbf{K(i) = K(1) = 7,421,960\,€} \quad \mathbf{0 \leq i \leq 1 \;\; Years} \\ \mathbf{Y(i) = 0.802} & \mathbf{K(i) = K(2) = 5,953,160\,€} \quad \mathbf{1 < i \;\; Years} \end{cases}, \; sub-case\; A(1)_5 \quad (7c)$$

The total Loss Function L(A(1)_SF,Y(i)) refers to the annual stable investment cost on tailings management processing. The additional at-risk cost expressed through the L(A(0),Y(i)) for the sub-cases $A(1)_{1\text{-}4}$ corresponds to the unexpected conditions of non-conformance despite investing in a closed system of industrial units to manage the hazardous waste. The additive at-risk cost for the sub-case $A(1)_5$ is considered to be proportional to the quality of equipment to be used. Therefore, $Y(i)_t$. equals the $Y(i)_a$ indicator, in this sub-case.

Despite investing in metal recovery to minimize risks of environmental pollution from free disposal while conforming to the legal obligations that are mandatory for the mining industry, it is considered that there is a potential risk of non-conformance even if there is any leachate detection. This potential probability of non-conformance refers only to the partial probability $\theta_{(3)}$. That may correspond to an additional at-risk cost for the mining company. So, based on the possible natural conditions that may describe actual circumstances during the annual environmental inspections in Case Scenario A(1), separated Loss Functions and Probability Density Functions are applied to describe the potential additive at-risk negative cost expressed by the theoretical Goal Function $G(A(1)_j,Y(i))_t$. The parameter of each applied Probability Density Function is symbolized as f(Y(i)).

In Sub-Cases $A(1)_1$-$A(1)_5$, it is considered that the potential stable at-risk cost is configured as declared in the national legislation, implying the legal obligations of the corresponding European legislation (9) (10) (64) and global regulation (5). Therefore, in these sub-cases, the at-risk Loss Function equals the L(A(0),Y(i)). Sub-case $A(1)_5$ refers to the maximum risky condition in Case Scenario A(1), whereas the potential at-risk cost is proportional to the money loss due to investment. Hence, the at-risk Loss Function is described as shown in equation (7). Applying the Probability Density Function is necessary to assess this potential additional cost. This additive cost is expressed through the theoretical Goal Function $G(A(1)_j,Y(i))_t$ (as shown in equation 8), which ensures the financial risk assessment even if there is a potential risk of non-full conformance despite the full legal conformance.

The total negative cost that describes the financial risk for each sub-case of the Case Scenario A(1) is calculated by the sum of the real Goal Function $G(A(1)_j,Y(i))_r$ (that equals the L(A(1)_SF,Y) and expresses the annual cost of investment in metal recovery to manage the produced mass of tailings as shown in equation 9) with the theoretical Goal Function $G(A(1)_j,Y(i))_t$. Thus, the total cost, including the stable cost of investment and the potential at-risk cost due to unexpected conditions, is configured by the sum of those partial Goal Functions as shown in equation 10. The indicator j refers to each sub-case of the Case Scenario A(1).

$$\mathbf{G}\left(\mathbf{A(1)_j},\mathbf{Y(i)}\right)_{\mathbf{t}} = \begin{cases} \mathbf{L}\big(\mathbf{A(0)},\mathbf{Y(i)}\big) * \mathbf{f}\left(\mathbf{Y(i)}\right), & \text{sub}-\text{cases } A(1)_1 - A(1)_4 \quad \mathbf{0 \le i \le 30} \quad \mathbf{Years} \\ \mathbf{L}\big(\mathbf{A(1)} - \mathbf{SF},\mathbf{Y(i)}\ \big) * \mathbf{f}\left(\mathbf{Y(i)}\right), & \text{sub}-\text{case } A(1)_5 \quad \mathbf{0 \le i \le 30} \quad \mathbf{Years} \end{cases} \quad (8)$$

$$\mathbf{G}\left(\mathbf{A(1)_j},\mathbf{Y(i)}\right)_{\mathbf{r}} = \mathbf{L}\big(\mathbf{A(1)\_SF},\mathbf{Y(i)}\big),\ \mathbf{0 \le i \le 30} \quad \mathbf{Years}\ (9)$$

$$\mathbf{G}\big(\mathbf{A(1)_j},\mathbf{Y(i)}\big)\mathbf{Total} = \mathbf{G}\left(\mathbf{A(1)}_{\boldsymbol{j}},\mathbf{Y(i)}\right)_{\mathbf{t}} + \mathbf{G}\left(\mathbf{A(1)}_{\boldsymbol{j}},\mathbf{Y(i)}\right)_{\mathbf{r}},\ \mathbf{0 \le i \le 30} \quad \mathbf{Years}(10)$$

6.4.2.1.1. Sub-Case $A(1)_1$ of Stable Negative Targeting. Potential Detection of Distributed Environmental Pollution in Random Locations inside the Closed System of Industrial Units

Sub-case $A(1)_1$ refers to the potential additional cost due to the risk of detected environmental pollution during audit control, despite investing in metal recovery and legal compliance with the mandatory obligations of the mining industry. In this sub-case, it is considered that within a 30-year period, distributed environmental pollution will be detected in random locations within the field of the closed system with industrial units due to unexpected leachates. To describe the potential financial risk expressed through the corresponding theoretical Goal Function, the negative cost expressed by the L(A(0),Y(i)) (as shown in equations (2, 7a)) is multiplied by the Probability Density Function of the binomial distribution (as shown in equation (3)). It is essential to mention that in this sub-case, the total non-conformance is calculated considering that $\theta_{(0)}=\theta_{(3)max}$. So, the range of the total non-conformance equals 0-0.25.

6.4.2.1.1.1. Probability Density Function-At Risk Conditions

During the audit control by the authorized environmental lead auditors, the potential non-conformance due to leachate detection or distributed environmental pollution in random locations might or might not be detected. Therefore, the binomial distribution is the most suitable density function for the detection of the total at-risk probability. The applied Probability Density Function in this sub-case is shown by equation

3. The factor of negative targeting Y(i) equals the numerical value "1" due to non-consecutive detection of potential non-conformance inside the field.

The Probability Density Function describes the total probability of the potential additive at-risk cost described through the Loss Function L(A(0),Y(i)) , which is estimated to increase for the consecutive years after the first non-compliance in sub-case $A(1)_1$. N refers to the audit control sites per year, estimated to be at least 4. Audit control sites for the consecutive years are increasing in an arithmetical progression (due to unexpected, inefficient environmental protection measures). In contrast, the stable factor equals the numerical value 4. Thus, audit control sites for the second year are at least 8, audit control sites for the third year are at least 12, and so on.

The natural meaning of PDF is to determine the incremental additive of growth for negative cost (160). Hence, the PDF considers the estimated future negative cost in terms of the constant corresponding negative cost in the case of 4 audit control positions per year. So, the N parameter, placed in the numerator function of the f(Y), refers to the additive control sites year by year after the first non-compliance. The N parameter, placed in the denominator function of the f(Y), refers to the stable numerical value of audit control sites in the arithmetical progression.

E[Y] refers to the estimated numerical value for Y(i). E[Y] is exported by multiplying N and probability $\theta_{(0)}$. Parameter N refers to the additive control sites year by year after the first non-compliance. Standard deviation σ^2 refers to the combination of the N factor (additive control sites year by year after the first non-compliance) and total probability for non-conformance ($\theta_{(0)max}=\theta_{(3)max}=0.25$ in sub-case $A(1)_1$), as shown by the equations (3-5).

6.4.2.1.1.2. Total Goal Function configured by the Theoretical (At Risk) Goal Function and Real Goal Function

The theoretical Goal Function $G(A(1)_1,Y(i))_t$ expresses the potential additive at-risk cost due to at-risk conditions in sub-case $A(1)_1$ as shown in equation 8. Further analysis of the configuration for the theoretical Goal Function is provided through equation 8a, including the determined numerical values of its parameters. The real Goal Function $G(A(1)_1,Y(i))_r$ expresses the total cost of investment in metal recovery processing. Thus, the total Goal Function $G(A(1)_1,Y(i))_{Total}$ determines the annual sum of the investment cost plus the potential at-risk cost in case of non-full conformance, as shown in equations 8a, 9a, 10a, respectively.

$$\mathbf{G\big(A(1)_1, Y(i)\big)_t = L\big(A(0), Y(i)\big) * f\,(Y(i))\{Y(i)_t = 1 \quad K(i) = 374{,}400\,€ \quad f\big(Y(i)\big) = 0.422, \quad 0 \le i \le 30\ Years} \quad (8a)$$

$$\mathbf{G\big(A(1)_1, Y(i)\big)_r = L\big(A(1)SF, Y(i)\big)\begin{cases} Y(i)_a = 1, & K(i) = K(1) = 7{,}421{,}960\,€ \quad 0 \le i \le 1 \quad Years \\ Y(i)_a = 0.802, & K(i) = K(2) = 5{,}953{,}160\,€ \quad 1 < i \quad Years \end{cases}} \quad (9a)$$

$$\mathbf{G\big(A(1)_1, Y(i)\big)_{Total} = G\big(A(1)_1, Y(i)\big)_t + G\big(A(1), Y(i)\big)_r \ \{0 \le i \le 30 \ \ Years} \quad (10a)$$

The results of the financial risk assessment through the CBA methodology are shown in Table A8 in the Appendix Section.

6.4.2.1.2. Sub-Case $A(1)_2$ of Unstable Negative Targeting. Potential Detection of Environmental Pollution in Specific Locations inside the Closed System of Industrial Units

Sub-case $A(1)_2$ refers to the potential additional cost due to the risk of detected environmental pollution during audit control, despite investing in metal recovery and legal compliance with the mandatory obligations of the mining industry. In this sub-case, it is considered that within a period of 30 years, environmental pollution in specific locations inside the field of the closed system with industrial units is detected due to unexpected leachate. To describe the potential financial risk expressed through the corresponding theoretical Goal Function, the negative cost expressed by the L(A(0),Y(i)) (as shown in equations (2, 7a)) is multiplied by the Probability Density Function of the binomial distribution (as shown in equation 3). It is essential to mention that in this sub-case, the total non-conformance is calculated considering that $\theta_{(0)}=\theta_{(3)max}$. So, the range of the total non-conformance equals 0-0.25.

6.4.2.1.2.1. Probability Density Function

During the audit control by the authorized environmental lead auditors, the potential non-conformance due to leachate detection or distributed environmental pollution in specific locations might or might not be detected. Therefore, the binomial distribution is the most suitable density function for the detection of the total at-risk probability. The applied Probability Density Function in this sub-case is shown by equation 3. The factor of negative targeting Y(i) equals ranges between numerical values "1-8.17" (as indicated by equation 2 and Table A1) due to the consecutive detection of potential non-conformance inside the field.

The Probability Density Function describes the total probability of the potential additive at-risk cost described through the Loss Function L(A(0),Y(i)) , which is estimated to increase for the consecutive years after the first non-compliance in sub-case $A(1)_2$. N refers to the audit control sites per year, estimated to be at least 4. Audit control sites for the consecutive years are increasing in an arithmetical progression (due to unexpected, inefficient environmental protection measures). In contrast, the stable factor equals the numerical value 4. Thus, audit control sites for the second year are at least 8, audit control sites for the third year are at least 12, and so on.

The natural meaning of PDF is to determine the incremental additive of growth for negative cost (160). Hence, the PDF considers the estimated future negative cost in terms of the constant corresponding negative cost in the case of 4 audit control positions per year. So, the N parameter, placed in the numerator function of the f(Y), refers to the additive control sites year by year after the first non-compliance. The N parameter, placed in the denominator function of the f(Y), refers to the stable numerical value of audit control sites in the arithmetical progression.

E[Y] refers to the estimated numerical value for Y(i). E[Y] is exported by multiplying N and probability θ(0). Parameter N refers to the additive control sites year by year after the first non-compliance. Standard deviation σ^2 refers to the combination of the N factor (additive control sites year by year after the first non-compliance) and total probability for non-conformance ($\theta_{(0)max}=\theta_{(3)max}=0.25$ in sub-case $A(1)_2$), as shown by the equations (3-5).

6.4.2.1.2.2. Total Goal Function configured by the Theoretical (At Risk) Goal Function and Real Goal Function

The theoretical Goal Function $G(A(1)_2,Y(i))_t$ expresses the potential additive at-risk cost due to at-risk conditions in sub-case $A(1)_2$ as shown in equation 8. Further analysis of the configuration for the theoretical Goal Function is provided through equation 8b, including the determined numerical values of its parameters. The real Goal Function $G(A(1)_2,Y(i))_r$ expresses the total cost of investment in metal recovery processing. Thus, the total Goal Function $G(A(1)_2,Y(i))_{Total}$ determines the annual sum of the investment cost plus the potential at-risk cost in case of non-full conformance, as shown in equations 8b, 9b, 10b, respectively.

$$G(A(1)_2, Y(i))_t = L(A(0), Y(i)) * \\ f\,(Y(i)) \begin{cases} Y(i)_t = 1 \quad K(i) = 374,400\,€ \quad f(Y(i)) = 0.422, \quad 0 \le i \le 1 \text{ Years} \\ Y(i)_t = 3.27 \quad K(i) = 1,223,165\,€ \quad f(Y(i)) = 0.035, \quad 1 < i \le 4 \text{ Years} \\ Y(i)_t = 8.17 \quad K(i) = 3,057,912\,€ \quad f(Y(i)) = 0.004, \quad 4 < i \le 30 \text{ Years} \end{cases} \quad (8b)$$

$$G(A(1)_2, Y(i))_r = L(A(1)SF, Y(i)) \begin{cases} Y(i)_a = 1, \quad K(i) = K(1) = 7,421,960\,€ \quad 0 \le i \le 1 \quad \text{Years} \\ Y(i)_a = 0.802, \quad K(i) = K(2) = 5,953,160\,€ \quad 1 < i \quad \text{Years} \end{cases} \quad (9b)$$

$$G(A(1)_2, Y(i))_{Total} = G(A(1)_2, Y(i))_t + G(A(1), Y(i))_r \ \{0 \le i \le 30 \ \text{ Years} \quad (10b)$$

The results of the financial risk assessment through the CBA methodology are shown in Table A9 in the Appendix Section.

6.4.2.1.3. Sub-Case $A(1)_3$ of Stable Negative Targeting. Expected Detection of Distributed Environmental Pollution in Random Locations inside the Closed System of Industrial Units

Sub-case $A(1)_3$ refers to the potential additional cost due to the risk of detected environmental pollution during audit control, despite investing in metal recovery and legal compliance with the mandatory obligations of the mining industry. In this sub-case, at least one annual non-conformance within a period of 30 years is expected due to environmental pollution caused by unexpected leachates in random locations inside the field of the closed system with industrial units for metal recovery. To describe the potential financial risk expressed through the corresponding theoretical Goal Function, the negative cost expressed by the L(A(0),Y(i)) (as shown in equation (7b)) is multiplied by the Probability Density Function of the Erlang distribution (as shown in equation (11)). It is essential to mention that in this sub-case, the total non-conformance is calculated considering that $\theta_{(0)}=\theta_{(3)max}$. So, the range of the total non-conformance equals 0-0.25.

6.4.2.1.3.1. Probability Density Function

During the audit control by the authorized environmental lead auditors, the non-conformance due to leachate detection or distributed environmental pollution in random locations is expected to be at least one. Therefore, the Erlang distribution is the most suitable density function for the detection of the total at-risk probability. The applied Probability Density Function in this sub-case is shown by equation 11.

$$f\,(x, k, \lambda) = [\lambda^k] * [x^{k-1}] * [e^{-\lambda x}] * [[k-1]!]^{-1} \quad (11)$$

Erlang Probability Density Function is applied to calibrate the at-risk Loss Function L(A(0),Y(i)) in case of identified environmental pollution despite the fact of implementing close industrial units to recover metal materials and protect from huge free mining waste disposal. This calibration outputs the theoretical goal function $G(A(1)_3, Y(i))_t$, which refers to the negative cost for non-compliance with environmental protective measures. Thus, L(A(1)-SF, Y(i)) refers to the real goal function $G(A(1)_3, Y(i))_r$, about the total annual investment costs as they are shown in Table A7. In contrast, the $G(A(1)_3, Y(i))_t$ refers to the probable non-compliance due to other reasons (non-efficient recovery method and identified pollution).

Factor λ refers to the stable number of audit control sites (4), factor k refers to the sum of audit control sites per year (4,8,12,…n), and factor x refers to the maximum probability of $\theta_{(0)}=\theta_{(3)max}=0.25$. Erlang PDF weights the at-risk Loss Function L(A(0), Y(i)) to extract the theoretical goal function due to identified environmental pollution. The physical meaning of this calibration is to determine the probable additive financial cost in case of identified pollution despite implementing environmentally friendly procedures.

The factor of negative targeting Y(i) equals the numerical value "1" due to expected non-conformance in random locations inside the field of the closed system with industrial units (there is no negative targeting).

The Probability Density Function describes the total probability of the expected additive at-risk cost described through the Loss Function L(A(0),Y(i)) , which is estimated to increase for the consecutive years after the first non-compliance in sub-case $A(1)_3$. N refers to the audit control sites per year, estimated to be at least 4. Audit control sites for the consecutive years are increasing in an exponential progression (due to unexpected, inefficient environmental protection measures). In contrast, the stable factor equals the numerical value 4. Thus, audit control sites for the second year are at least 8, audit control sites for the third year are at least 12, etc.

The natural meaning of PDF is to determine the incremental additive of growth for negative cost (119) (157).

Hence, the PDF considers the estimated future negative cost in terms of the constant corresponding negative cost in the case of 4 audit control positions per year. So, the k parameter, placed in the numerator function of the f(Y), refers to the additive control sites year by year after the first non-compliance. The λ parameter, placed in the f(Y), refers to the stable numerical value of audit control sites per year of operation.

E[Y] refers to the estimated numerical value for Y(i). E[Y] is exported by multiplying N and probability θ(0) as shown in equation 12.

$$\mathbf{E[Y] = N * \theta(0)} \quad (12)$$

Parameter N refers to the additive control sites year by year after the first non-compliance. The maximum total probability for non-conformance is $\theta_{(0)max}$ ($\theta_{(0)max}=\theta_{(3)max}=0.25$ in sub-case $A(1)_3$).

6.4.2.1.3.2. Total Goal Function configured by the Theoretical (At Risk) Goal Function and Real Goal Function

The theoretical Goal Function $G(A(1)_3,Y(i))_t$ expresses the potential additive at-risk cost due to at-risk conditions in sub-case $A(1)_3$ as shown in equation 8. Further analysis of the configuration for the theoretical Goal Function is provided through equation 8c, including the determined numerical values of its parameters. The real Goal Function $G(A(1)_3,Y(i))_r$ expresses the total cost of investment in metal recovery processing. Thus, the total Goal Function $G(A(1)_3,Y(i))_{Total}$ determines the annual sum of the investment cost plus the potential at-risk cost in case of non-full conformance, as shown in equations 8c, 9c, 10c, respectively.

$$\mathbf{G(A(1)_3, Y(i))_t = L(A(0), Y(i)) * f(Y(i)) \quad \{Y(i)_t = 1 \quad K(i) = 374,400\ € \quad 2.66E-197 \le f(Y(i)) \le 2.47E-01, \quad 0 \le i \le 30\ Years} \quad (8c)$$

$$\mathbf{G(A(1)_3, Y(i))_r = L(A(1)SF, Y(i)) \begin{cases} Y(i)_a = 1, & K(i) = K(1) = 7,421,960\ € \quad 0 \le i \le 1 \quad Years \\ Y(i)_a = 0.802, & K(i) = K(2) = 5,953,160\ € \quad 1 < i \quad Years \end{cases}} \quad (9c)$$

$$\mathbf{G(A(1)_3, Y(i))_{Total} = G(A(1)_3, Y(i))_t + G(A(1), Y(i))_r \quad \{0 \le i \le 30 \quad Years} \quad (10c)$$

The results of the financial risk assessment through the CBA methodology are shown in Table A10 in the Appendix Section.

6.4.2.1.4. Sub-Case $A(1)_4$ of Unstable Negative Targeting. Expected Detection of Environmental Pollution in Specific Locations inside the Closed System of Industrial Units

Sub-case $A(1)_4$ refers to the potential additional cost due to the risk of detected environmental pollution during audit control, despite investing in metal recovery and legal compliance with the mandatory obligations of the mining industry. In this sub-case, at least one annual non-conformance is expected within a 30-year period due to environmental pollution caused by unexpected leachates in specific locations within the field of the closed system, which includes industrial units for metal recovery. To describe the potential financial risk expressed through the corresponding theoretical Goal Function, the negative cost expressed by the L(A(0),Y(i)) (as shown in equation (7b)) is multiplied by the Probability Density Function of the Erlang distribution (as shown in equation (11)) including the increase of the negative targeting Y(i). It is essential to mention that in this sub-case, the total non-conformance is calculated considering that $\theta_{(0)}=\theta_{(3)max}$. So, the range of the total non-conformance equals 0-0.25.

6.4.2.1.4.1. Probability Density Function

During the audit control by the authorized environmental lead auditors, the non-conformance due to leachate detection or distributed environmental pollution in random locations is expected to be at least one. Therefore, the Erlang distribution is the most suitable density function for the detection of the total at-risk probability. The applied Probability Density Function in this sub-case is shown by equation 11.

The Erlang Probability Density Function is applied to calibrate the at-risk Loss Function L(A(0),Y(i)) in case of identified environmental pollution despite the fact of implementing close industrial units to recover metal materials and protect from huge free mining waste disposal. This calibration outputs the theoretical goal function $G(A(1)_4, Y(i))_{t,}$ which refers to the negative cost for non-compliance with environmental protective measures. Thus, L(A(1)-SF, Y(i)) refers to the real goal function $G(A(1)_4, Y(i))_r$, about the total annual investment costs as they are shown in Table A7. In contrast, the $G(A(1)_4, Y(i))_t$ refers to the probable non-compliance due to other reasons (non-efficient recovery method and identified pollution).

Factor λ refers to the stable number of audit control sites (4), factor k refers to the sum of audit control sites per year (4,8,12,…n), and factor x refers to the maximum probability of $\theta_{(0)}=\theta_{(3)max}=0.25$. Erlang PDF weights the at-risk Loss Function L(A(0), Y(i)) to extract the theoretical goal function due to identified environmental pollution. The physical meaning of this calibration is to determine the probable additive financial cost in case of identified pollution despite implementing environmentally friendly procedures.

The factor of negative targeting Y(i) ranges between the numerical values "1-8.17" due to expected non-conformance in specific locations inside the field of the closed system with industrial units (there is no negative targeting) .

The Probability Density Function describes the total probability of the expected additive at-risk cost described through the Loss Function L(A(0),Y(i)) , which is estimated to increase for the consecutive years after the first non-compliance in sub-case $A(1)_4$. N refers to the audit control sites per year, estimated to be at least 4. Audit control sites for the consecutive years are increasing in an exponential progression (due to unexpected, inefficient environmental protection measures). In contrast, the stable factor equals the numerical value 4. Thus, audit control sites for the second year are at least 8, audit control sites for the third year are at least 12, etc.

The natural meaning of PDF is to determine the incremental additive of growth for negative cost (119) (157) (158). Hence, the PDF considers the estimated future negative cost in terms of the constant corresponding negative cost in the case of 4 audit control positions per year. So, the k parameter, placed in the numerator function of the f(Y), refers to the additive control sites year by year after the first non-compliance. The λ parameter, placed in the f(Y), refers to the stable numerical value of audit control sites per year of operation.

E[Y] refers to the estimated numerical value for Y(i). E[Y] is exported by multiplying N and probability θ(0) as shown in equation 12. Parameter N refers to the additive control sites year by year after the first non-compliance. The maximum total probability for non-conformance is $\theta_{(0)max}$ ($\theta_{(0)max}=\theta_{(3)max}=0.25$ in sub-case $A(1)_4$).

6.4.2.1.4.2. Total Goal Function configured by the Theoretical (At Risk) Goal Function and Real Goal Function

The theoretical Goal Function $G(A(1)_4,Y(i))_t$ expresses the potential additive at-risk cost due to at-risk conditions in sub-case $A(1)_4$ as shown in equation 8. Further analysis of the configuration for the theoretical Goal Function is provided through equation 8d, including the determined numerical values of its parameters. The real Goal Function $G(A(1)_4,Y(i))_r$ expresses the total cost of investment in metal recovery processing. Thus, the total Goal Function $G(A(1)_4,Y(i))_{Total}$ determines the annual sum of the investment cost plus the potential at-risk cost in case of non-full conformance, as shown in equations 8d, 9d, 10d, respectively.

$$\mathbf{G(A(1)_4, Y(i))_t = L(A(0), Y(i)) *}$$
$$\mathbf{f(Y(i))} \begin{cases} \mathbf{Y(i)_t = 1 \quad K(i) = 374,400\,€ \quad f(Y(i)) = 2.47E-01, \quad 0 \le i \le 1\ Years} \\ \mathbf{Y(i)_t = 3.27 \quad K(i) = 1,223,165\,€ \quad 1.13E-12 < f(Y(i)) \le 2.94E-04, \quad 1 < i \le 4\ Years} \\ \mathbf{Y(i)_t = 8.17 \quad K(i) = 3,057,912\,€ \quad 2.66E-197 < f(Y(i)) \le 1.22E-17, \quad 4 < i \le 30\ Years} \end{cases} \quad (8d)$$

$$\mathbf{G(A(1)_4, Y(i))_r = L(A(1)SF, Y(i))} \begin{cases} \mathbf{Y(i)_a = 1, \quad K(i) = K(1) = 7,421,960\,€ \quad 0 \le i \le 1 \quad Years} \\ \mathbf{Y(i)_a = 0.802, \quad K(i) = K(2) = 5,953,160\,€ \quad 1 < i \quad Years} \end{cases} \quad (9d)$$

$$\mathbf{G(A(1)_4, Y(i))_{Total} = G(A(1)_4, Y(i))_t + G(A(1), Y(i))_r \ \{0 \le i \le 30 \ \ Years} \quad (10d)$$

The results of the financial risk assessment through the CBA methodology are shown in Table A11 in the Appendix Section.

6.4.2.1.5. Sub-Case $A(1)_5$ of Unstable Negative Targeting. Expected Detection of Distributed Environmental Pollution in Specific Locations inside the Closed System of Industrial Units. Financial Risk is calculated based on the Annual Stable Total Investment Cost

Sub-case $A(1)_5$ refers to the potential additional cost due to the risk of detected environmental pollution during audit control, despite investing in metal recovery and legal compliance with the mandatory obligations of the mining industry. In this sub-case, at least one annual non-conformance is expected within a 30-year period due to environmental pollution caused by unexpected leachates in specific locations within the field of the closed system, which includes industrial units for metal recovery. Based on the practical application meaning of this sub-case, it is considered that the expected additive money loss due to detected leachates or misshaping of the equipment to be used is proportional to the total investment cost. It is observed that the total cost of investment to waste treat mining tailings in Case Scenario A(1), expressed through the actual L(A(1)_SF,Y(i)) or $G(A(1),Y(i))_r$ is approximately equaled to the additive penalty cost of non-compliance in Case Scenario A(0) that refers to the period from 1st to 5th Year of Operation.

So, from a clear, practical point of view, the expected additional at-risk cost should be decreasing annually while investing in metal recovery operations and avoiding free tailings disposal. To describe the expected financial risk expressed through the corresponding theoretical Goal Function, the negative cost expressed by the L(A(1)_SF,Y(i)) (as shown in equation (7c)) is multiplied by the Probability Density Function of the Erlang distribution (as shown in equation (11)) including the negative targeting Y(i) depending on the annual stable money loss. It is essential to note that in this sub-case, the total non-conformance is calculated considering that $\theta_{(0)}=\theta_{(3)max}$. So, the range of the total non-conformance equals 0-0.25.

6.4.2.1.5.1. Probability Density Function

During the audit control by the authorized environmental lead auditors, the non-conformance due to leachate detection or distributed environmental pollution in random locations is expected to be at least one. Therefore, the Erlang distribution is the most suitable density function for the detection of the total at-risk probability. The applied Probability Density Function in this sub-case is shown by equation 11.

The Erlang Probability Density Function is applied to calibrate the actual Loss Function L(A(1)_SF,Y(i)) and calculate the expected additive negative cost. This calibration outputs the theoretical goal function $G(A(1)_5, Y(i))_t$, which refers to the negative cost for non-compliance with environmental protective measures, depending on the total investment cost (in case of identified environmental pollution despite the fact of implementing close industrial units to recover metal materials and protect from huge free mining waste disposal). Thus, L(A(1)-SF, Y(i)) refers to the real goal function $G(A(1)_5, Y(i))_r$, about the total annual investment costs as they are shown in Table A7. In contrast, the $G(A(1)_5, Y(i))_t$ refers to the probable non-compliance due to other reasons (non-efficient recovery method and identified pollution).

Factor λ refers to the stable number of audit control sites (4), factor k refers to the sum of audit control sites per year (4,8,12,…n), and factor x refers to the maximum probability of θ(0)=θ(3)max=0.25. Erlang PDF weights the actual Loss Function L(A(1)_SF, Y(i)) to extract the theoretical goal function due to identified environmental pollution. The physical meaning of this calibration is to determine the probable additive financial cost, which is proportional to the total investment cost.

The factor of negative targeting Y(i) ranges between the numerical values "1-0.802" as shown in equation 7, due to expected non-conformance that is proportional to the quality and efficiency of the equipment to be used inside the field of the closed system with industrial units. Therefore, negative targeting is expected to decrease year by year.

The Probability Density Function describes the total probability of the expected additive at-risk cost described through the theoretical Goal Function $G(A(1)_5, Y(i))_t$, which is estimated to decrease for the consecutive years after the first non-compliance in sub-case $A(1)_5$. N refers to the audit control sites per year, estimated to be at least 4. Audit control sites for the consecutive years are increasing in an exponential progression (due to unexpected, inefficient environmental protection measures). In contrast, the stable factor equals the numerical value 4. Thus, audit control sites for the second year are at least 8, audit control sites for the third year are at least 12, etc.

The natural meaning of PDF is to determine the incremental additive of growth for negative cost (119) (157) (158). Hence, the PDF considers the estimated future negative cost in terms of the constant corresponding negative cost in the case of 4 audit control positions per year. So, the k parameter, placed in the numerator function of the f(Y), refers to the additive control sites year by year after the first non-compliance. The λ parameter, placed in the f(Y), refers to the stable numerical value of audit control sites per year of operation.

E[Y] refers to the estimated numerical value for Y(i). E[Y] is exported by multiplying N and probability θ(0) as shown in equation 12. Parameter N refers to the additive control sites year by year after the first non-compliance. The maximum total probability for non-conformance is θ(0)max (θ(0)max=θ(3)max=0.25 in sub-case $A(1)_5$).

6.4.2.1.5.2. Total Goal Function configured by the Theoretical (At Risk) Goal Function and Real Goal Function

The theoretical Goal Function $G(A(1)_5,Y(i))_t$ expresses the expected additive at-risk cost due to at-risk conditions in sub-case $A(1)_5$ as shown in equation 8. Further analysis of the configuration for the theoretical Goal Function is provided through equation 8e, including the determined numerical values of its parameters. The real Goal Function $G(A(1)_5,Y(i))_r$ expresses the total cost of investment in metal recovery processing. Thus, the total Goal Function $G(A(1)_5,Y(i))_{Total}$ determines the annual sum of the investment cost plus the potential at-risk cost in case of non-full conformance, as shown in equations 8e, 9e, 10e, respectively.

$$\mathbf{G\big(A(1)_5, Y(i)\big)_t = L\big(A(1)_{SF}, Y(i)\big) * f\,(Y(i)) \begin{cases} Y(i)_{a,t} = 1 \quad K(i) = K(1) \quad f\big(Y(1)\big) = 0.298 \quad 0 \le i \le 1 \text{ Years} \\ Y(i)_{a,t} = 0.802 \quad K(i) = K(2) \quad f\big(Y(2)\big) = 0 \quad 1 < i \text{ Years} \end{cases}} \quad (8e)$$

$$\mathbf{G\big(A(1)_5, Y(i)\big)_r = L\big(A(1)\_SF, Y(i)\big) \begin{cases} Y(i)_{a,t} = 1, \quad K(i) = K(1) \quad 0 \le i \le 1 \text{ Years} \\ Y(i)_{a,t} = 0.802, \quad K(i) = K(2) \quad 1 < i \text{ Years} \end{cases}} \quad (9e)$$

$$\mathbf{G\big(A(1)_5, Y(i)\big)_{Total} = K(i) * Y(i) * [1 + f(Y(i))] \begin{cases} Y(i)_{a,t} = 1 \quad K(i) = K(1) \quad f\big(Y(1)\big) = 0.298 \quad 0 \le i \le 1 \text{ Years} \\ Y(i)_{a,t} = 0.802 \quad K(i) = K(2) \quad f\big(Y(2)\big) = 0 \quad 1 < i \text{ Years} \end{cases}}$$
(10e)

The results of the financial risk assessment through the CBA methodology are shown in Table A12 in the Appendix Section.

7. Stochastic Financial Risk Assessment Methodology and Results

7.1. Overview of the SRA in Mining Waste Management

The SRA methodology is applied to estimate the degree of financial riskassessment, which refers to the financial risk assessment escalation provided from the deterministic approach of CBA through the total goal functions of each examined case. Considering the posterior conditions determined by the cost functions of CBA, the SRA methodology estimates the prior economic risk grade that caused them, implementing the Bayesian analysis simulation.

As a purely scientific approach, SRA focuses on the risk probability distribution inside the working period in each case's actual condition. The total risk might or might not be detected during the environmental inspection at a mining company's sites. So, based on the axiom that the detection of non-conformance may or may not be identified, the false-trial methodology is implemented to describe risky conditions (69) (70). According to these actual circumstances, to assess the total risk except for the primary cost function configured by CBA, the corresponding binomial stochastic Probability Density Function and its prior Beta distribution expressing the fluctuation of risk probability are needed (53) (55) (69) (70).

To ensure the sensitivity of the SRA approach, theoretical sub-cases of Case Scenario A(1) are evaluated. Sub-cases $A(1)_{1,3,4}$ refer to theoretical conditions of potential detection of widespread environmental pollution, expected detection of environmental pollution in random locations inside the mining area, and expected detection of environmental pollution in specific points inside the field, respectively, despite investing in mining waste management through a closed system of unit plants and in their maintenance, annually.

Sub-cases $A(1)_{2,5}$ refer to actual conditions of a potential risk detection. Sub-case $A(1)_2$ refers to potential environmental pollution detection in specific points inside the mining area. Sub-case $A(1)_5$ refers to the expected detection of environmental pollution proportional to the quality of equipment used to manage mining waste.

In sub-cases where the detection of environmental pollution might or might not be detected, the binomial stochastic Probability Density Function and its prior, Beta distribution are considered for the potential risk detection. In sub-cases where the detection of environmental pollution is expected during the working period, the Erlang stochastic Probability Density Function and its mixture with Beta distribution are considered for the potential risk detection. Despite the prior distribution of the binomial Probability Density Function being the Gamma, considering the potentiality of risk, the most appropriate distribution is BetE (Beta-Erlang mixture of Sarmanov family distributions). Evaluating the actual conditions, the Sarmanov family distribution is the most optimal to assess the potential risk, especially in finance and engineering (42) (55) (87).

The SRA applies the stochastic risk integral, including as a differential, the risk probability of non-conformance, to estimate the numerical value calculated from the generating function of multiplying the variables of cost functions from CBA—stochastic Probability Density Functions—and prior Beta distribution. Two types of risk indicators are calculated. The first type of stochastic financial risk indicator that refers to the grade of financial risk, including monetary terms, is symbolized as R(A(0 or 1),Y(i)). The second type of stochastic risk indicator, including only the probability of non-conformance, is symbolized as $R(A(0 \text{ or } 1),\theta_{(0)})$.

Applying SRA allows a comparative assessment of its results with CBAs. The theoretical sub-cases are identified, and a deterministic risk matrix is structured by comparing the stochastic financial risk ratios of all cases. SRA methodology verifies CBA's results and which of the examined sub-cases are unlikely to occur in actual circumstances.

7.2. Stochastic Financial Risk Assessment in Case Scenario A(0)

This section focuses on the stochastic risk assessment of Case Scenario A(0). The SRA is applied to assess the Risk Indicator R, considering the mandatory legal obligations of the mining industry regarding tailings management. The stochastic indicator R shows the grade of economic sustainability risk that describes Case Scenario A(0) based on the terms and definitions of the Circular Economy's principles that are expressed through the corresponding legal obligations. The calculated risk indicator R(A(0),Y(i)) describes the stochastic financial risk of the Case Scenario A(0), including monetary terms. The corresponding risk indicator R(A(0),$\theta_{(0)}$) shows the grade of the stochastic financial risk growth in Case Scenario A(0).

7.2.1. Application of Stochastic Risk Integral

The corresponding Risk Integral, as shown in equation 13c, calculates the Stochastic Financial Risk of the Case Scenario A(0) through Bayesian Analysis. In Case Scenario A(0), the Risk Integral limits' range is determined by the addition of $\theta_{(0)}$ partial probabilities, as shown in Table 3. Thus, considering the minimum risky conditions, the limit down equals 0.75, and the limit up equals 1. Total Stochastic Financial Risk is exported by Equations 13a-13c (95) (86).

Equations 13a to 13c demonstrate the configuration of the total Stochastic Risk Integral. Equation 14 analyzes the configuration of partial risks that structure the total financial risk in Case Scenario A(0) (95) (86).

$$\mathbf{R(A(0), Y(i))} = \sum_{i=1}^{n} \mathbf{R(A(0), Y(i)} \quad (13a)$$

$$\mathbf{R(A(0), Y(i))} = \mathbf{R(A(0), Y(1))} + \mathbf{R(A(0), Y(2,3,4))} + \mathbf{R(A(0), Y(5))} \quad (13b)$$

$$\mathbf{R(A(0), Y(i))} = \int_{0.75}^{1} \mathbf{E[G(A(0), \theta_{(0)})] * Beta(t, r, \theta_{(0)}) d\theta(0)} \quad (13c)$$

$$\mathbf{R(A(0), \quad Y(i))} = \int_{0.75}^{1} \mathbf{(K(1) * E(Y(1)^2) * f(Y(1)) + K(1) * E(Y(2,3,4)^2) * f(Y(2,3,4)) + K(1) * E(Y(5)^2) * f(Y(5))) * Beta(t, r, \theta_{(0)})\, d\theta_{(0)}} \quad (14)$$

The estimated value of the Y(i)^2 factor is symbolized as E[Y(i)^2], and its calculation is given using the following equations (15-18).

a) $\mathbf{E[Y^2] = (m(\theta_{(0)}))^2 + \sigma^2}$ (15), whereas

b) $\mathbf{m(\theta_{(0)}) = \frac{N(Audit\ Control\ Sites)}{N(Working\ Days)} = \frac{4}{300} = 0.013}$ (16) and

$\mathbf{\sigma^2 = N(Aud. Control\ Sites = 4, 16, . \geq 20) * \theta_{(0)}(1 - \theta_{(0)})}$ (17).

c) $\mathbf{\sigma^2 = N * \theta_{(0)}(1 - \theta_{(0)})}$ (18)

Considering the stochastic parameters described in equations 15-18 the following Stochastic Risk Integrals are determined. **R(A(0), Y(1))** refers to the first classification of non-conformance (first year) as shown in equation 19, **R(A(0), Y(2,3,4))** refers to the second classification of non-conformance (second, third, and fourth year) as shown in equation 20, and **R(A(0), Y(5))** refers to the stable risk for the classification after fourth year as shown in equation 21.

$$R(A(0),Y(1))=\int_{0.75}^{1}(\mathbf{K(1)} * (\mathbf{0.000174} + \mathbf{4} * \theta_{(0)} * (\mathbf{1} - \theta_{(0)})) * \left[\left(\frac{1}{(4*\theta_{(0)})!}\right)\right] * \left(\left[\frac{1}{(4-4*\theta_{(0)})!}\right]\right) * \mathbf{4!} * (\theta_{(0)}{}^{4*\theta_{(0)}}) * (\mathbf{1} - \theta_{(0)})^{4*(1-\theta_{(0)})}) * \mathbf{Beta}(t, r, \theta_{(0)})d\theta_{(0)} \quad (19)$$

$$R(A(0),Y(2,3,4)) \quad =\int_{0.75}^{1}(\mathbf{K(1)} * (\mathbf{0.000174} + \mathbf{16} * \theta_{(0)} * (\mathbf{1} - \theta_{(0)})) * \left[\left(\frac{1}{(16*\theta_{(0)})!}\right)\right] * \left(\left[\frac{1}{(16-16*\theta_{(0)})!}\right]\right) * \mathbf{16!} * (\theta_{(0)}{}^{16*\theta_{(0)}}) * (\mathbf{1} - \theta_{(0)})^{16*(1-\theta_{(0)})}) * \mathbf{Beta}(t, r, \theta_{(0)})d\theta_{(0)} \quad (20)$$

$$R(A(0),Y(5)) \quad =\int_{0.75}^{1}(\mathbf{K(1)} * (\mathbf{0.000174} + \mathbf{20} * \theta_{(0)} * (\mathbf{1} - \theta_{(0)})) * \left[\left(\frac{1}{(20*\theta_{(0)})!}\right)\right] * \left(\left[\frac{1}{(20-20*\theta_{(0)})!}\right]\right) * \mathbf{20!} * (\theta_{(0)}{}^{20*\theta_{(0)}}) * (\mathbf{1} - \theta_{(0)})^{20*(1-\theta_{(0)})}) * \mathbf{Beta}(t, r, \theta_{(0)})d\theta_{(0)} \quad (21)$$

7.2.2. Beta Distribution and its Practical Application

Beta distribution is the prior of the applied Probability Density Function of the binomial distribution. Equation 22 shows the Beta distribution, and its physical meaning inside Risk Integral is to calibrate the stochastic goal function.

$$\mathbf{Beta}(t, r, \theta_{(0)}) = \left[\frac{1}{(r-1)!}\right] * \left[\frac{1}{(t-r-1)!}\right] * (t-1)! * \left[(\theta_{(0)})^{r-1}\right] * \left[(1-\theta_{(0)})^{t-r-1}\right] \quad (22)$$

t refers to the annual working days=300**, and r** refers to the minimum audit control sites per year=4. Equation 23 configures the beta Function based on the numerical values of t and r.

$$\mathbf{Beta}(t, r, \theta_{(0)}) = \left(\frac{299!}{3!*295!}\right) * (\theta_{(0)})^{3} * ((1-\theta_{(0)})^{295}) \quad (23)$$

Accounting for all these parameters, the Stochastic Financial Risk Integral is identified. Table A13 (in the Appendix) summarizes all the applied parameters to describe the at-risk situation in Case Scenario A(0). In this case, the false–trial methodology is applied to assess the potential stochastic financial risk due to non-compliance with the legal obligations that express the policy of Circular Economy.

7.3. Stochastic Financial Risk Assessment in Case Scenario A(1)

This section focuses on the stochastic risk assessment of all sub-cases in Case Scenario A(1). The SRA is applied to assess the Risk Indicator R, considering the mandatory legal obligations of the mining industry, regarding the tailings management. The stochastic indicator R shows the grade of economic sustainability risk that describes Case Scenario A(1) based on the terms and definitions of the Circular Economy's principles that are expressed through the corresponding legal obligations. The calculated risk indicator $R(A(1)_j,Y(i))$ describes the stochastic financial risk of the Case Scenario A(1), including monetary terms. The corresponding risk indicator $R(A(1)_j,\theta_{(0)})$ shows the grade of the stochastic financial risk growth in Case Scenario A(1).

7.3.1. Application of Stochastic Risk Integral

The corresponding Risk Integral, as shown in equation 24, calculates the Stochastic Financial Risk of all sub-cases in Case Scenario A(1) through Bayesian Analysis. In Case Scenario A(1), the Risk Integral limits' range is determined by the addition of $\theta_{(0)}$ partial probabilities, as shown in Table 3. Thus, considering the maximum risky conditions, the limit down equals 0, and the limit up equals 0.25. Total Stochastic Financial Risk is exported by equation 24 (86). Equations 24, 25 demonstrate the configuration of the total Stochastic Risk Integral for all sub-cases of the Case Scenario A(1). Equations 26 and 27 analyze the configuration of partial risks that structure the total financial risk in the sub-cases of the Case Scenario A(1) (42) (56) (55) (86) (95) (164).

$$\mathbf{R}\left(\mathbf{A(1)_j, Y(i)}\right) = \sum_{i=1}^{n} \mathbf{R(A(1)_j, Y(i)} \quad (24)$$

$$\mathbf{R}\left(\mathbf{A(1)_j, Y(i)}\right) = \int_{0}^{0.25} \mathbf{E[G(A(1)_j, \theta_{(0)})] * Beta(t, r, \theta_{(0)}) d\theta(0)} \quad (25)$$

The factor $E[G(A(1)_j,\theta_{(0)}]$ refers to the stochastic total annual goal function that describes each sub-case. This stochastic annual total goal function is configured by multiplying the total annual loss function, exported from CBA, with the stochastic Probability Density Function that describes each sub-case. So, in sub-cases $A(1)_1$ and $A(1)_2$, the binomial Probability Density Function is applied to identify the potential stochastic goal function. In contrast, the Erlang Probability Density Function is applied to identify the expected stochastic goal function.

$R(A(1)_j, Y(i))$ refers to the annual classification of non-conformance (in each sub-case of Case Scenario A(1)) as shown in equations 26 and 27. L(i) refers to the total annual loss function that is structured by the sum of the actual and the theoretical loss function, including SF. Equation 26 describes the $R(A(1)_j,Y(i))$ in sub-cases $A(1)_1$ and $A(1)_2$. Equation 27 describes the $R(A(1)_j,Y(i))$ in sub-cases $A(1)_3$ and $A(1)_4$.

$$\mathbf{R(A(1)_j,Y(i))} = \int_{0}^{0.25} \mathbf{L(i) * (0.000174 + Ni * \theta_{(0)} * (1-\theta_{(0)})) * \left[\left(\frac{1}{(Ni*\theta_{(0)})!}\right)\right] * \left(\left[\frac{1}{(Ni - Ni*\theta_{(0)})!}\right]\right) * Ni! * \left({\theta_{(0)}}^{Ni*\theta_{(0)}}\right) * (1-\theta_{(0)})^{Ni*(1-\theta_{(0)})}) * Beta(t, r, \theta_{(0)}) d\theta_{(0)}} \quad (26)$$

$$\mathbf{R(A(1)_j,Y(i))} = \int_{0}^{0.25} \mathbf{L(i) * Ni * \theta_{(0)} * \left\{\left[(Ni)^{Ni\theta_{(0)}}\right] * \left[{\theta_{(0)}}^{Ni\theta_{(0)}-1}\right] * \frac{e^{[-Ni\theta_{(0)}]}}{[4-1]!}\right\} * Beta(t, r, \theta_{(0)}) d\theta_{(0)}} \quad (27)$$

Ni refers to the numerical value of the sum of audit control sites corresponding to each specific negative targeting class. So, in sub-cases $A(1)_1$ and $A(1)_3$ Ni equals 4 due to non-increase of the negative targeting Y(i). In sub-case $A(1)_2$, and $A(1)_4$ Ni equals 4, 16, 20 for the first, second, and third classes of negative targeting Y(i) as shown in equation 28.

$$\mathbf{Ni} = \begin{cases} \mathbf{4,} & \mathbf{Y(i) = 1} \\ \mathbf{16,} & \mathbf{Y(i) = 3.27} \\ \mathbf{20,} & \mathbf{Y(i) = 8.17} \end{cases} \quad (28)$$

Equations 29, 30 describe the partial $R(A(1)_j,Y(i))$ indicators for the sub-case $A(1)_5$. These indicators structure the estimated stochastic financial risk in sub-case $A(1)_5$. In this sub-case, it is considered that the loss function inside the stochastic risk integral is expressed by the total goal function that is structured by the sum of the actual and the theoretical goal function. This axiom is adopted because the total goal function calculated by the CBA may deviate from the actual expected conditions (even if these conditions refer to the total investment cost or the expected penalty cost despite implementing environmental safety procedures).

$$\mathbf{R(A(1),Y(1))=\int_0^{0.25} G\big(A(1)_5,Y(1)\big)_{Total} * 4 * \theta_{(0)} * \left\{\left[(4)^{4\theta_{(0)}}\right] * \left[\theta_{(0)}{}^{4\theta_{(0)}-1}\right] * \frac{e^{\left[-4\theta_{(0)}\right]}}{[4-1]!}\right\} * Beta\big(t,r,\theta_{(0)}\big)d\theta_{(0)}} \quad (29)$$

$$\mathbf{R(A(1),Y(2,3,4,\ldots n))=\int_0^{0.25} G\big(A(1)_5,Y(2)\big)_{Total} * 8 * \theta_{(0)} * \left\{\left[4^{(8,12,16\ldots n)\theta_{(0)}}\right] * \left[\theta_{(0)}{}^{(8,12,16\ldots n)\theta(0)-1}\right] * \frac{e^{\left[-4\theta_{(0)}\right]}}{[(8,12,16\ldots n)-1]!}\right\} * Beta\big(t,r,\theta_{(0)}\big)\,d\theta_{(0)}} \quad (30)$$

7.3.2. Beta Distribution and its Practical Application

Beta distribution is the prior of the applied Probability Density Function of the binomial distribution. Equation 22 shows the beta distribution, and its physical meaning inside Risk Integral is to calibrate the stochastic goal function to assess the potential stochastic financial risk. Accounting for all these parameters, the Stochastic Financial Risk Integral is identified.

Table A14 (in the Appendix) summarizes all the applied parameters to describe the at-risk situation in sub-cases $A(1)_1$ and $A(1)_2$. In these cases, the false–trial methodology is applied to assess the potential stochastic financial risk due to non-compliance with the legal obligations that express the policy of Circular Economy. Table A15 (in the Appendix) summarizes all the applied parameters to describe the at-risk situation in sub-cases $A(1)_3$ to $A(1)_5$. In these cases the linear approach is applied to assess the expected stochastic financial risk due to non-compliance with the legal obligations that express the policy of Circular Economy.

8. Results and Discussion

8.1. Techno-Economic Assessment, under the Principles of Circular Economy and Environmental Risks

8.1.1. Key Factors Affecting the Sustainability Level in Case Scenario A(0)

Considering the legal obligations that configure the negative cost expressed by the annual Penalty Cost K(i) of the Case Scenario A(0), it is observed that the key factors are involved in the parameter K(i) (as shown in equation 1 and Table A1). These factors refer to a) the standard penalty cost, b) the annual revenue growth of each company, and c) the weighting factor ΣΒ depending on the classification of waste risk. The crucial factors refer to a) the annual $\Sigma\varepsilon\varepsilon\pi_i$ indicator that configures the corresponding indicator of negative targeting Y(i), and b) the total probability of non-conformance. It is essential to mention that, based on the legislation for environmental protection, the potential detected environmental pollution corresponds only to a quarter (25%) of total non-conformance, as shown in Table 4. This study takes into account the axiom that there is at least one environmental inspection per year, while the minimum risky conditions describe the conditions of Case Scenario A(0). So, the techno-economic assessment described by the Penalty Cost refers to the best-case conditions ($\theta_{(0)}$=75%) in case of non-legal conformance with environmental legislative requirements. Figure 14 shows the behavior of the crucial factor Y(i) and the total estimated negative cost expressed by the Penalty Cost K(i) per year, within a period of 30 years.

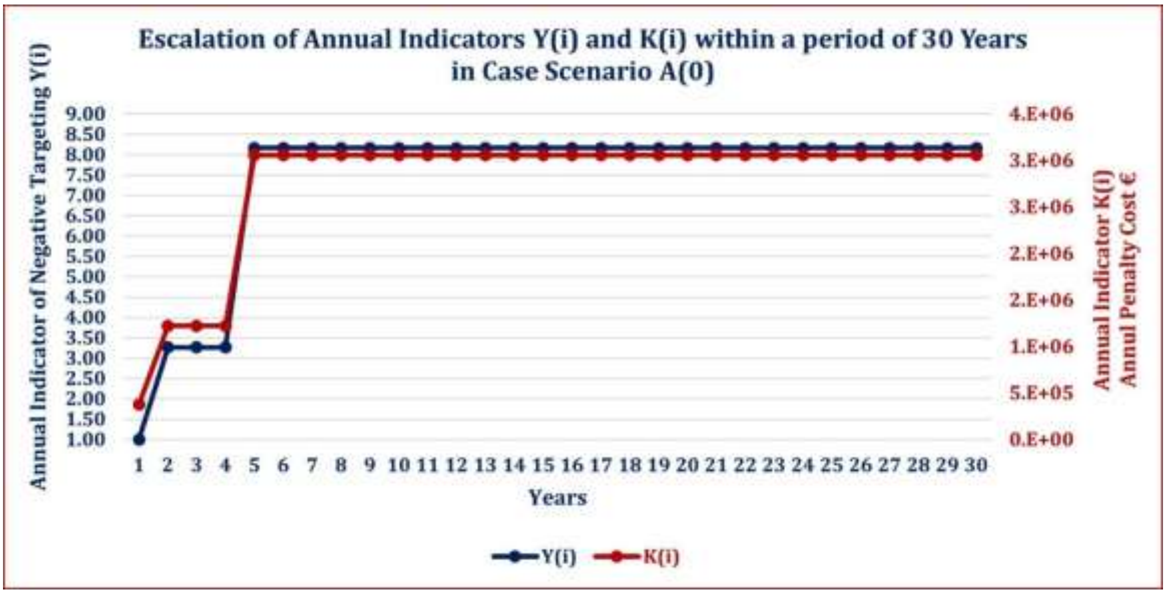


Figure 14 Escalation of Y(i) and K(i) in Case Scenario A(0), within a period of 30 Years

Figure 15 shows the behavior of the crucial factor Y(i) and the sum of total estimated negative cost expressed by the Penalty Cost K(i) per year, within a period of 30 years.

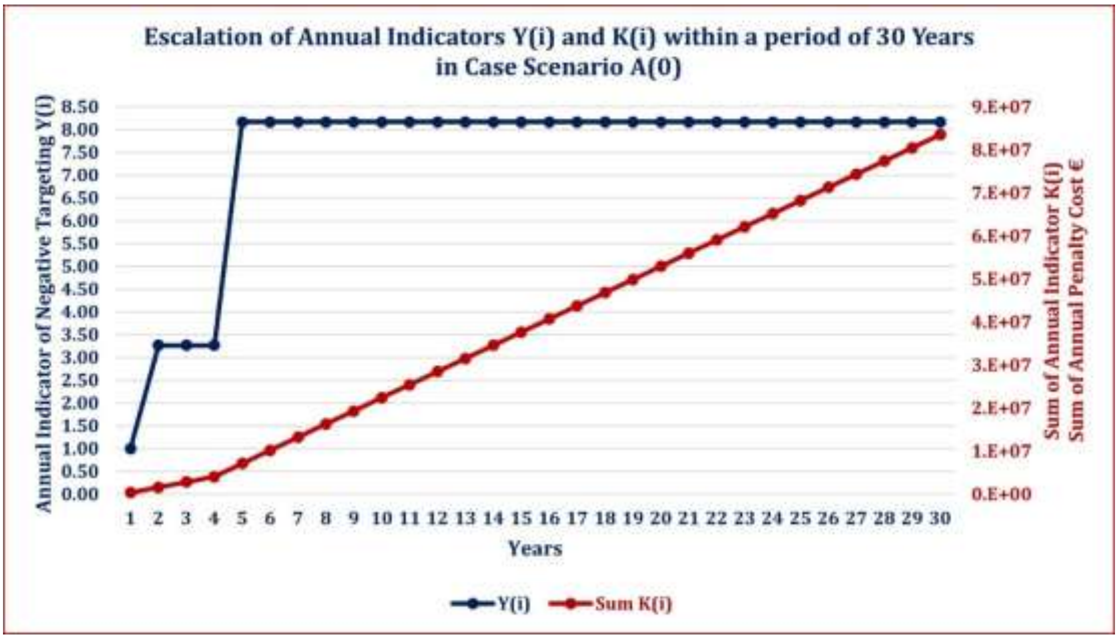


Figure 15 Escalation of Y(i) and Sum K(i) in Case Scenario A(0), within a period of 30 Years

Figures 14 and 15 demonstrate a primary draft techno-economic assessment of the penalty cost in Case Scenario A(0). To assess the estimated actual negative cost in Case Scenario A(0), considering the negative targeting Y(i) and its consequences that correspond to an increase of non-conformances during environmental inspection, the results calculated by the Loss, Probability Density, and Goal Functions as shown in equations 2, 3-6, respectively, are required. The Loss Function expresses the correlation of total penalty cost K(i) with the negative targeting Y(i). The role of PDF is to accurately estimate the total actual consecutive increase in the probability of negative achievement. The Goal Function calculates the total actual financial risk. Further description of these and CBA's partial indicators is provided in section 8.2.

8.1.2. Key Factors Affecting the Sustainability Level in Case Scenario A(1)

8.1.2.1. Case Scenario A(1). Sub-Cases $A(1)_1$ – $A(1)_4$

Considering the legal obligations that configure the negative cost expressed by the potential annual Penalty Cost K(i) of the sub-cases $A(1)_1$ and $A(1)_2$ and the expected Penalty Cost K(i) of the sub-cases $A(1)_3$ and $A(1)_4$, it is observed that the key factors are involved in the parameter K(i) (as shown in equation 1 and Table A1). These factors refer to a) the standard penalty cost, b) the annual revenue growth of each company, and c) the weighting factor ΣB depending on the classification of waste risk. The crucial factors refer to a) the annual $\Sigma\varepsilon\varepsilon\pi_i$ indicator that configures the corresponding indicator of negative targeting Y(i), and b) the total probability of non-conformance. It is essential to mention that sub-cases $A(1)_1$-$A(1)_5$ consider the maximum risky conditions despite fully-legal conformance with the principles of Circular economy by investing to establish a closed system of industrial units to manage 150 ktonnes of tailings. So, based on the legislation for environmental protection, the total non-conformance refers to only the potential detected environmental pollution. In these sub-cases, the total probability of non-conformance corresponds only to a quarter (25%) of total non-conformance as shown in Table 3. The primary draft techno-economic assessment focuses on calculating the total estimated negative cost $K(i)_{Total}$ corresponding to the total investment cost with the maximum penalty cost of non-conformance. This study considers the axiom that there is at least one environmental inspection per year while the maximum risky conditions describe the conditions of sub-cases $A(1)_1$-$A(1)_5$. So, the techno-economic assessment described by the Penalty Cost K(i), as shown in Tables A8-A11, refers to the worst-case conditions ($\theta_{(0)}$=25%) in case of non-conformance due to unexpected environmental risks (misshaping of the equipment, leachate detections, inefficient recovery procedures, etc.). Figure 16 shows the behavior of the crucial factor Y(i) and the annual estimated negative cost expressed only by the Penalty Cost K(i) per year, within a period of 30 years.

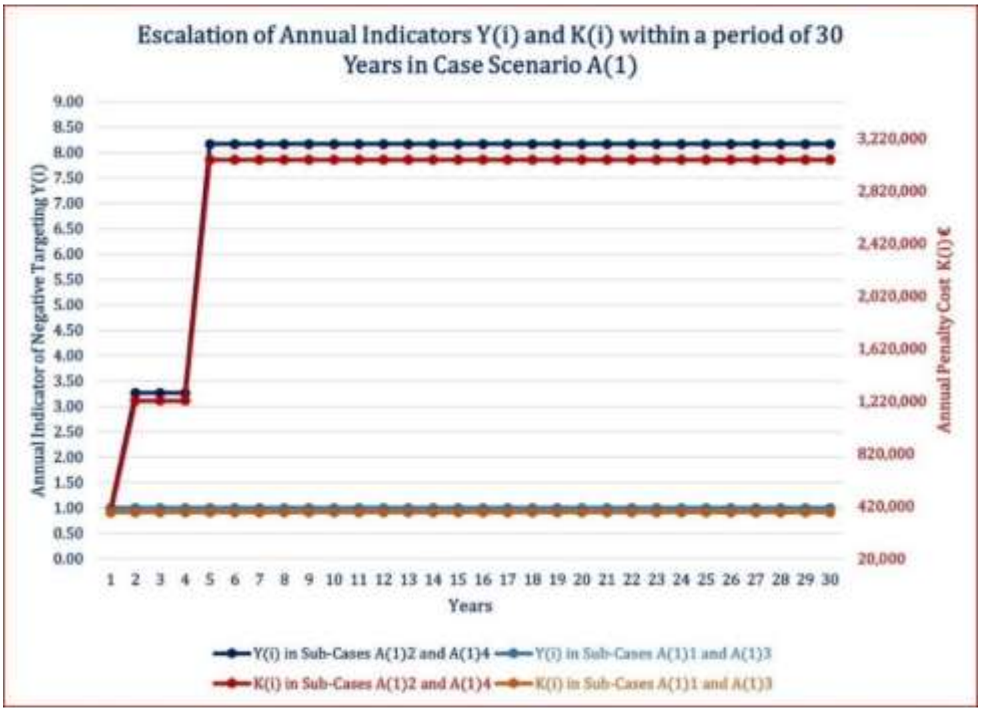


Figure 16 Annual Escalation of Y(i) and K(i) Indicators within a period of 30 Years

In contrast, Figure 17 demonstrates the behavior of Y(i) negative targeting indicator, with the total estimated annual negative cost, including the total cost of investment and the potential additive penalty cost in case of non-conformance due to only the probability of detected environmental pollution.

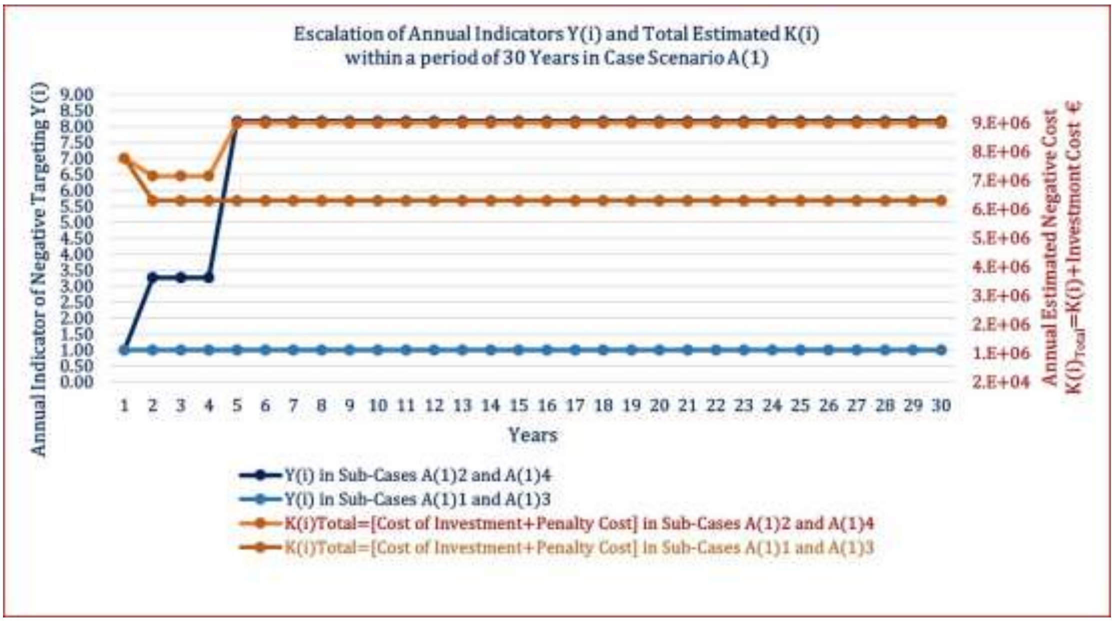


Figure 17 Annual Escalation of Y(i) and $K(i)_{Total}$ Indicators for Sub-cases $A(1)_1$-$A(1)_4$, within a period of 30 Years

As it is observed, the indicators K(i) and K(i)Total are proportional to the behavior of the indicator Y(i). This happens because in sub-cases of consecutive non-conformance, when the negative targeting increases, the corresponding negative cost increases, too. Figure 17 presents the fluctuation of $K(i)_{Total}$. The decrease in price value K(i)Total, which is observed in the 2nd, 3rd, and 4th years of operation, refers to the decrease in the total investment cost, as shown in Table A7.

Figures 16 and 17 refer to a primary draft estimation of techno-economic assessment for the sub-cases $A(1)_1$-$A(1)_4$. To assess the estimated actual negative cost in all these sub-cases of the Case Scenario A(1), considering the negative targeting Y(i) and its consequences that correspond to an increase of non-conformances during environmental inspection, are the results calculated by the Loss, Probability Density, and Goal Functions as shown in equations 2-6, and 7(a-d)-8(a-d), and tailored to each sub-case respectively, are required. Figures 14 and 15 present a preliminary techno-economic assessment of the penalty cost in Case Scenario A(0). The Loss Function expresses the correlation of total penalty cost K(i) with the negative targeting Y(i). The role of PDF is to accurately estimate the total actual consecutive increase in the probability of negative achievement. The Goal Function calculates the total actual financial risk. Further description of these and CBA's partial indicators is provided in Section 8.2.

8.1.2.2. Case Scenario A(1). Sub-Case $A(1)_5$

Considering the actual conditions that configure the negative cost expressed by the expected annual negative Cost K(i) of the sub-case $A(1)_5$, it is observed that the key factors are proportional to the partial costs that structure the total investment cost to treat 150 ktonnes of tailings (as shown in Tables A6 and A7).

These costs refer to the Cost of Primary Installation of equipment for metal recovery and the annual cost of Maintenance (CIM), Cost of Chemical Reagents (CCR), Cost of Energy (EC), and the estimated Benefit (BV) from the utilization of the recovered metal mixture. Observing that the total negative cost of sub-case $A(1)_5$ is approximately fifteen to twenty times higher than the corresponding calculated cost that refers to the sub-cases $A(1)_1$-$A(1)_4$, it is shown that the sub-case $A(1)_5$ describes the maximum risky conditions of non-conformance ($\theta_{(0)}=\theta_{(3)}=25\%$) despite implementing the principles of Circular Economy.

At the same time, the negative targeting indicator Y(i) is configured by the ratio of the total negative cost for each year to the first. Therefore, considering that the metal recovery operation, which adopts the Circular Economy and meets legal obligations, is funded annually over a 30-year period, the Y(i) remains stable and decreases for all years. Despite this axiom, to configure the K(i), an additive cost corresponding to plus 20% is calculated as the safety factor of the techno-economic assessment.

Figure 18 shows the behavior of the negative targeting indicator Y(i) with the K(i) negative cost that refers to the total cost of investment considering the estimated benefit by the utilization of recovered metal mixture.

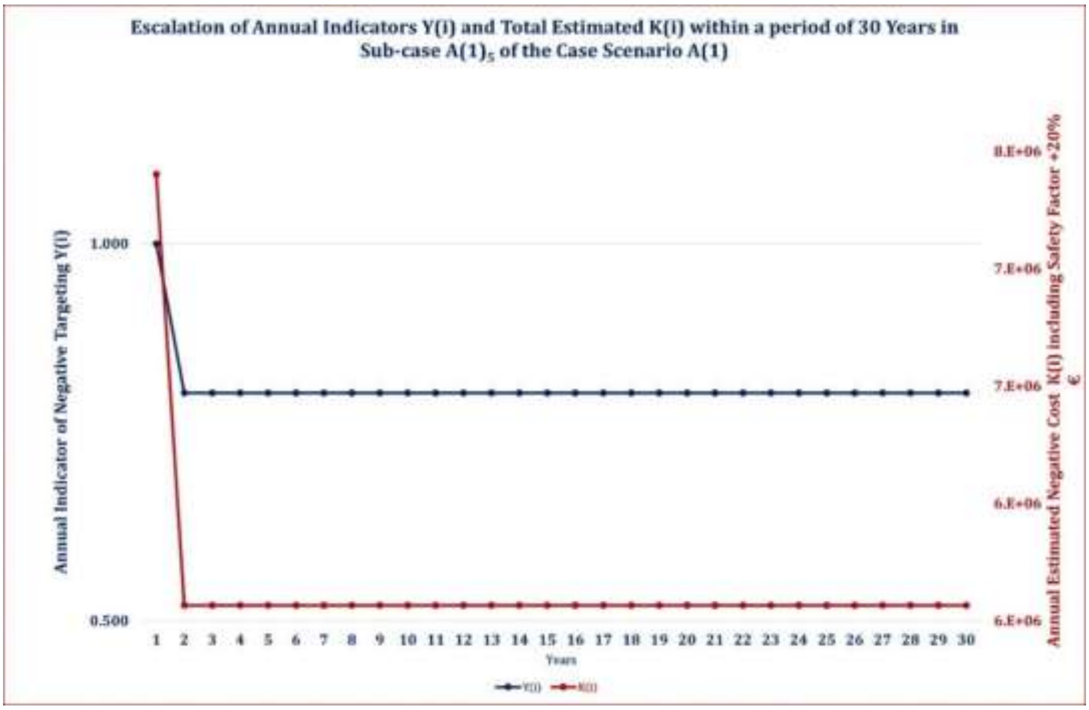


Figure 18 Annual Escalation of Y(i) and K(i) Indicators for Sub-case$A(1)_5$, within a period of 30 Years

To assess the total annual negative cost, including the corresponding at-risk cost due to the maximum expected non-conformance, the determination of Loss, Probability Density, and Goal Functions is required. The K(i) refers only to the actual Loss Function that describes the total annual investment cost minus benefit through the correlation of total negative cost K(i) with the negative targeting Y(i). The theoretical Loss Function is required to determine the at-risk cost. The role of PDF is to accurately estimate the total actual consecutive fluctuation in the probability of negative achievement. The Goal Function calculates the total actual financial risk, considering the real direct cost of investment and the theoretical additional cost due to the expected non-conformance despite implementing the Circular Economy principles. Further descriptions of these and CBA's partial indicators are provided in Section8.2.2.

8.2. Financial Risk Assessment through Cost-Benefit Analysis – Key Findings, Indices of Sustainability in Terms of Circular Economy and Results

8.2.1. Financial Risk Assessment – CBA Results of the Case Scenario A(0)

The CBA methodology is applied to provide the financial risk assessment by calculating the Goal Function. Results of the G(A(0),Y(i)) consider the corresponding results of the multiplication among L(A(0),Y(i)) and PDF (f(Y(i))) as shown in equations 2-6 and Tables A1 and A2.

Figure 19 shows the escalation of Y(i) and PDF(i) indicators over the period of thirty years. Figure 20 demonstrates the behavior of the annual Penalty Cost K(i), Loss Function L(A(0),Y(i)), and Goal Function G(A(0),Y(i)) with the indicators of Probability Density Function PDF, and Negative Targeting Y(i) within a period of thirty years.

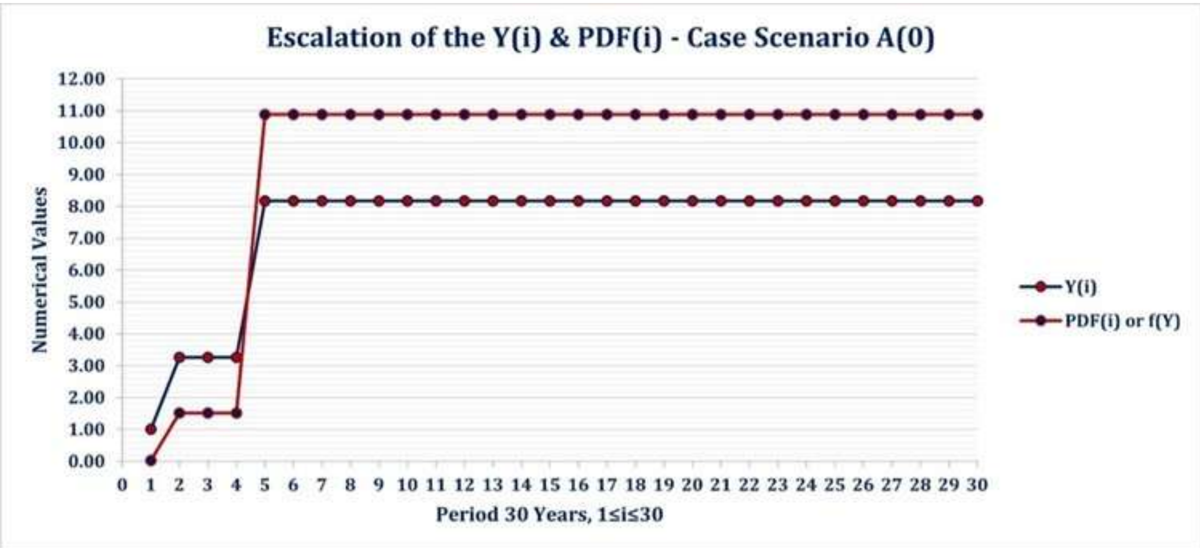


Figure 19 Escalation of the Y(i) and PDF(i) within a period of 30 Years in Case Scenario A(0)

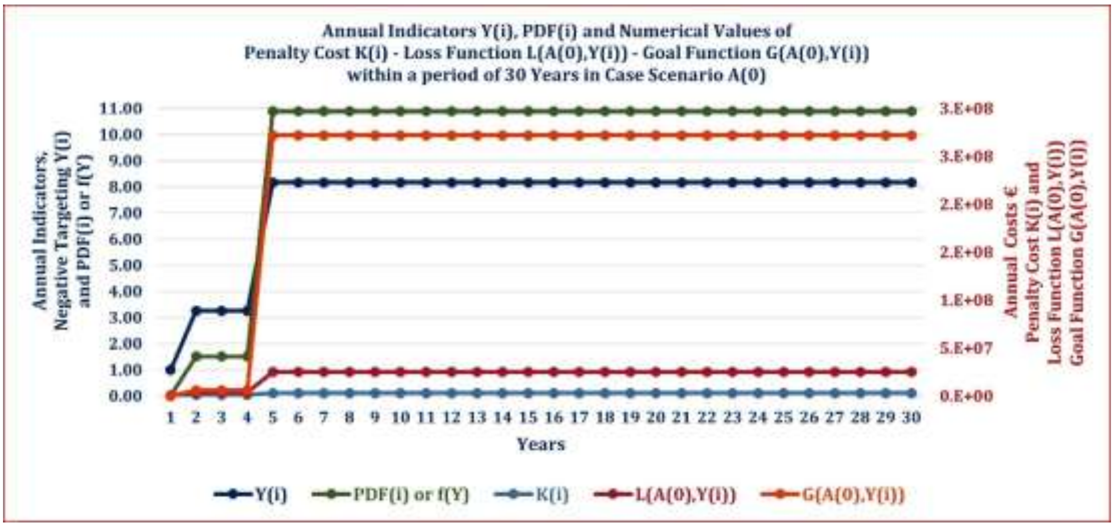


Figure 20 Annual Financial Risk Assessment in Case Scenario A(0) including the Annual Indicators Y(i)-PDF(i) and the Corresponding Negative Costs K(i), L(A(0),Y(i)) and-G(A(0),Y(i))

Figure 21 demonstrates the total financial risk assessment through CBA methodology for a period of thirty years, considering the sum of the annual Penalty Cost K(i), Loss Function L(A(0),Y(i)), and Goal Function G(A(0),Y(i)) that describe the relevant money loss. The total negative cost, expressed as the Sum of G(A(0),Y(30)), refers to the estimated financial risk of approximately 7.088 billion euros, which describes the actual conditions converted into monetary terms in Case Scenario A(0).

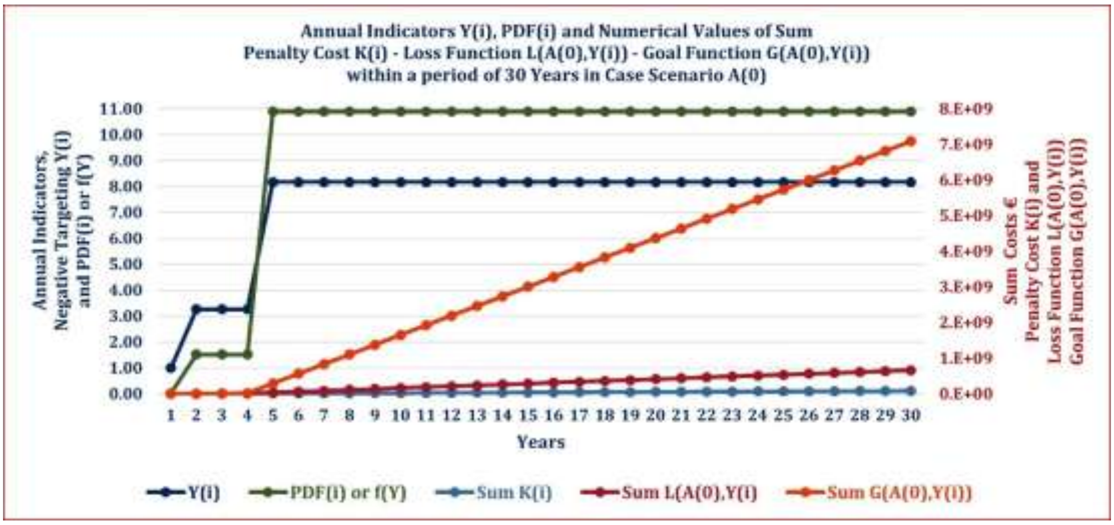


Figure 21 Financial Risk Assessment in Case Scenario A(0) including the Annual Indicators Y(i)-PDF(i) and the Sum of the Corresponding Negative Costs L(A(0),Y(i))-G(A(0),Y(i))

To make a business decision on what the optimal industrial case conditions are to follow, by the side of a start-up mining company's internal business consultants, the calculated financial risk corresponding to Case Scenario A(0) should be compared with the opposite sub-cases' financial risks that refer to Case Scenario A(1). The indicators of LTP, ERR, and CBA that show the sustainability grade between the comparable Case Scenario A(0) and all the sub-cases of Case Scenario A(1) are needed to optimize the decision-making.

8.2.2. Financial Risk Assessment – CBA Results of the Case Scenario A(1)

8.2.2.1. Sub-Case $A(1)_1$

The CBA methodology is applied to provide the financial risk assessment by calculating the total Goal Function. Results of the $G(A(1)_1,Y(i))_{Total}$ consider the corresponding results of the $G(A(1)_1,Y(i))_t$ and $G(A(1)_1,Y(i))_r$ as shown in the equations 8a,9a, and 10a. The $G(A(1)_1,Y(i))_t$ expresses the potential additive cost that corresponds to non-conformance due to unexpected conditions, and its results are configured as shown in equations 2-6 and 8a by multiplying the theoretical Loss Function equaled L(A(0),Y(i)) (that is determined by the consequences of non-conformance with the legal obligations) with the PDF (f(Y(i))). The theoretical $Y(i)_t$ included by the L(A(0),Y(i)) of sub-case $A(1)_1$ refers to the potential negative targeting due to unexpected conditions despite investing in the Circular Economy principles (as shown in Table A8) considering that the maximum probability of non-conformance equals 25% due to only the potential detected environmental pollution ($\theta_{(0)max}=\theta_{(3)max}=0.25$). The $G(A(1)_1,Y(i))_r$ expresses the stable investment cost L(A(1)_SF,Y(i)) including SF as shown in equations 7 and 9a. The actual negative targeting $Y(i)_a$, included by the L(A(1)_SF,Y(i)) refers to the ratio of annual investment cost to the investment cost of the first year (as shown in Table A7). Figure 22 demonstrates the behavior of the annual Loss, Probability Density, and Goal Functions compared with the escalation of both actual $Y(i)_a$, theoretical $Y(i)_t$, and PDF(i) (f(Y(i))) indicators within a period of thirty years.

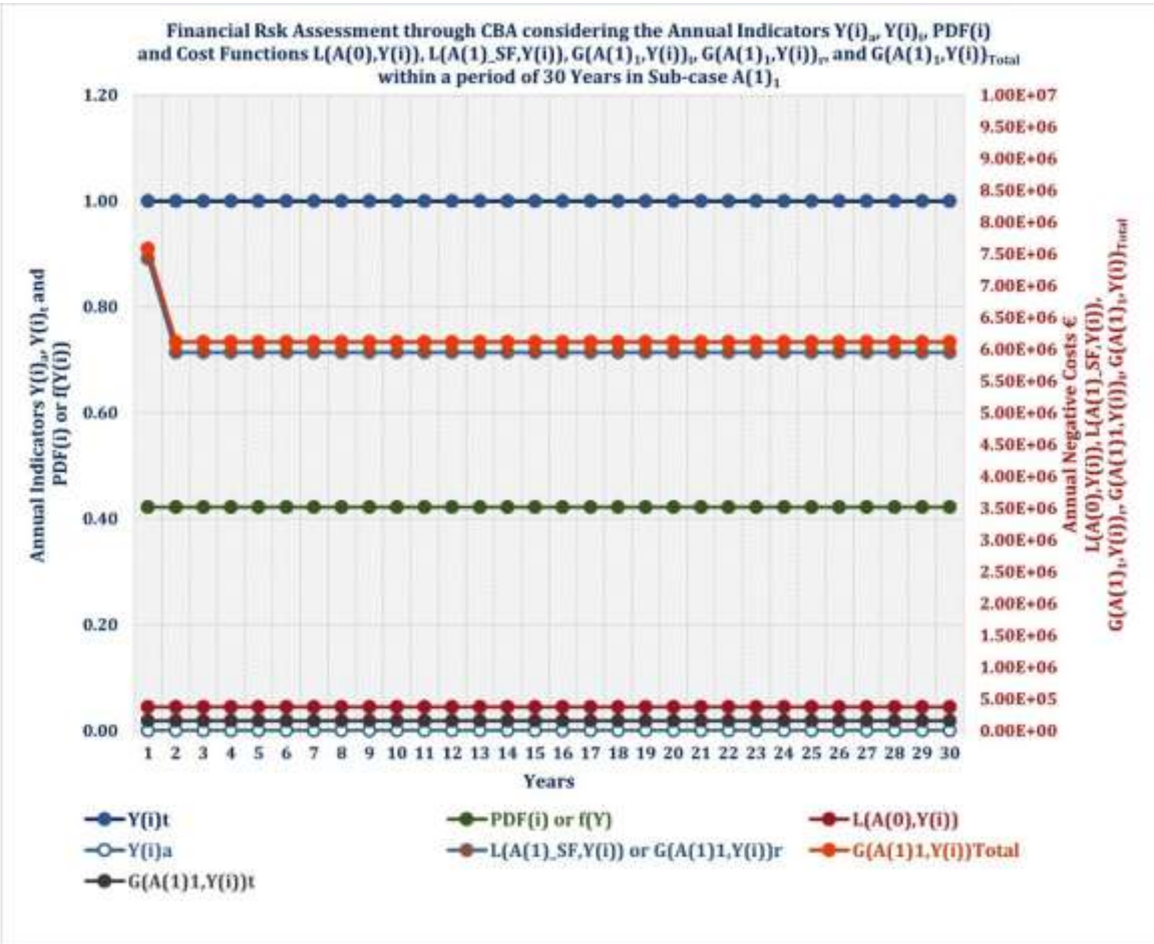


Figure 22 Financial Risk Assessment in sub-case $A(1)_1$ of Case Scenario A(1) including the Annual Indicators of Y(i)-PDF(i) and the Corresponding Negative Cost Functions L(A(0),Y(i)), L(A(1)_SF,Y(i)),-$G(A(1)_1,Y(i))_t$, $G(A(1)_1,Y(i))_r$, $G(A(1)_1,Y(i))_{Total}$

Figure 23 illustrates the total financial risk assessment using the CBA methodology over a thirty-year period, considering the sum of the annual Loss and Goal Functions that describe the relevant negative costs. Total negative cost expressed by the Sum $G(A(1)_1,Y(30))_{Total}$ refers to the estimated financial risk of approximately 185 m€ that describes the actual conditions converted into monetary terms in the sub-case $A(1)_1$ of Case Scenario A(1). The total theoretical negative cost, which refers to the potential penalty for non-conformance, equals approximately 4.7 million euros. At the same time, the total negative cost that refers to the investment cost including a safety factor (+20%) is estimated at 180 m€.

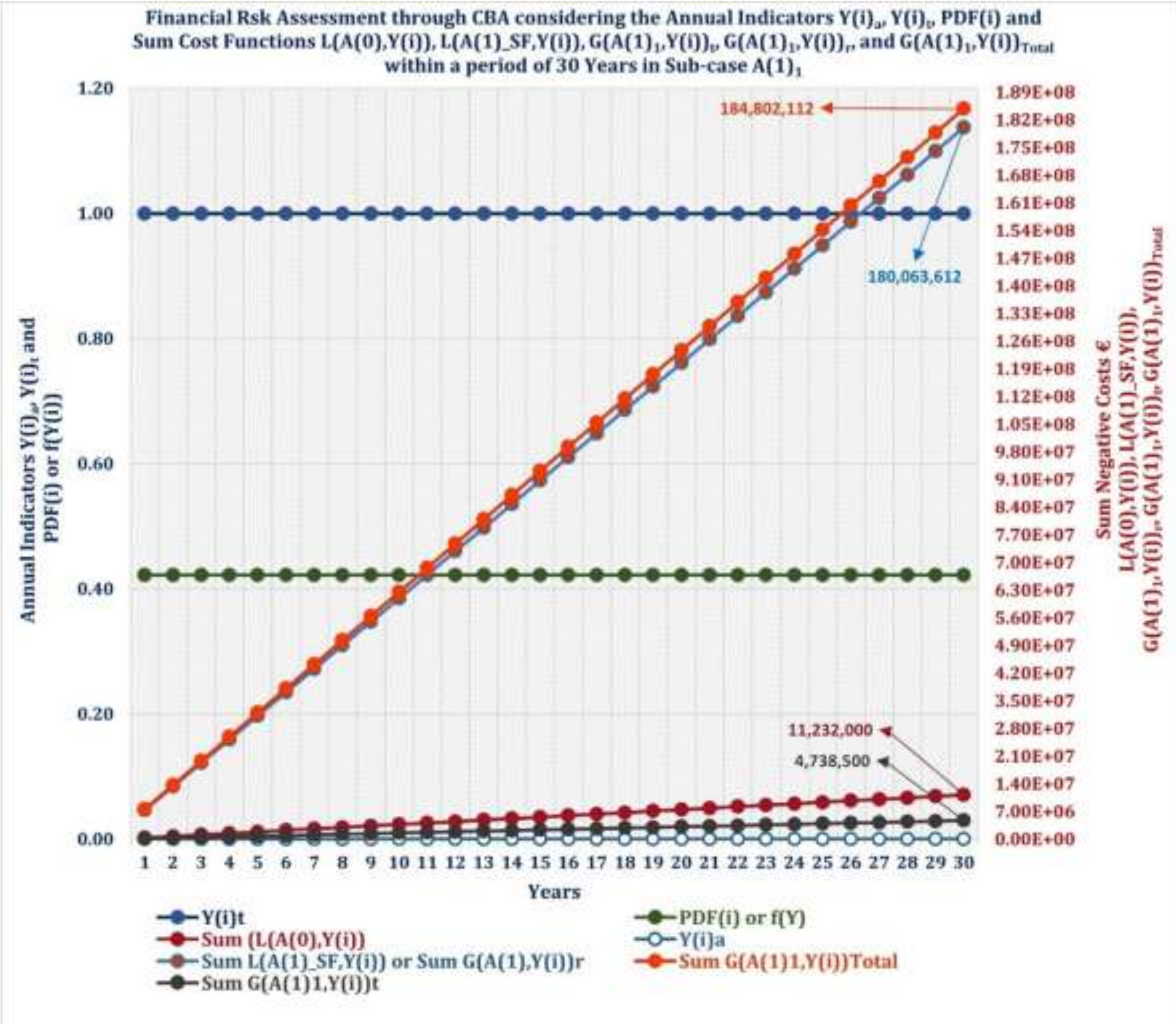


Figure 23 Financial Risk Assessment in Sub-case A(1)1 of Case Scenario A(1) including Annual Indicators of Y(i)-PDF(i) with the Sum of Corresponding Negative Cost Functions L(A(0),Y(i)), L(A(1)_SF,Y(i)),-$G(A(1)_1,Y(i))_t$, $G(A(1)_1,Y(i))_r$, and $G(A(1)_1,Y(i))_{Total}$

To make a business decision on what the optimal industrial case conditions to follow, by the side of a start-up mining company's internal business consultants, the calculated financial risk corresponding to sub-case $A(1)_1$ should be compared with the opposite financial risk that refers to Case Scenario A(0). The indicators of LTP, ERR, and CBA that show the sustainability grade between the comparable sub-case $A(1)_1$ of Case Scenario A(1) with Case Scenario A(0) are needed to optimize the decision-making.

8.2.2.2. Sub-Case $A(1)_2$

The CBA methodology is applied to provide the financial risk assessment by calculating the total Goal Function. Results of the $G(A(1)_2,Y(i))$Total consider the corresponding results of the $G(A(1)_2,Y(i))_t$ and $G(A(1)_2,Y(i))_r$ as shown in equations 8b, 9b, and 10b. The $G(A(1)_2,Y(i))_t$ expresses the potential additive cost that corresponds to non-conformance due to unexpected conditions, and its results are configured as shown in equations 2-6 and 8b multiplying the L(A(0),Y(i)) with the PDF (f(Y(i))). The $G(A(1)_2,Y(i))_r$ expresses the stable investment cost L(A(1)_SF,Y(i)) including SF as shown in equations 7 and 9b. Figure 24 demonstrates the behavior of the annual Loss, Probability Density, and Goal Functions compared with the escalation of both actual $Y(i)_a$, theoretical $Y(i)_t$, and PDF(i) (f(Y(i))) indicators within a period of thirty years. The actual $Y(i)_a$ refers to the ratio of annual investment cost to the investment cost of the first year (as shown in Table A7). The theoretical $Y(i)_t$ refers to the potential negative targeting due to unexpected conditions despite investing in the Circular Economy principles (as shown in Table A9).

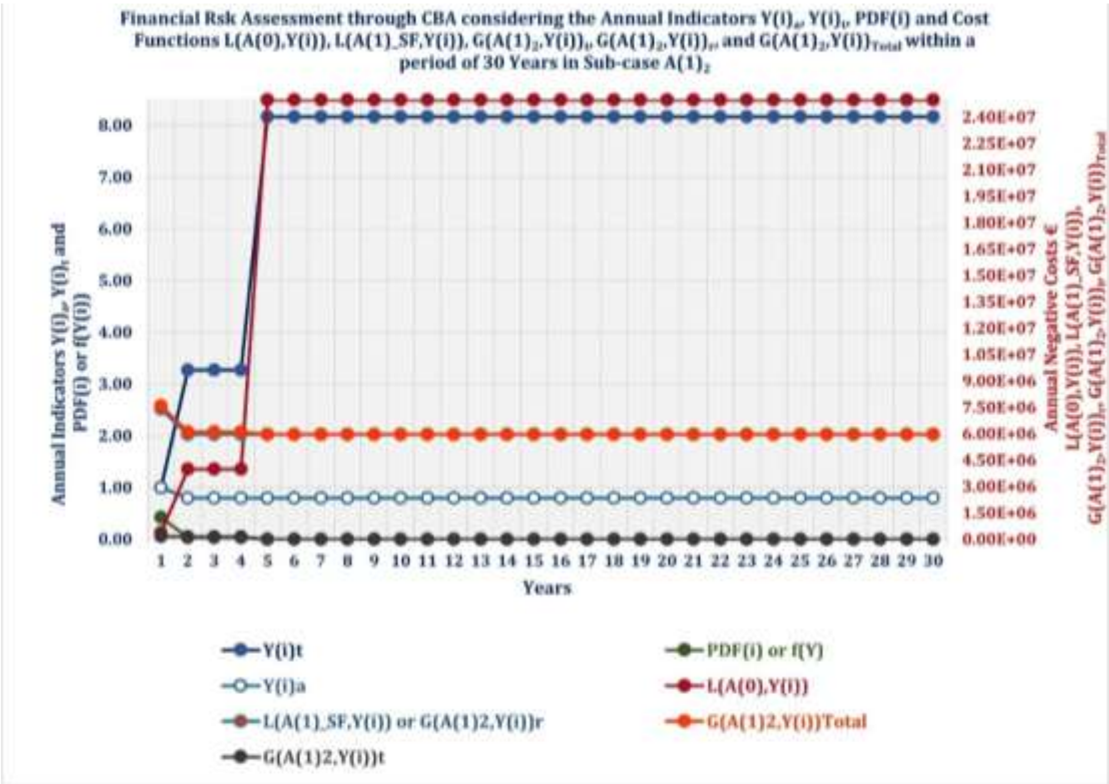


Figure 24 Financial Risk Assessment in sub-case $A(1)_2$ of Case Scenario A(1) including the Annual Indicators of Y(i)-PDF(i) and the Corresponding Negative Cost Functions L(A(0),Y(i)), L(A(1)_SF,Y(i)),-$G(A(1)_2,Y(i))_t$, $G(A(1)_2,Y(i))_r$, $G(A(1)_2,Y(i))_{Total}$

Figure 25 demonstrates the total financial risk assessment through CBA methodology for a period of thirty years, considering the sum of the annual Loss and Goal Functions that describe the relevant negative costs. Total negative cost expressed by the Sum $G(A(1)_2,Y(30))_{Total}$ refers to the estimated financial risk of approximately 180.6 m€ that describes the actual conditions converted into monetary terms in the sub-case $A(1)_2$ of Case Scenario A(1). The total theoretical negative cost, which refers to the potential penalty for non-conformance, equals approximately 577.000 €. At the same time, the total negative cost that refers to the investment cost including a safety factor (+20%) is estimated at 180 m€.

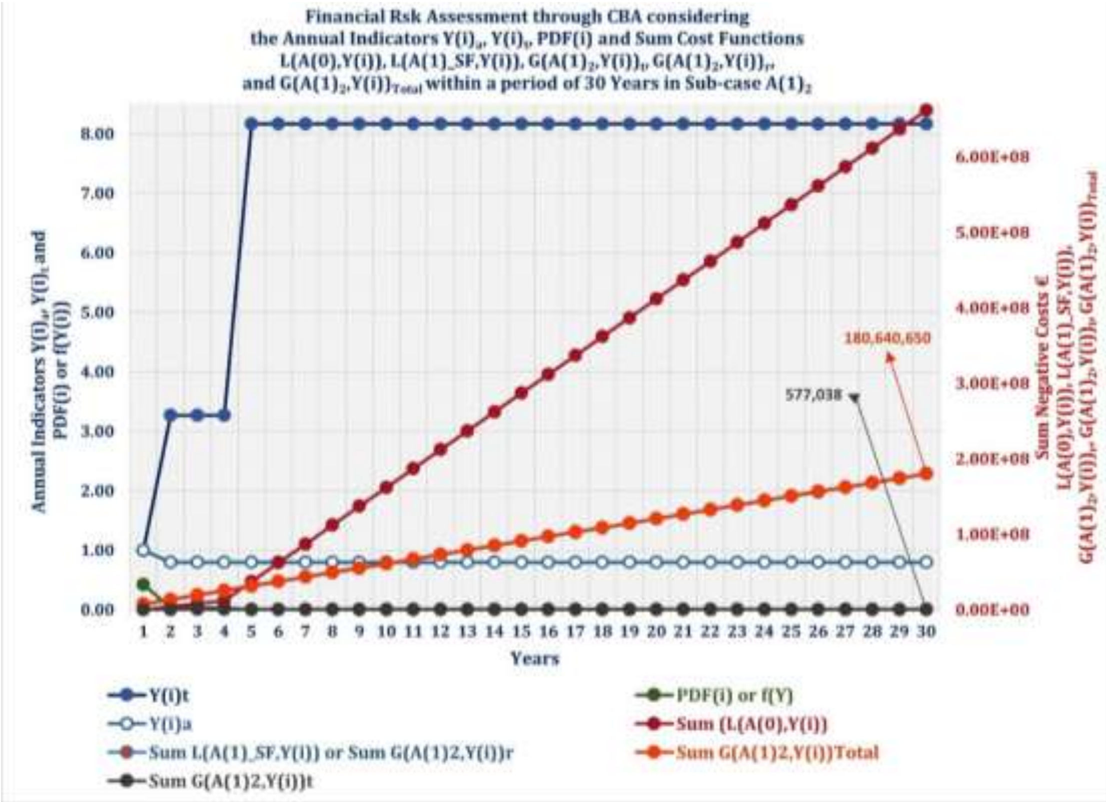


Figure 25 Financial Risk Assessment in Sub-case $A(1)_2$ of Case Scenario A(1) including Annual Indicators Y(i)-PDF(i) with the Sum of Corresponding Negative Cost Functions L(A(0),Y(i)), L(A(1)_SF,Y(i)),-$G(A(1)_2,Y(i))_t$, $G(A(1)_2,Y(i))_r$, and $G(A(1)_2,Y(i))_{Total}$

To make a business decision on what the optimal industrial case conditions to follow, by the side of a start-up mining company's internal business consultants, the calculated financial risk corresponding to sub-case $A(1)_2$ should be compared with the opposite financial risk that refers to Case Scenario A(0). The indicators of LTP, ERR, and CBA that show the sustainability grade between the comparable sub-case $A(1)_2$ of Case Scenario A(1) with Case Scenario A(0) are needed to optimize the decision-making.

8.2.2.3. Sub-Case $A(1)_3$

The CBA methodology is applied to provide the financial risk assessment by calculating the total Goal Function. Results of the $G(A(1)_3,Y(i))_{Total}$ consider the corresponding results of the $G(A(1)_3,Y(i))_t$ and $G(A(1)_3,Y(i))_r$ as shown in the equations 8c, 9c, and 10c. The $G(A(1)_3,Y(i))_t$ expresses the expected additive cost that corresponds to non-conformance due to negative consequences, and its results are configured as shown in equations 11, 12 and 8c, multiplying the L(A(0),Y(i)) with the PDF (f(Y(i))). The $G(A(1)_3,Y(i))_r$ expresses the stable investment cost L(A(1)_SF,Y(i)) including SF as shown in equations 7, and 9c. Figure 26 demonstrates the behavior of the annual Loss, Probability Density, and Goal Functions compared with the escalation of both actual $Y(i)_a$, theoretical $Y(i)_t$ and PDF(i) (f(Y(i))) indicators within a period of thirty years. The actual $Y(i)_a$ refers to the ratio of annual investment cost to the investment cost of the first year (as shown in Table A7). The theoretical $Y(i)_t$ refers to the potential negative targeting due to unexpected conditions despite investing in the Circular Economy principles (as shown in Table A10).

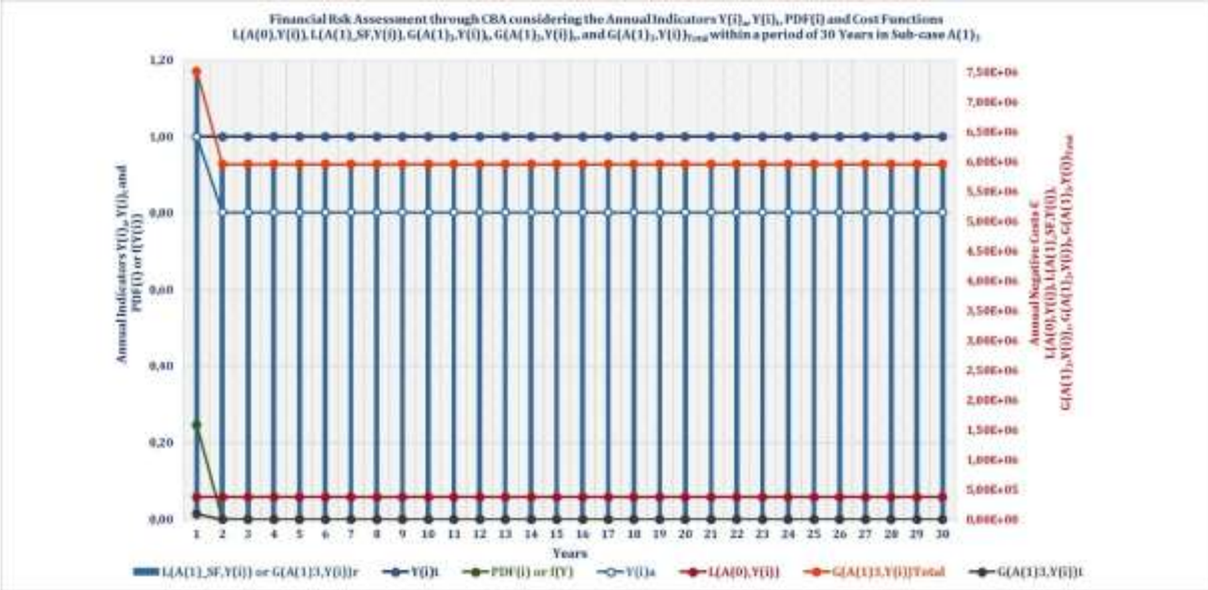


Figure 26 Financial Risk Assessment in sub-case A(1)$_3$ of Case Scenario A(1) including the Annual Indicators of Y(i)-PDF(i) and the Corresponding Negative Cost Functions L(A(0),Y(i)), L(A(1)_SF,Y(i)),-G(A(1)$_3$,Y(i))$_t$, G(A(1)$_3$,Y(i))$_r$, G(A(1)$_3$,Y(i))$_{Total}$

Figure 27 illustrates the total financial risk assessment using the CBA methodology over a thirty-year period, considering the sum of the annual Loss and Goal Functions that describe the relevant negative costs. Total negative cost expressed by the Sum G(A(1)$_3$,Y(30))$_{Total}$ refers to the estimated financial risk of approximately 180.15 m€ that describes the actual conditions converted into monetary terms in the sub-case A(1)$_3$ of Case Scenario A(1). The total theoretical negative cost, which refers to the potential penalty for non-conformance, equals approximately 92.555 €. At the same time, the total negative cost that refers to the investment cost including a safety factor (+20%) is estimated at 180 m€.

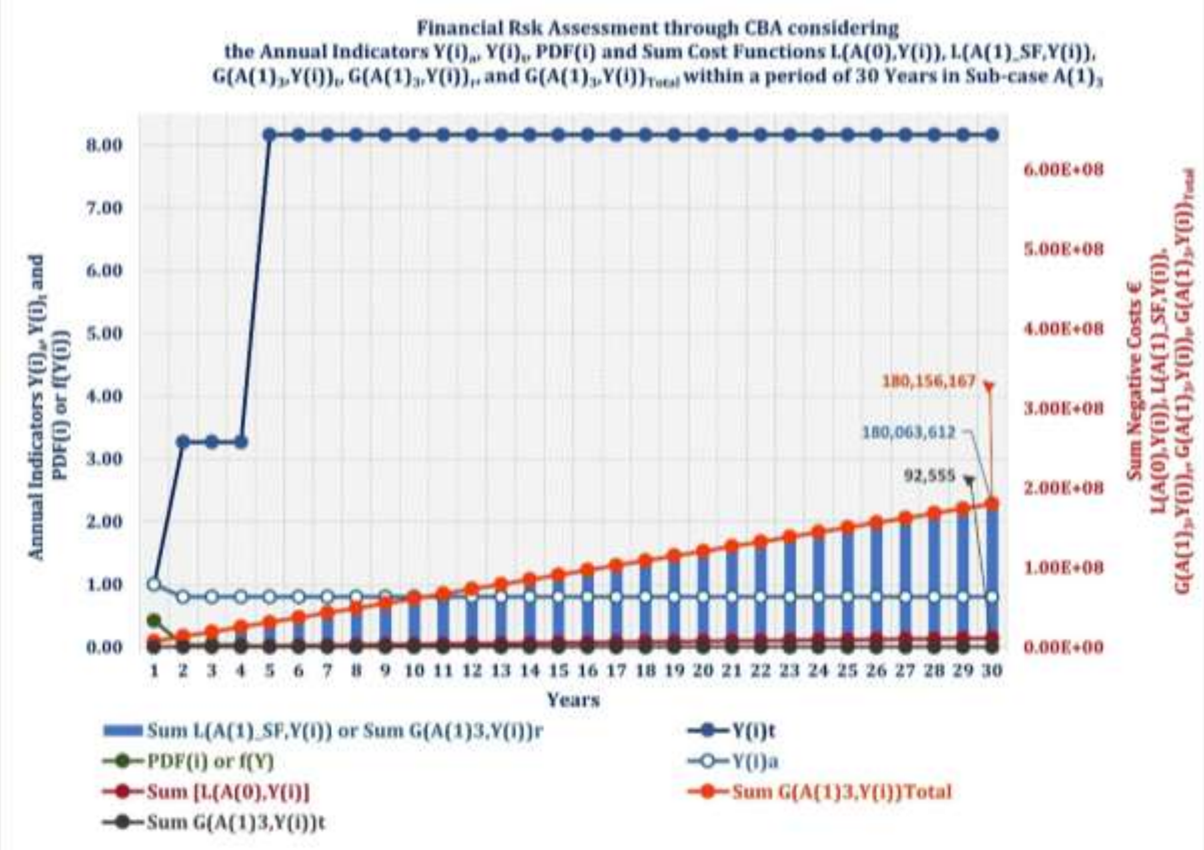


Figure 27 Financial Risk Assessment in Sub-case $A(1)_3$ of Case Scenario A(1) including Annual Indicators Y(i)-PDF(i) with the Sum of Corresponding Negative Cost Functions L(A(0),Y(i)), L(A(1)_SF,Y(i)),-$G(A(1)_3,Y(i))_t$, $G(A(1)_3,Y(i))_r$, and $G(A(1)_3,Y(i))_{Total}$

To make a business decision on what the optimal industrial case conditions to follow, by the side of a start-up mining company's internal business consultants, the calculated financial risk corresponding to sub-case $A(1)_3$ should be compared with the opposite financial risk that refers to Case Scenario A(0). The indicators of LTP, ERR, and CBA that show the sustainability grade between the comparable sub-case $A(1)_3$ of Case Scenario A(1) with Case Scenario A(0) are needed to optimize the decision-making.

8.2.2.4. Sub-Case $A(1)_4$

The CBA methodology is applied to provide the financial risk assessment by calculating the total Goal Function. Results of the $G(A(1)_4,Y(i))_{Total}$ consider the corresponding results of the $G(A(1)_4,Y(i))_t$ and $G(A(1)_4,Y(i))_r$ as shown in the equations 8d,9d, and 10d. The $G(A(1)_4,Y(i))_t$ expresses the expected additive cost that corresponds to non-conformance due to negative consequences, and its results are configured as shown in equations 11, 12 and 8d, multiplying the L(A(0),Y(i)) with the PDF (f(Y(i))). The $G(A(1)_4,Y(i))_r$ expresses the stable investment cost L(A(1)_SF,Y(i)) including SF as shown in equations 7 and 9d. Figure 28 illustrates the behavior of the annual Loss, Probability Density, and Goal Functions compared with the escalation of both actual $Y(i)_a$, theoretical $Y(i)_t$ and PDF(i) (f(Y(i))) indicators within a period of thirty years. The actual $Y(i)_a$ refers to the ratio of annual investment cost to the investment cost of the first year (as shown in Table A7). The theoretical $Y(i)_t$ refers to the potential negative targeting due to unexpected conditions despite investing in the Circular Economy principles (as shown in Table A11).

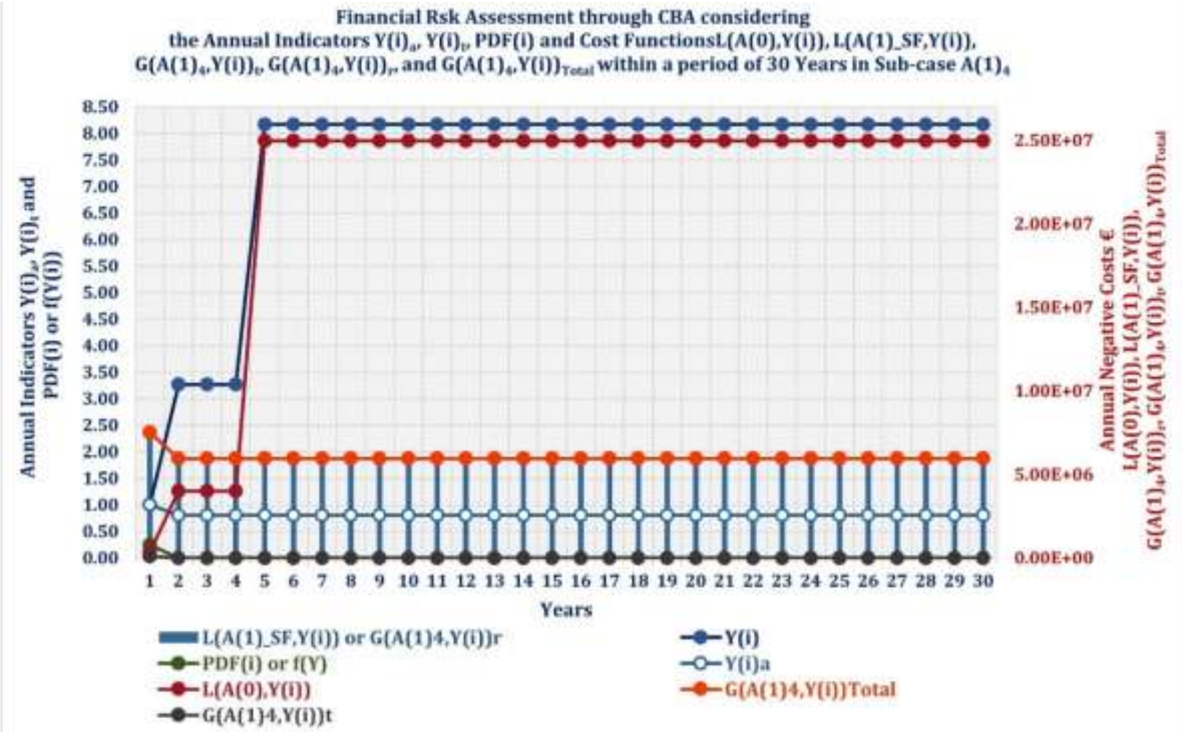


Figure 28 Financial Risk Assessment in sub-case A(1)$_4$ of Case Scenario A(1) including the Annual Indicators of Y(i)-PDF(i) and the Corresponding Negative Cost Functions L(A(0),Y(i)), L(A(1)_SF,Y(i)),-G(A(1)$_4$,Y(i))$_t$, G(A(1)$_4$,Y(i))$_r$, G(A(1)$_4$,Y(i))$_{Total}$

Figure 29 illustrates the total financial risk assessment using the CBA methodology over a thirty-year period, considering the sum of the annual Loss and Goal Functions that describe the relevant negative costs. Total negative cost expressed by the Sum G(A(1)$_4$,Y(30))$_{Total}$ refers to the estimated financial risk of approximately 180.15 m€ that describes the actual conditions converted into monetary terms in the sub-case A(1)$_4$ of Case Scenario A(1). The total theoretical negative cost, which refers to the potential penalty for non-conformance, equals approximately 93.619 €. At the same time, the total negative cost that refers to the investment cost including a safety factor (+20%) is estimated at 180 m€.

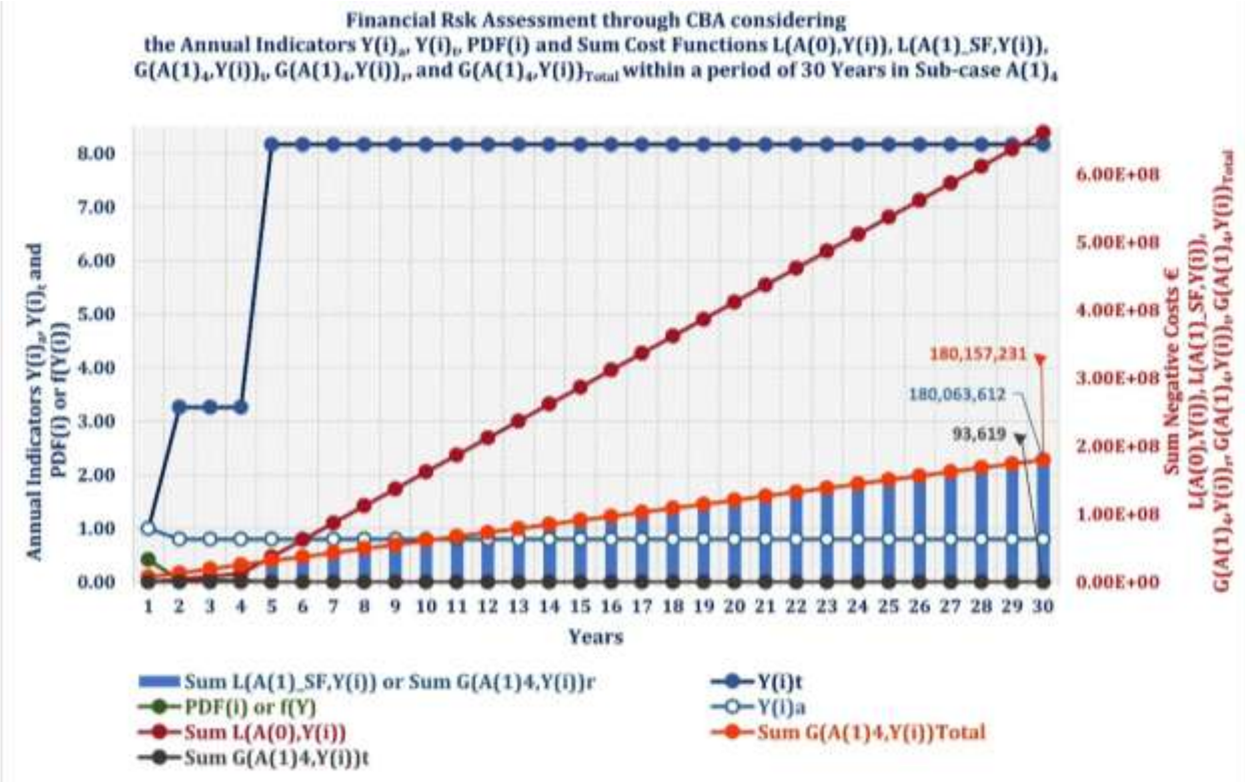


Figure 29 Financial Risk Assessment in Sub-case $A(1)_4$ of Case Scenario A(1) including Annual Indicators Y(i)-PDF(i) with the Sum of Corresponding Negative Cost Functions L(A(0),Y(i)), L(A(1)_SF,Y(i)),-$G(A(1)_4,Y(i))_t$, $G(A(1)_4,Y(i))_r$, and $G(A(1)_4,Y(i))_{Total}$

To make a business decision on what the optimal industrial case conditions to follow, by the side of a start-up mining company's internal business consultants, the calculated financial risk corresponding to sub-case $A(1)_4$ should be compared with the opposite financial risk that refers to Case Scenario A(0). The indicators of LTP, ERR, and CBA that show the sustainability grade between the comparable sub-case $A(1)_4$ of Case Scenario A(1) with Case Scenario A(0) are needed to optimize the decision-making.

8.2.2.5. Sub-Case $A(1)_5$

The CBA methodology is applied to provide the financial risk assessment by calculating the total Goal Function. Results of the $G(A(1)_5,Y(i))_{Total}$ consider the corresponding results of the $G(A(1)_5,Y(i))_t$ and $G(A(1)_5,Y(i))_r$ as shown in the equations 8e,9e, and 10e. The $G(A(1)_5,Y(i))_t$ expresses the expected additive cost that corresponds to non-conformance due to negative consequences, and its results are configured as shown in equations 11,12 and 8e multiplying the L(A(1)_SF,Y(i)) with the PDF (f(Y(i))). The $G(A(1)_5,Y(i))_r$ expresses the stable investment cost L(A(1)_SF,Y(i)) including SF as shown in equations 7 and 9e.

Figure 30 illustrates the behavior of the annual Loss, Probability Density, and Goal Functions compared with the escalation of both actual $Y(i)_a$, theoretical $Y(i)_t$ and PDF(i) (f(Y(i))) indicators within a period of thirty years. The actual $Y(i)_a$ refers to the ratio of annual investment cost to the investment cost of the first year (as shown in Table A7). The theoretical $Y(i)_t$ refers to the potential negative targeting due to unexpected conditions despite investing in the Circular Economy principles and is proportional to the money loss of total investment (as shown in Table A12). Therefore, both indices $Y(i)_a$ and $Y(i)_t$ refer to the same grade of negative targeting.

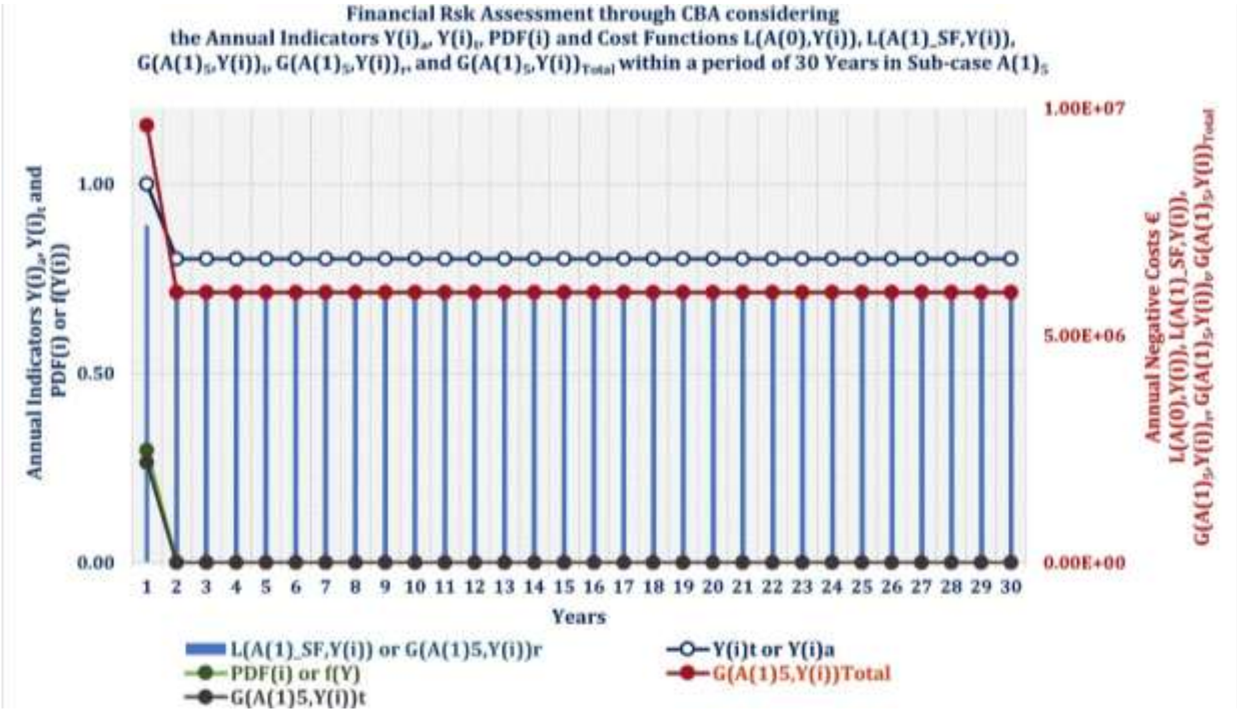


Figure 30 Financial Risk Assessment in sub-case $A(1)_5$ of Case Scenario A(1) including the Annual Indicators of Y(i)-PDF(i) and the Corresponding Negative Cost Functions $L(A(1)\_SF,Y(i))$,-$G(A(1)_5,Y(i))_t$, $G(A(1)_5,Y(i))_r$, $G(A(1)_5,Y(i))_{Total}$

Figure 31 illustrates the total financial risk assessment using the CBA methodology over a thirty-year period, considering the sum of the annual Loss and Goal Functions that describe the relevant negative costs. Total negative cost expressed by the Sum $G(A(1)_5,Y(30))_{Total}$ refers to the estimated financial risk of approximately 182.2 m€ that describes the actual conditions converted into monetary terms in the sub-case $A(1)_5$ of Case Scenario A(1). The total theoretical negative cost, which refers to the potential penalty for non-conformance, equals approximately 2.2 million euros. At the same time, the total negative cost that refers to the investment cost including a safety factor (+20%) is estimated at 180 m€.

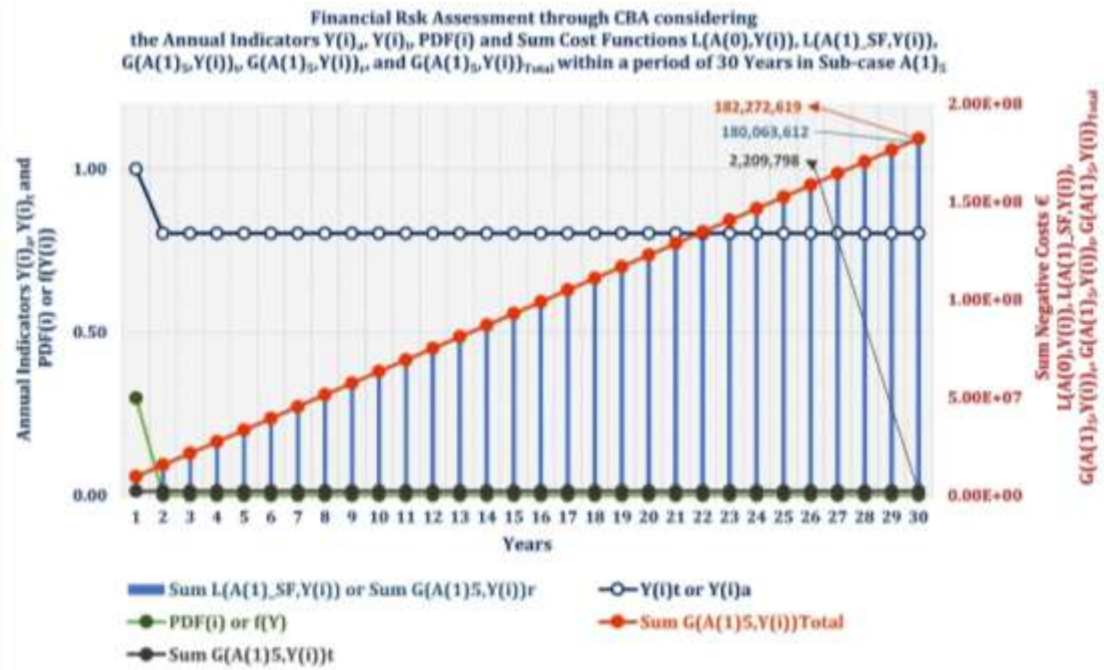


Figure 31 Financial Risk Assessment in Sub-case $A(1)_5$ of Case Scenario A(1) including Annual Indicators Y(i)-PDF(i) with the Sum of Corresponding Negative Cost Functions L(A(1)_SF,Y(i)), $G(A(1)_5,Y(i))_t$, $G(A(1)_5,Y(i))_r$, and $G(A(1)_5,Y(i))_{Total}$

To make a business decision on what the optimal industrial case conditions to follow, by the side of a start-up mining company's internal business consultants, the calculated financial risk corresponding to sub-case $A(1)_5$ should be compared with the opposite financial risk that refers to Case Scenario A(0). The indicators of LTP, ERR, and CBA that show the sustainability grade between the comparable sub-case $A(1)_5$ of Case Scenario A(1) with Case Scenario A(0) are needed to optimize the decision-making.

8.2.3. Financial Risk Assessment – Comparative Evaluation of CBA Indices between Case Scenario A(0) and Sub-cases of Case Scenario A(1)

The CBA indices that assess the efficiency of each case while meeting the requirements of the CBA guidance on investment projects are the LTP, ERR, and CBA. LTP refers to the impact of the whole project on the environmental protection requirements in terms of the Circular Economy, considering the corresponding legislation. The LTP index is extracted by the ratio between each sub-case of the Case Scenario A(1) Sum $G(A(1)_{1\text{-}5}, Y(i))_{Total}$ with the corresponding Sum G(A(0),Y(i)) that describes the Case Scenario A(0), as shown in equations 31 and 32 for each case scenario, respectively. The coefficient a% refers to the weighting factor that each company attributes to the LTP indicator. Results of the LTP indicator comparing all sub-cases of Case Scenario A(1) with Case Scenario A(0) are shown in Table A16.

$$LTP[A(0)] = a\% * \frac{\text{Sum } G\big(A(0),Y(i)\big)_{Total}}{\text{Sum } G\big(A(1)_{1-5},Y(i)\big)_{Total}}, \quad i = 1,2,\ldots n \text{ years} \ (31)$$

$$LTP[A(1)_{1-5}] = a\% * \frac{\text{Sum } G\big(A(1)_{1-5},Y(i)\big)_{Total}}{\text{Sum } G\big(A(0),Y(i)\big)_{Total}}, \quad i = 1,2,\ldots n \text{ years} \ (32)$$

This study, following the legal obligations, adopts the axiom that a% coefficient of LTP has the minimum permissible numerical value equaled 50%. As a result, there is direct correlation between LTP and ERR as shown in the equations 33, and 34. ERR refers to the potential revenue from reusing recovered metal materials due to their importance in present and future industrial needs. The ERR index is extracted based on the LTP's numerical value. Equations 30 and 31 show the configuration of ERR for each case scenario, respectively. Results of the ERR indicator comparing all sub-cases of Case Scenario A(1) with Case Scenario A(0) are shown in Table A17.

$$\mathrm{ERR}[A(0)] = (1-a)\% * \frac{\mathrm{Sum\ G}(A(0),Y(i))_{\mathrm{Total}}}{\mathrm{Sum\ G}(A(1)_{1-5},Y(i))_{\mathrm{Total}}}, \quad i = 1,2,\ldots n \text{ years } (33)$$

$$\mathrm{ERR}[A(1)_{1-5}] = (1-a)\% * \frac{\mathrm{Sum\ G}(A(1)_{1-5},Y(i))_{\mathrm{Total}}}{\mathrm{Sum\ G}(A(0),Y(i))_{\mathrm{Total}}}, \quad i = 1,2,\ldots n \text{ years } (34)$$

The total CBA index is extracted by the ratio between the Benefit and the Cost of each examined case, as shown in equations 35-37. The IRR indicator, which stochastically simulates the efficiency grade for all sub-cases of the Case Scenario A(1) against the Case Scenario A(0), is calculated as shown in the supplementary equation of CBA's formula 36a. In this study, the CBA index is determined by considering that ERR's effect equals LTP's impact on the project's total sustainability. Each company could configure the correlation between those indices according to its corporate policy; however, the LTP index always has a greater impact than ERR. So, by weighting both indices with the same factor, an additional level of safety is achieved. Results of the CBA indicator comparing all sub-cases of Case Scenario A(1) with Case Scenario A(0) are shown in Table A18. Results of the supplementary indicator IRR indicator comparing all sub-cases of Case Scenario A(1) with Case Scenario A(0), which are presented in Table A18, show the differential investment among each sub-case of the Case Scenario A(1) with the opposite Case Scenario A(0), too.

$$\mathrm{CBA}\ [A(0)] = \frac{\mathrm{Benefit}}{\mathrm{Sum\ (G}(A(0),Y(i))_{\mathrm{Total}}} \ (35)$$

$$\mathrm{CBA}\ [A(1)_{1-5}] = \frac{\mathrm{Benefit}}{\mathrm{Sum\ (G}(A(1)_{1-5},Y(i))_{\mathrm{Total}}} \ (36)$$

$$\mathrm{NPV}_{\mathrm{Net\ Present\ Value}} = \sum_{t=1}^{30} \frac{PV(Inflows-Outflows)}{(1+IRR)^t} \ (36a)$$

$$\mathrm{PV}_{\mathrm{Present\ Value}} = Cash\ Flow\ at\ the\ time\ (t) \ (36b)$$

$$\mathrm{Benefit} = \left|\mathrm{Sum\ G}\left(A(1)_{1-5}, Y(i)\right) - \mathrm{Sum\ G}(A(0), Y(i))\right| \ (37)$$

Benefit of the Case Scenario A(0) is calculated as shown in Equation 37a.

Benefit of the Case Scenario A(1) is calculated as shown in the equation 37b.

$$\mathrm{Benefit}_{A(0)} = \mathrm{Sum\ G}\left(A(1)_{1-5}, Y(i)\right) - \mathrm{Sum\ G}(A(0), Y(i)) \ (37a)$$

$$\mathrm{Benefit}_{A(1)_{1-5}} = \mathrm{Sum\ G}\left(A(0), Y(i)\right) - \mathrm{Sum\ G}\left(A(1)_{1-5}, Y(i)\right) \ (37b)$$

The sub-sections 8.2.3.1-8.2.3.5 demonstrate the assessment of LTP, ERR, and final CBA indices comparing each sub-case of Case Scenario A(1) with Case Scenario A(0). Hence, separately, the financial risk assessment is provided through CBA, accounting for the total financial risks calculated by the corresponding Sum of the Goal Functions for all sub-cases $A(1)_{1\text{-}5}$ and Case A(0).

8.2.3.1. Escalation of the Financial Risk Assessment and partial Indices LTP, ERR, CBA comparing sub-case $A(1)_1$ and Case Scenario A(0)

Figure 32 illustrates the comparative evaluation of LTP indicators, which refer to the risk grade of sub-case A(1)1 and Case A(0), over a 30-year period, respectively.

Figure 32 Comparative Evaluation for the Escalation of LTP Indicators that refer to the Grade of Long Term Perspective Risk for the sub-case $A(1)_1$ and Case Scenario A(0), respectively

Figure 33 shows the comparative evaluation for the escalation of ERR indicators that refer to the rate of revenue for the sub-case $A(1)_1$ and Case A(0) within a period of 30 years, respectively.

Figure 33 Comparative Evaluation for the Escalation of ERR Indicators that describe the Economic Rate of Return Grade for the sub-case $A(1)_1$ and Case Scenario A(0), respectively

Figure 34 demonstrates the comparative evaluation for the escalation of final CBA indicators that describe the total grade of sustainability for the sub-case $A(1)_1$ and Case A(0) within a period of 30 years, respectively.

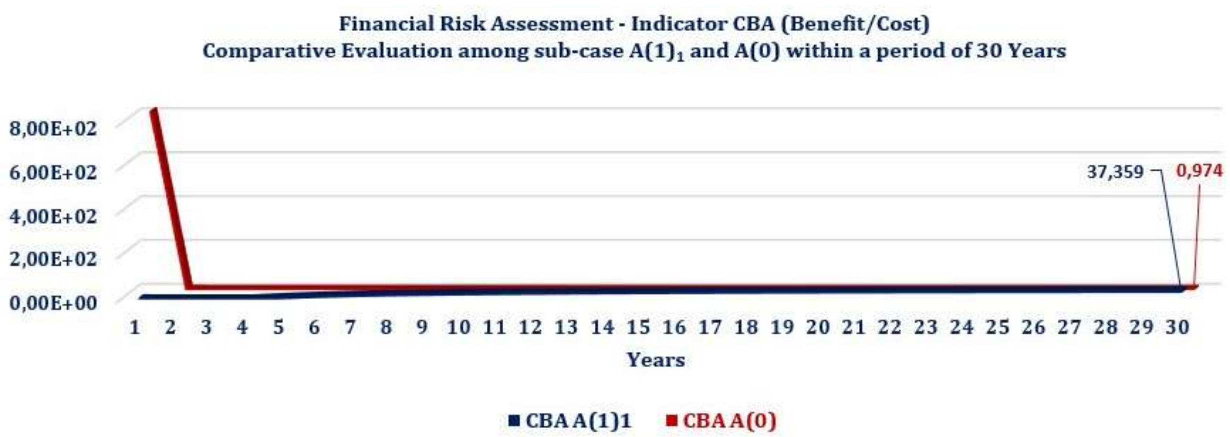


Figure 34 Comparative Evaluation for the Escalation of final CBA Indicators that describe the Grade of Sustainability for the sub-case $A(1)_1$ and Case Scenario A(0), respectively

Figure 35 summarizes the total Financial Risk Assessment, which is calculated by the corresponding Sum of Goal Functions that describe sub-case $A(1)_1$ and A(0), respectively.

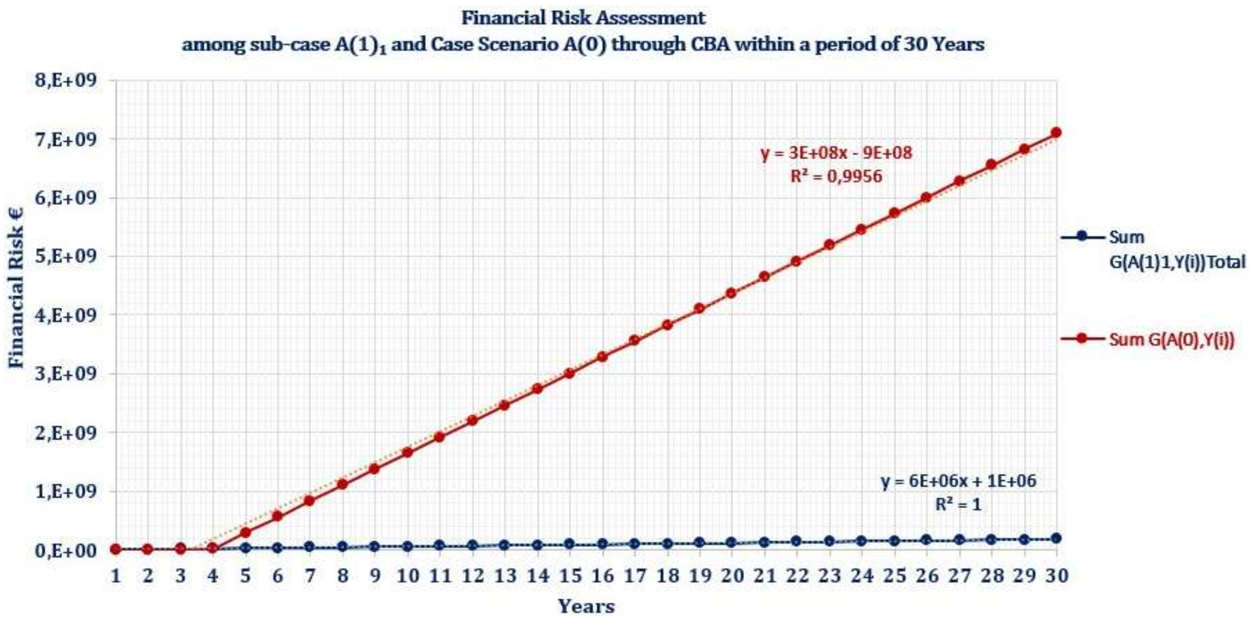


Figure 35 Financial Risk Assessment through CBA, illustrating the Total Grade of Economic Sustainability converted into Monetary Terms for the sub-case $A(1)_1$ and Case Scenario A(0), respectively

The unique solution among the simulative equations for each examined case, shown in Figure 35, identifies the point of efficiency of the sub-case $A(1)_1$ to the opposite Case Scenario A(0). Hence, the theoretical point of efficiency is assessed by the first eighteen days after the third working year.

8.2.3.2. Escalation of the Financial Risk Assessment and partial Indices LTP, ERR, CBA comparing sub-case $A(1)_2$ and Case Scenario A(0)

Figure 36 demonstrates the comparative evaluation for the escalation of LTP indicators that refer to the risk grade of the sub-case $A(1)_2$ and Case A(0) within a period of 30 years, respectively.

Figure 36 Comparative Evaluation for the Escalation of LTP Indicators that refer to the Grade of Long Term Perspective Risk for the sub-case $A(1)_2$ and <u>Case</u> Scenario A(0), respectively

Figure 37 shows the comparative evaluation for the escalation of ERR indicators that refer to the rate of revenue for the sub-case $A(1)_2$ and Case A(0) within a period of 30 years, respectively.

Figure 37 Comparative Evaluation for the Escalation of ERR Indicators that describe the Economic Rate of Return Grade for the sub-case $A(1)_2$ and Case Scenario A(0), respectively

Figure 38 illustrates the comparative evaluation of the escalation of final CBA indicators, which describe the total grade of sustainability for sub-case A(1)1 and Case A(0) over a 30-year period, respectively.

Figure 38 Comparative Evaluation for the Escalation of final CBA Indicators that describe the Grade of Sustainability for the sub-case $A(1)_2$ and Case Scenario A(0), respectively

Figure 39 summarizes the total Financial Risk Assessment that is calculated by the corresponding Sum of Goal Functions that describe the sub-case $A(1)_1$ and A(0), respectively.

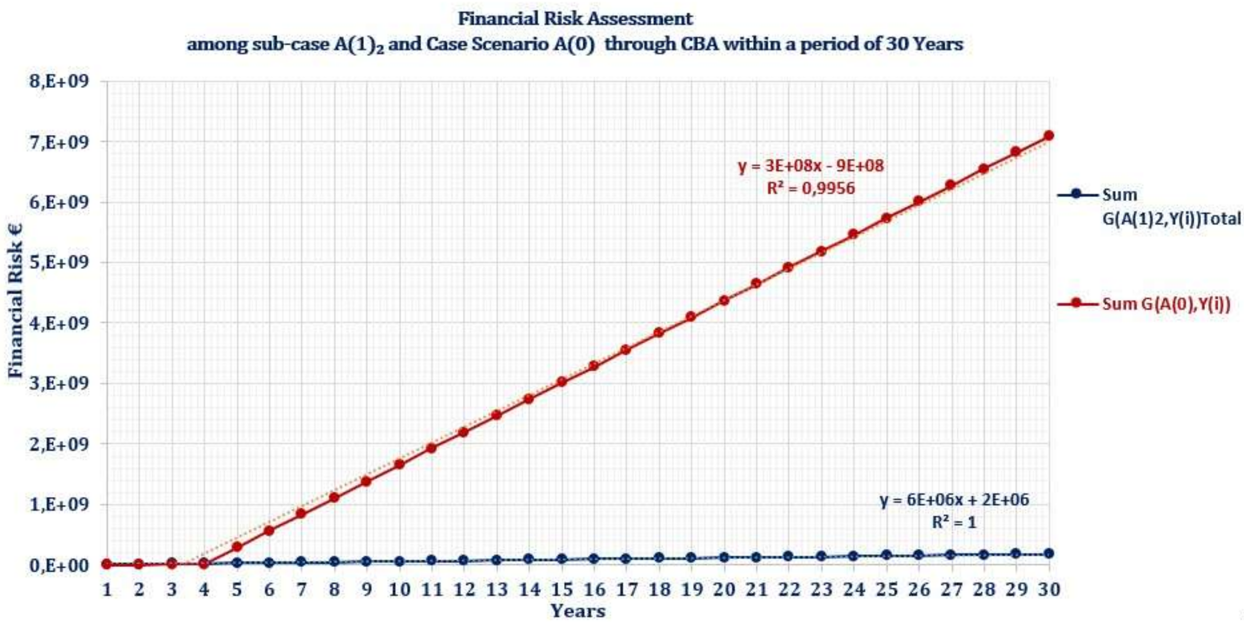


Figure 39 Financial Risk Assessment through CBA illustrating the Total Grade of Economic Sustainability converted into Monetary Terms for the sub-case $A(1)_2$ and Case Scenario A(0), respectively

The unique solution among the simulative equations for each examined case shown in Figure 39, identifies the point of efficiency of the sub-case $A(1)_2$ to the opposite Case Scenario A(0). Hence, the theoretical point of efficiency is assessed by the first eighteen days after the third working year.

8.2.3.3. Escalation of the Financial Risk Assessment and partial Indices LTP, ERR, CBA comparing sub-case $A(1)_3$ and Case Scenario A(0)

Figure 40 demonstrates the comparative evaluation for the escalation of LTP indicators that refer to the risk grade of the sub-case $A(1)_3$ and Case A(0) within a period of 30 years, respectively.

Figure 40 Comparative Evaluation for the Escalation of LTP Indicators that refer to the Grade of Long Term Perspective Risk for the sub-case $A(1)_3$ and Case Scenario A(0), respectively

Figure 41 presents a comparative evaluation of the escalation of ERR indicators, specifically the rate of revenue, for sub-case A(1)3 and Case A(0) over a 30-year period, respectively.

Figure 41 Comparative Evaluation for the Escalation of ERR Indicators that describe the Economic Rate of Return Grade for the sub-case $A(1)_3$ and Case Scenario A(0), respectively

Figure 42 illustrates the comparative evaluation of the escalation of final CBA indicators, which describe the total grade of sustainability for sub-case A(1)3 and Case A(0) over a 30-year period, respectively.

Figure 42 Comparative Evaluation for the Escalation of final CBA Indicators that describe the Grade of Sustainability for the sub-case $A(1)_3$ and Case Scenario A(0), respectively

Figure 43 summarizes the total Financial Risk Assessment, which is calculated by the corresponding Sum of Goal Functions that describe sub-case $A(1)_3$ and A(0), respectively.

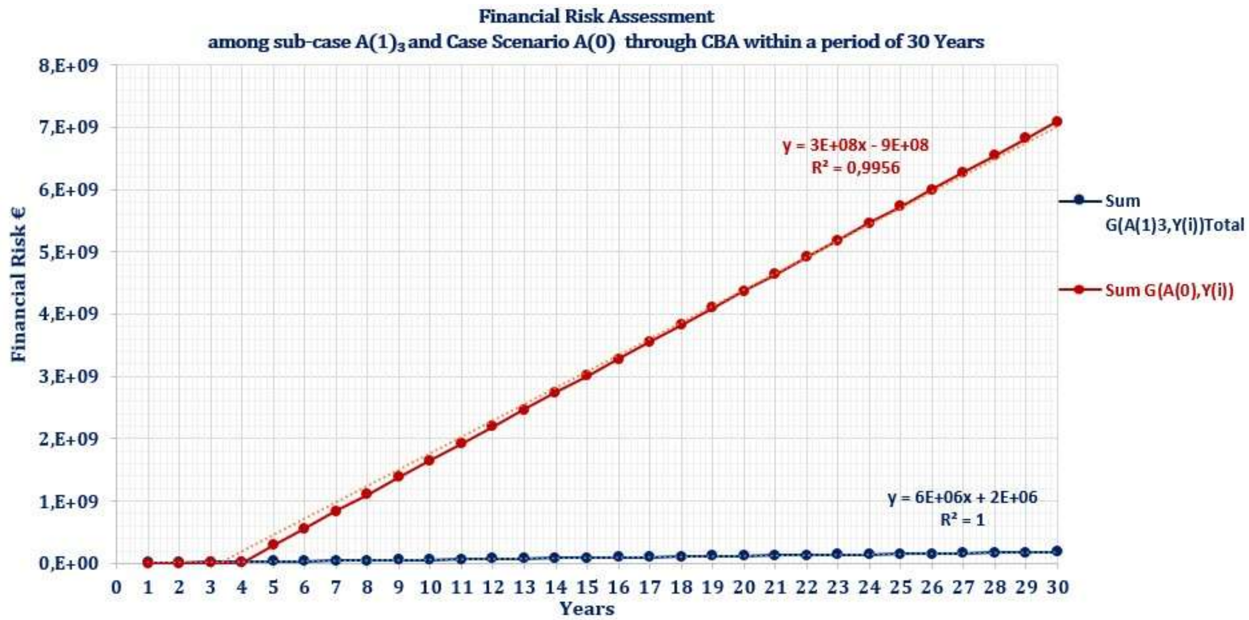


Figure 43 Financial Risk Assessment through CBA, illustrating the Total Grade of Economic Sustainability converted into Monetary Terms for the sub-case $A(1)_3$ and Case Scenario A(0), respectively

The unique solution among the simulative equations for each examined case, shown in Figure 43, identifies the point of efficiency of the sub-case $A(1)_3$ to the opposite Case Scenario A(0). Hence, the theoretical point of efficiency is assessed by the first eighteen days after the third working year.

8.2.3.4. Escalation of the Financial Risk Assessment and partial Indices LTP, ERR, CBA comparing sub-case $A(1)_4$ and Case Scenario A(0)

Figure 44 illustrates the comparative evaluation of LTP indicators, which refer to the risk grade of sub-case A(1)4 and Case A(0), over a 30-year period, respectively.

Figure 44 Comparative Evaluation for the Escalation of LTP Indicators that refer to the Grade of Long Term Perspective Risk for the sub-case $A(1)_4$ and Case Scenario A(0), respectively

Figure 45 presents a comparative evaluation of the escalation of ERR indicators, specifically the rate of revenue, for sub-case A(1)4 and Case A(0) over a 30-year period, respectively.

Figure 45 Comparative Evaluation for the Escalation of ERR Indicators that describe the Economic Rate of Return Grade for the sub-case $A(1)_4$ and Case Scenario A(0), respectively

Figure 46 illustrates the comparative evaluation of the escalation of final CBA indicators, which describe the total grade of sustainability for sub-case A(1)4 and Case A(0) over a 30-year period, respectively.

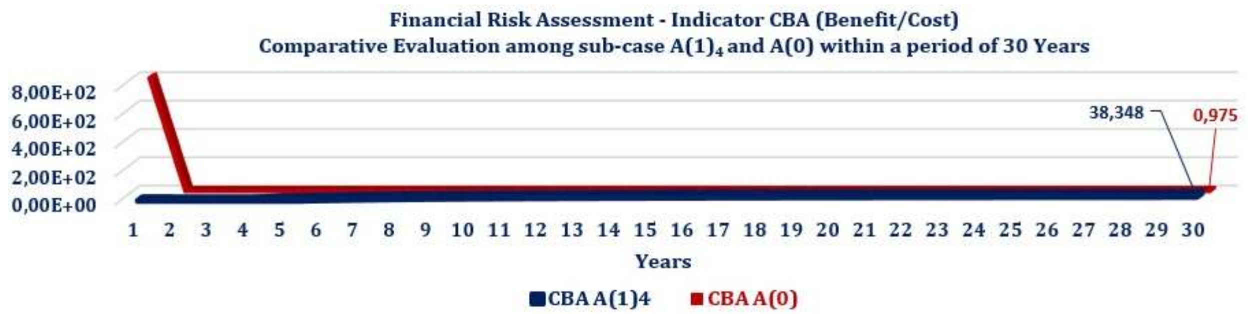


Figure 46 Comparative Evaluation for the Escalation of final CBA Indicators that describe the Grade of Sustainability for the sub-case $A(1)_4$ and Case Scenario A(0), respectively

Figure 47 summarizes the total Financial Risk Assessment, which is calculated by the corresponding Sum of Goal Functions that describe sub-case $A(1)_4$ and A(0), respectively.

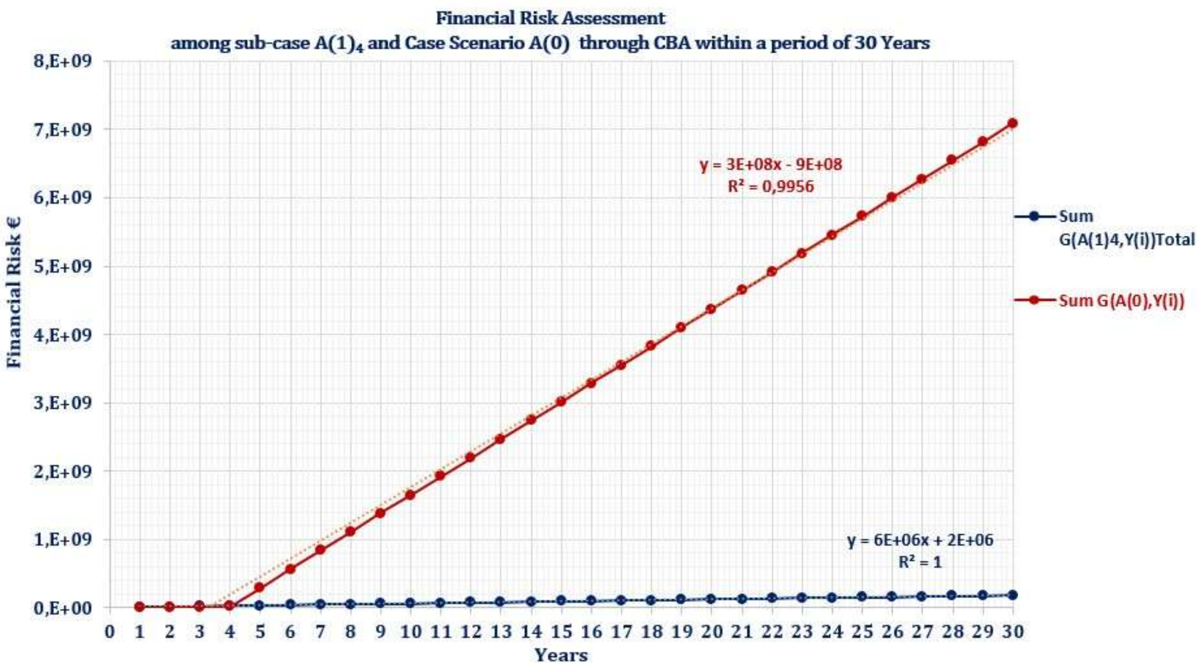


Figure 47 Financial Risk Assessment through CBA illustrating the Total Grade of Economic Sustainability converted into Monetary Terms for the sub-case $A(1)_4$ and Case Scenario A(0), respectively

The unique solution among the simulative equations for each examined case, shown in Figure 47, identifies the point of efficiency of the sub-case $A(1)_4$ to the opposite Case Scenario A(0). Hence, the theoretical point of efficiency is assessed by the first eighteen days after the third working year.

8.2.3.5. Escalation of the Financial Risk Assessment and partial Indices LTP, ERR, CBA comparing sub-case $A(1)_5$ and Case Scenario A(0)

Figure 48 illustrates the comparative evaluation of LTP indicators, which refer to the risk grade of sub-case A(1)4 and Case A(0), over a 30-year period, respectively.

Figure 48 Comparative Evaluation for the Escalation of LTP Indicators that refer to the Grade of Long Term Perspective Risk for the sub-case $A(1)_5$ and Case Scenario A(0), respectively

Figure 49 shows the comparative evaluation of the escalation of ERR indicators, which refer to the rate of revenue for sub-case A(1)5 and Case A(0) over a 30-year period, respectively.

Figure 49 Comparative Evaluation for the Escalation of ERR Indicators that describe the Economic Rate of Return Grade for the sub-case $A(1)_5$ and Case Scenario A(0), respectively

Figure 50 illustrates the comparative evaluation of the escalation of final CBA indicators, which describe the total grade of sustainability for sub-case A(1)5 and Case A(0) over a 30-year period, respectively.

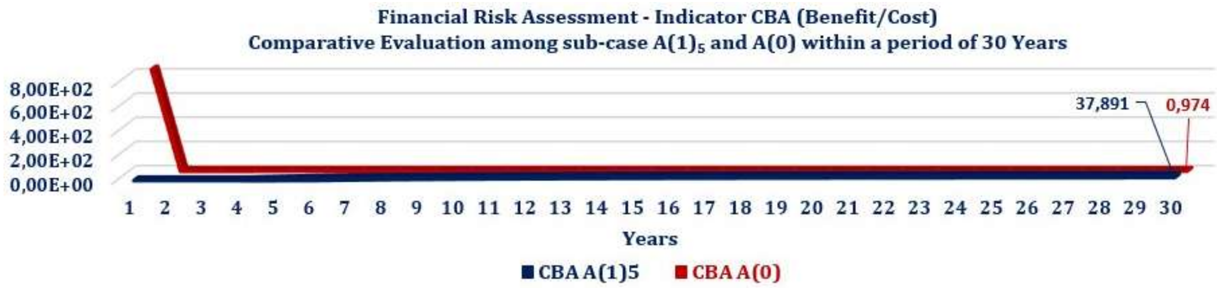


Figure 50 Comparative Evaluation for the Escalation of final CBA Indicators that describe the Grade of Sustainability for the sub-case $A(1)_5$ and Case Scenario A(0), respectively

Figure 51 summarizes the total Financial Risk Assessment, which is calculated by the corresponding Sum of Goal Functions that describe sub-case $A(1)_5$ and A(0), respectively.

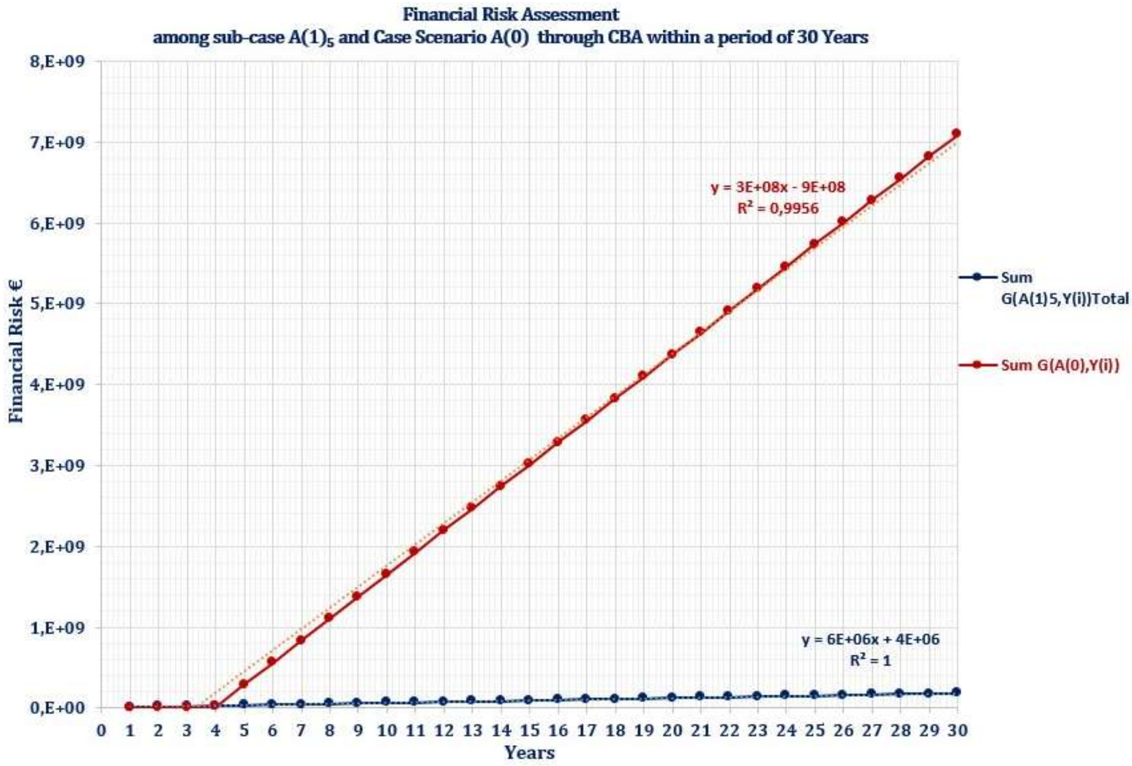


Figure 51 Financial Risk Assessment through CBA illustrating the Total Grade of Economic Sustainability converted into Monetary Terms for the sub-case $A(1)_5$ and Case Scenario A(0), respectively

The unique solution among the simulative equations for each examined case, shown in Figure 51, identifies the point of efficiency of the sub-case $A(1)_5$ to the opposite Case Scenario A(0). Hence, the theoretical point of efficiency is assessed by the twenty-first day after the third working year.

8.3. Stochastic Financial Risk Assessment through Stochastic Risk Assessment of Bayesian Analysis – Key Findings, Indices and Results

Stochastic financial risk assessment, considering the corresponding cost functions as shown in equations 13c and 25, detects the Stochastic Risk for both case scenarios A(0) and A(1), respectively, accounting for the total Goal Functions through CBA. The results of equations 19-21 and 26 describe the potential Stochastic Financial Risk, including probable additive monetary terms that are based on the potential non-conformance with the legal requirements, referring to the Case Scenario A(0), and sub-cases $A(1)_1$ and $A(1)_2$ of the Case Scenario A(1), respectively.

The results of equation 27 refer to the potential Stochastic Financial Risk, involving expectedly additive monetary values that are configured based on the legal obligations, referring to the sub-cases $A(1)_3$ and $A(1)_4$. Results of equations 29-30 describe the potential Stochastic Financial Risk, including the expectedly additive costs that are based on the total investment cost that overlaps the determined penalty cost configured by the legislation, describing sub-case $A(1)_5$.

Except for the Stochastic Financial Risk Assessment, which contains monetary values to describe the stochastic grade of sustainability in each condition, SRA aims to assess the Stochastic Risk Indicator R that demonstrates the rate of the Stochastic Financial Risk Assessment for each case. Hence, the R indicator that is symbolized as R(A(0),Y(i)) and $R(A(1)_{1\text{-}5},Y(i))$ refers to the Stochastic Financial Risk Indicator converted into monetary terms, containing the relevant costs as shown in equations 19-21, 26-27, and 29-30, for the Case Scenario A(0) and all the sub-cases of the Case Scenario A(1), respectively. The R indicator symbolized as $R(A(0),\theta_{(0)})$ and $R(A(1)_{1\text{-}5},\theta_{(0)})$ refers to the Stochastic Risk Indicator without accounting the relevant costs as shown in equations 19-21, 26-27, and 29-30, and describes the grade of the Stochastic Financial Risk Indicators R(A(0),Y(i)) and $R(A(1)_{1\text{-}5},Y(i))$, respectively.

Moreover, a comparative evaluation of the Stochastic Financial Risk Indicators $\sum R(A(1)_{1\text{-}5},Y(i))$ and $\sum R(A(0),Y(i))$, between all sub-cases of the Case Scenario A(1) and Case Scenario A(0), is occurred to calculate the Stochastic Financial Risk Assessment ratio symbolized as CBA Risk $[A(1)_{1\text{-}5}\text{-}A(0)]$ or CBA Risk $[A(0)\text{-}A(1)_{1\text{-}5}]$. CBA Risk $[A(1)_{1\text{-}5}\text{-}A(0)]$ evaluates the sustainability grade of $\sum R(A(1)_{1\text{-}5},Y(i))$ against $\sum R(A(0),Y(i))$, and via versa.

Sections 8.3.1 to 8.3.3 present the results of the SRA calculated through the Mathcad software application, while a further analysis is provided. Table A19 summarizes all the SRA results, considering the corresponding R indicators R(A,Y(i)), R(A,θ(0)), and the CBA Risk Ratios. Table A20 shows the deviations between the results from SRA and CBA. Tables A21-A26 demonstrate the calculations of the Stochastic Risk Integrals using MathCad Software Application Version 10.0.1.0.

8.3.1. Stochastic Financial Risk Assessment Results

Figure 52 demonstrates the Stochastic Financial Risk indicators (including price values) for all the examined cases. It is essential to mention that sub-case $A(1)_4$ refers to the highest risk conditions compared to all the other sub-cases of Case Scenario A(1) and Case Scenario A(0). Sub-case $A(1)_4$ corresponds to the worst-case conditions, while despite investing in environmentally friendly procedures to manage tailings, the negative targeting is increasing annually due to unexpected circumstances (inefficient industrial unit, leachate detection, mishaps, etc.). In actual conditions, this sub-case cannot occur because the annual cost of maintenance aims to provide solutions and consecutive improvement of the equipment to be used to prevent inefficient circumstances systematically. It is accounted only to describe the stochastic financial risk in the maximum risky conditions of Case Scenario A(1) through an absolute theoretical mode of view.

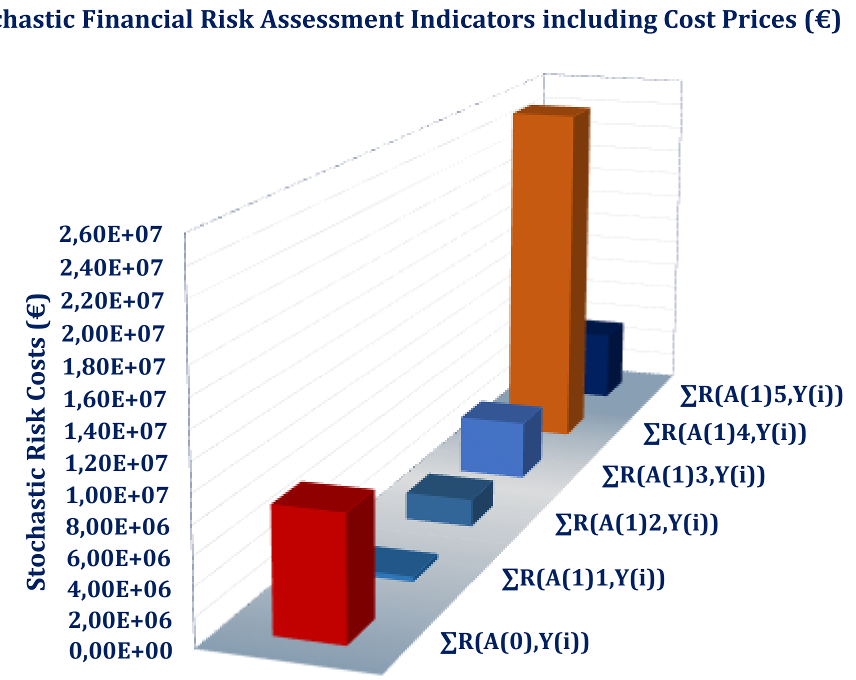
Hence, considering the role of the sub-case $A(1)_4$, it is observed that in actual conditions, Case Scenario A(0) corresponds to the highest stochastic financial risk estimated approximately at 8.6 m€/yr for a mining company that does not conform to the legal obligations of sustainable mining.

Sub-case $A(1)_5$ refers to actual conditions of maximum risk, whereas the stochastic financial risk equals approximately 5.3 m€/yr for a mining company adopting a potential level of non-conformance ranging from 0-25%. In this sub-case, the axiom is that the total efficiency level in sustainable mining practices is proportional to the total investment cost, which characterizes the quality of the equipment used. Therefore, considering that the total negative cost of investment is significantly greater than the corresponding negative cost of penalties, as calculated in accordance with legal obligations, a higher level of safety ensures the assessed grade of final sustainability.

Sub-case $A(1)_3$ refers to actual conditions of intermediate risk, whereas the stochastic financial risk equals approximately 4.2 m€/yr for a mining company adopting a potential level of non-conformance ranging from 0-25%. In this sub-case, the axiom is that the total efficiency level in sustainable mining practices is proportional to minimizing non-conformance grade. Therefore, considering that despite investing in tailings management, there is a risk of non-conformance limited to 25% due to unexpected conditions (such as mishaps, leachate detection, or inefficient recovery, etc.). The potential total stochastic financial risk indicator considers this additive stochastic negative risk cost. The SRA approach is based on the axiom that, despite conforming with a range of 75%, one annual non-conformance ranging from 0-25% is still expected to occur in stable adverse targeting conditions.

Sub-case $A(1)_2$ refers to actual conditions of low risk, whereas the stochastic financial risk equals approximately 1.9 m€/yr for a mining company adopting a potential level of non-conformance ranging from 0-25%. In this sub-case, the axiom is that the total efficiency level in sustainable mining practices is proportional to the detection probability of non-conformance during the audit control. So, considering that despite investing in tailings management, there is a risk of non-conformance limited to 25% due to unexpected conditions (mishaps, leachate detection, non-efficient recovery, etc.). The potential total stochastic financial risk indicator considers this additive stochastic negative risk cost. The SRA approach is based on the axiom that, despite conforming to a range of 75%, one annual non-conformance, ranging from 0 to 25%, may occur in a consecutive increase of negative targeting conditions.

Sub-case $A(1)_1$ refers to actual conditions of extremely low risk, whereas the stochastic financial risk equals approximately 0.335 m€/yr for a mining company adopting a potential level of non-conformance ranging from 0 to 25%. In this sub-case, the axiom is that the total efficiency level in sustainable mining practices is proportional to the detection probability of non-conformance during the audit control. Therefore, considering that despite investing in tailings management, there is a risk of non-conformance limited to 25% due to unexpected conditions (such as mishaps, leachate detection, or inefficient recovery, etc.). The potential total stochastic financial risk indicator considers this additive stochastic negative risk cost. The SRA approach is based on the axiom that, despite conforming to a range of 75%, one annual non-conformance, ranging from 0 to 25%, may occur in stable negative targeting conditions.



∑R(A(0),Y(i))	8.611.200
∑R(A(1)1,Y(i))	335.243
∑R(A(1)2,Y(i))	1.917.133
∑R(A(1)3,Y(i))	4.210.034
∑R(A(1)4,Y(i))	25.723.738
∑R(A(1)5,Y(i))	5.290.509

Figure 52 Stochastic Financial Risk Indicators, including Cost Prices (€) for the Case Scenario A(0) and all sub-cases $A(1)_{1-5}$ of the Case Scenario A(1)

8.3.2. Stochastic Risk Assessment Results

Figure 53 demonstrates the Stochastic Risk indicators configuring the stochastic financial risk monetary indices for all the cases examined, correlated with the corresponding total negative costs expressed by their total goal functions supported by CBA.

It is essential to mention that sub-case $A(1)_4$ refers to the highest risky conditions compared to all the other sub-cases of Case Scenario A(1) and Case Scenario A(0), which equaled approximately 3.94. As previously mentioned, sub-case $A(1)_4$ corresponds to the worst-case conditions, while despite investing in environmentally friendly procedures to manage tailings, the negative targeting is increasing annually due to unexpected circumstances (inefficient industrial unit, leachate detection, mishaps, etc.). In actual conditions, this sub-case cannot occur. It is accounted only to describe the stochastic financial risk in the maximum risky conditions of Case Scenario A(1) through an absolute theoretical mode of view. Therefore, in this case, the stochastic risk indicator considers the degree of risk corresponding to the systematic increase of negative targeting while investing in tailings management and expecting at least one non-conformance annually.

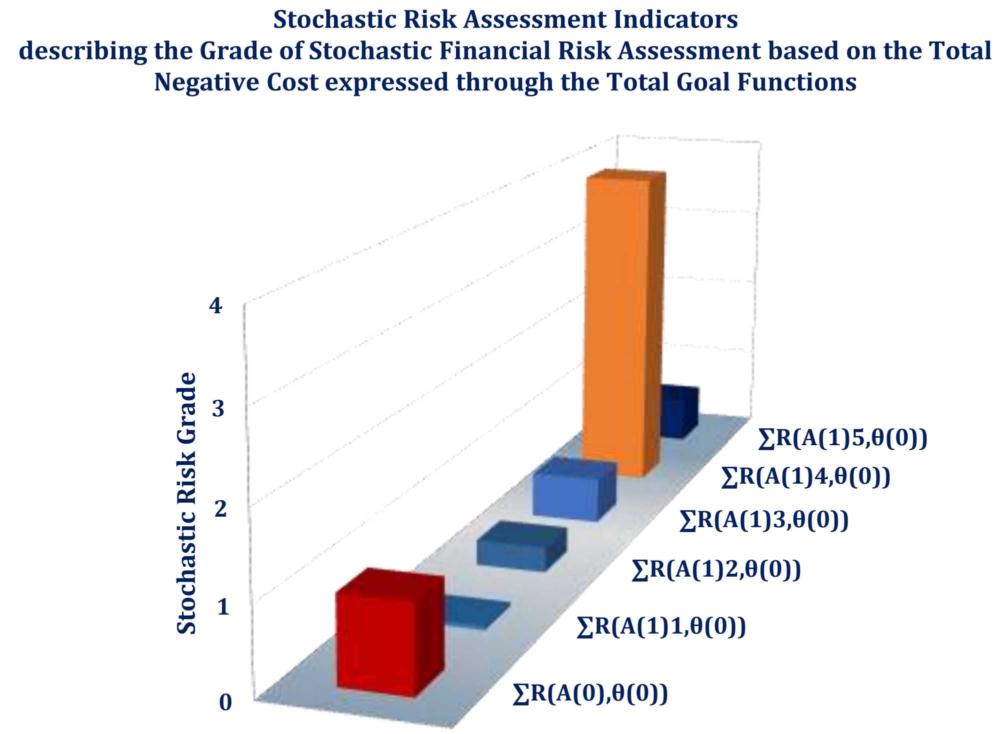
Hence, considering the role of the sub-case $A(1)_4$, it is observed that in actual conditions, Case Scenario A(0) corresponds to the highest stochastic risk estimated approximately at 1 for a mining company that does not conform to the legal obligations of sustainable mining. In this case, the stochastic risk indicator considers the degree of risk that corresponds to the systematic increase of negative targeting, while the potential probability of non-conformance may occur at least one time per year.

Sub-case $A(1)_5$ refers to actual conditions of maximum risk, whereas the stochastic risk equals approximately 5 for a mining company adopting a potential level of non-conformance ranging from 0-25%. In this sub-case, the axiom is that the total efficiency level in sustainable mining practices is proportional to the total investment cost, which characterizes the quality of the equipment used. Therefore, considering that the total negative cost of investment is significantly greater than the corresponding negative cost of penalties, as calculated in accordance with legal obligations, a higher level of safety ensures the assessed grade of final sustainability. It is essential to mention that the stochastic risk indicator considers the annual decrease of negative targeting while the maximum additive cost is expected, which refers to at least one expected non-conformance per year.

Sub-case $A(1)_3$ refers to actual conditions of intermediate risk, whereas the stochastic risk equals approximately 0.54 for a mining company adopting a potential level of non-conformance ranging from 0 to 25%. In this sub-case, the axiom is that the total efficiency level in sustainable mining practices is proportional to minimizing non-conformance grade. So, considering that despite investing in tailings management, there is a risk of non-conformance limited to 25% due to unexpected conditions (mishaps, leachate detection, non-efficient recovery, etc.). The potential total stochastic risk indicator considers this additive stochastic negative risk grade. The SRA approach is based on the axiom that, despite conforming with a range of 75%, one annual non-conformance ranging from 0-25% is still expected to occur in stable negative targeting conditions.

Sub-case $A(1)_2$ refers to actual conditions of low risk, whereas the stochastic risk equals approximately 0.3 for a mining company adopting a potential level of non-conformance ranging from 0 to 25%. In this sub-case, the axiom is that the total efficiency level in sustainable mining practices is proportional to the detection probability of non-conformance during the audit control. Therefore, considering that despite investing in tailings management, there is a risk of non-conformance, its probability is limited to 25% due to unexpected conditions (such as mishaps, leachate detection, or inefficient recovery, etc.). The potential total stochastic risk grade considers this additive stochastic negative risk index. The SRA approach is based on the axiom that, despite conforming to a range of 75%, one annual non-conformance, ranging from 0-25%, may occur in a consecutive increase of adverse targeting conditions.

Sub-case $A(1)_1$ refers to actual conditions of extremely low risk, whereas the stochastic risk equals approximately 0.04 for a mining company adopting a potential level of non-conformance ranging from 0-25%. In this sub-case, the axiom is that the total efficiency level in sustainable mining practices is proportional to the detection probability of non-conformance during the audit control. So, considering that despite investing in tailings management, there is a risk grade of non-conformance, its probability is limited to 25% due to unexpected conditions (mishaps, leachate detection, non-efficient recovery, etc.). The potential total stochastic risk grade considers this additive stochastic negative risk index. The SRA approach is based on the axiom that, despite conforming to a range of 75%, one annual non-conformance, ranging from 0 to 25%, may occur in stable adverse targeting conditions.

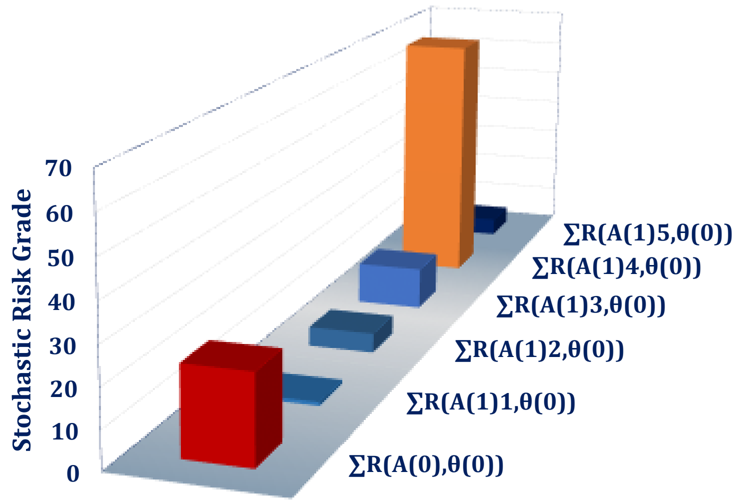


∑R(A(0),θ(0))	1,00
∑R(A(1)1,θ(0))	0,04
∑R(A(1)2,θ(0))	0,29
∑R(A(1)3,θ(0))	0,54
∑R(A(1)4,θ(0))	3,94
∑R(A(1)5,θ(0))	0,55

Figure 53 Stochastic Risk Indicators for the Case Scenario A(0) and all sub-cases $A(1)_{1\text{-}5}$ of the Case Scenario A(1) that demonstrate the Grade of the corresponding Stochastic Financial Risk Assessment based on the Total Goal Functions

Figure 54 illustrates the Stochastic Risk indicators that configure the stochastic financial risk monetary indices shown in Figure 52 for all examined cases, correlated with the corresponding negative costs expressed by the bases of their cost functions, as supported by the CBA.

Stochastic Risk Assessment Indicators describing the Grade of Stochastic Financial Risk Assessment based on the Potential/Expected Additive Negative Cost expressed through the Base of Theoretical Goal Functions in Case A(0), sub-Cases $A(1)_{1\text{-}4}$, and the Theoretical Goal Function of the sub-Case $A(1)_5$



∑R(A(0),θ(0))	23
∑R(A(1)1,θ(0))	0,90
∑R(A(1)2,θ(0))	5,12
∑R(A(1)3,θ(0))	11,24
∑R(A(1)4,θ(0))	68,71
∑R(A(1)5,θ(0))	4,79

Figure 54 Stochastic Risk Indicators for the Case Scenario A(0) and all sub-cases $A(1)_{1\text{-}5}$ of the Case Scenario A(1) that demonstrate the Grade of the corresponding Stochastic Financial Risk Assessment based on the defined at-Risk Cost Functions

8.3.3. Stochastic Financial Risk Assessment – Comparative Evaluation of SRA Indices between Case Scenario A(0) and Sub-cases of Case Scenario A(1)

Considering the results through the SRA methodology analyzed in sections 8.3.1 and 8.3.2, shown in Table A19, the CBA Risk Ratio Matrix is created. This data provides a comparative evaluation of the stochastic financial risk grades that describe Case Scenario A(0) and all sub-cases of Case A(1). Figure 54 presents a graphical representation of the comparative evaluation for the degrees of stochastic financial risk, which describe the examined cases.

As it is observed, Case Scenario A(0) adopts the highest grade of CBA Risk against all the sub-cases of Case Scenario A(1) except the sub-case $A(1)_4$. Considering that sub-case $A(1)_4$ corresponds to an absolute theoretical point of view without physical meaning in actual conditions, it is assessed that Case Scenario A(1) is more sustainable than Case Scenario A(0), as clearly shown in Figure 55, too.

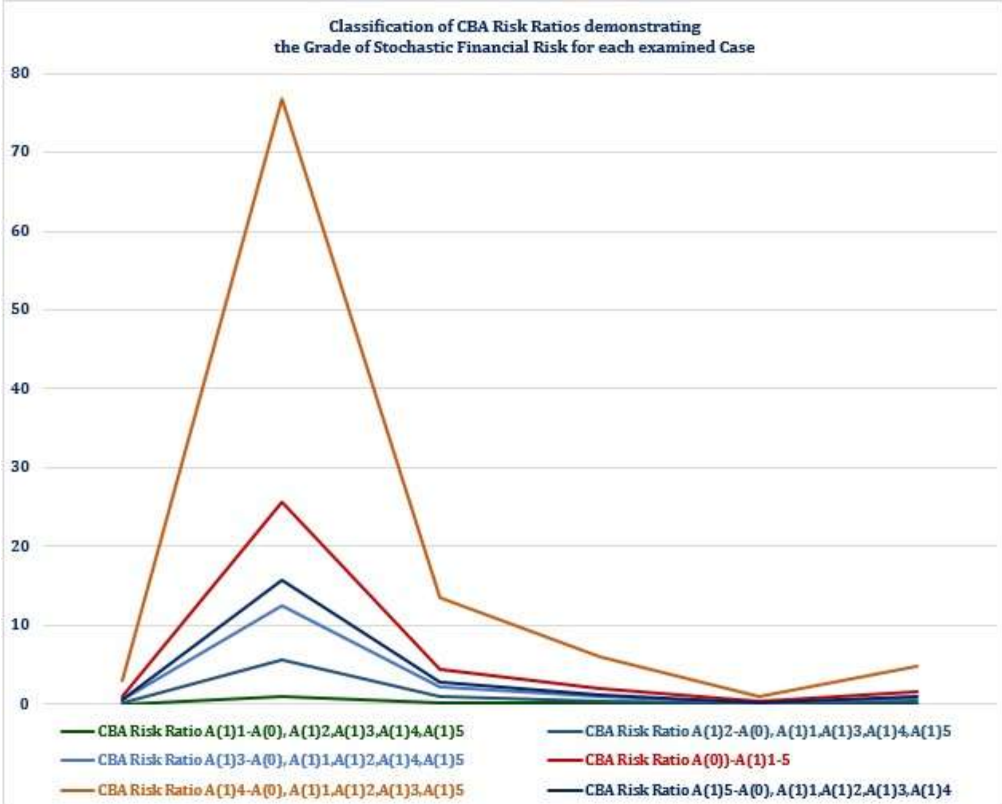
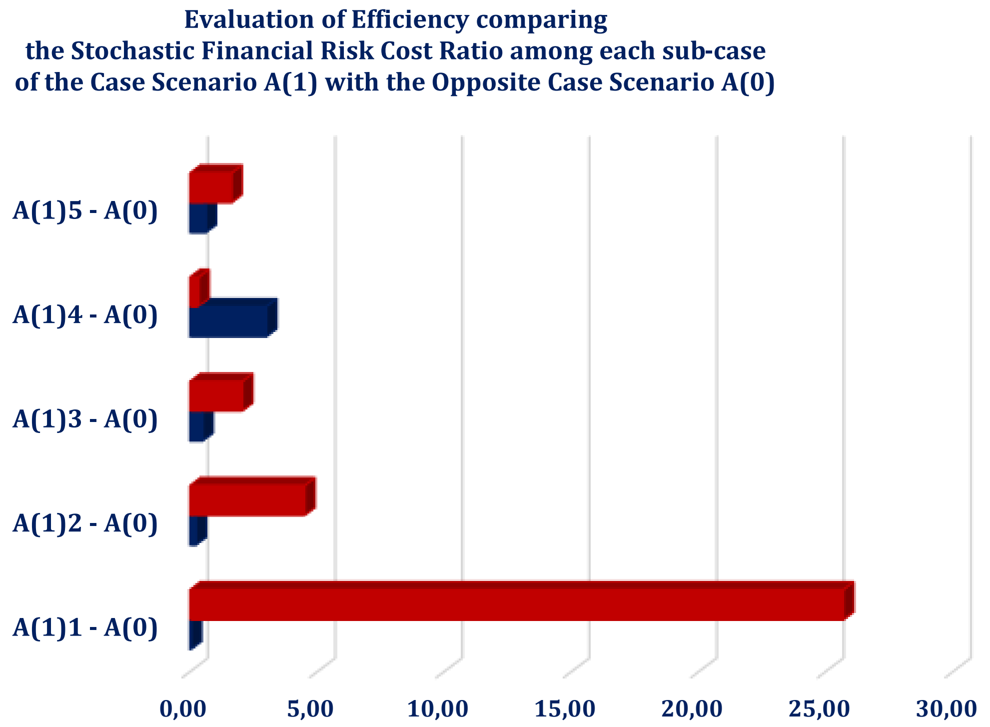


Figure 55 Graphical Description of the CBA Risk Ratios' Comparative Evaluation shown in Table A19

Observing both Figures 55 and 56, it is assessed that sub-case $A(1)_5$ refers to the maximum risky actual conditions of Case Scenario A(1). The sub-case $A(1)_3$ corresponds to the intermediate risk category, while sub-cases $A(1)_2$ and $A(1)_1$ describe the low-risk actual circumstances in Case Scenario A(1).

The physical meaning of this observation focuses on the separated axioms adopted by both CBA and SRA methodologies for risk grade detection. In sub-cases $A(1)_{1\text{-}3}$ it is accounted the potential/expected additive negative cost due to a range of non-conformance (0-25%), following the legal terms and definitions, to the total investment cost. In sub-case $A(1)_5$ the potential additive negative cost due to a range of expected non-conformance (0-25%) that is proportional to the total investment cost, is accounted to estimate both the financial and stochastic risk.



	A(1)1 - A(0)	A(1)2 - A(0)	A(1)3 - A(0)	A(1)4 - A(0)	A(1)5 - A(0)
■ CBA Risk Ratio [∑R(A(0),Y(i))/∑R(A(1)1-5,Y(i))]	25,686	4,492	2,045	0,335	1,628
■ CBA Risk Ratio [∑R(A(1)1-5,Y(i))/∑R(A(0),Y(i))]	0,039	0,223	0,489	2,987	0,614

Figure 56 Comparative Evaluation, supported by the Stochastic Financial Risk Cost Ratio, among the sub-cases A(1)$_{1\text{-}5}$ of the Case Scenario A(1) with the Case Scenario A(0), illustrating their Grade of Efficiency

8.4. Discussion

This study examines how project and risk management in mining waste management can be enhanced using the CBA methodology and SRA Bayesian analysis, supported by applicable mathematical models.

By following the legislative guidance on environmental protection, these norms extract the optimal business decision according to CBA's requirements. Additionally, Stochastic Risk Analysis enhances the accuracy of CBA by evaluating the project's overall efficiency.

This calibration is achieved by the integrals shown in Sections 7.2 and 7.3. The correlation between the total CBA's goal functions and the required weighting factor is described within the integral. This weighting factor is a Beta distribution. Due to the nature of work activities, the binomial distribution is configured as described in Sections 6.4.1 and 6.4.2.

<u>Case Scenario A(0)</u>

For the case scenario A(0), as shown in Figure 20, there are three different classes of Y(i) and the corresponding PDF(i) factors over the period of 30 years. This classification is displayed due to the implementation of legally mandatory obligations, and especially the increase of Σεεπi indicator due to negative targeting for consecutive non-conformance. The behavior of these two partial factors is a milestone for the configuration of the total potential negative goal function, which expresses the total potential negative cost.

Therefore, the annual escalation of the stable loss and goal functions shown in Figure 19 also adopts a classification mode of three different classes. Thus, as indicated in Figure 21, the final estimated costs for case scenario A(0) are 7.088 b€ over a period of 30 years. Considering the financial risk indicators (LTP, ERR, and CBA (B/C)) by CBA methodology, it is assessed that the point of inefficiency of the Case Scenario A(0) against sub-cases $A(1)_{1\text{-}5}$ is observed by the 5th year of operation.

Comparing the stochastic risk indicators between Case Scenario A(0) and sub-cases $A(1)_{1\text{-}5}$ through SRA, it is observed that grade of the stochastic risk that describes Case Scenario A(0) adopts higher numerical values than all sub-cases of the Case Scenario A(1) except for the sub-case $A(1)_4$ as shown in Table A19.

Case Scenario A(1)

For the case scenario A(1), five different sub-cases are taken into account. A summary of their results and physical meaning through a clear view of practical application on an industrial scale is provided below in this chapter.

Sub-cases $A(1)_{1\text{-}2}$

Sub-cases $A(1)_{1\text{-}2}$ focus on the false trial methodology considering that despite investing in tailings management providing legal conformance at least 75%, there is an additive risk of non-conformance ranging between (0-25%) due to potential detected environmental pollution during the audit control in mining sites of a start-up mining company. The sub-case $A(1)_1$ examines the conditions of a potential detected environmental pollution in random locations inside the industrial field. Therefore, the negative targeting adopts stable numerical values during years of operation. The sub-case $A(1)_2$ examines the conditions of potential detected environmental pollution in a specific location inside the industrial field. Therefore, the negative targeting adopts unstable numerical values that increase during the operating years.

The total financial risks through CBA that refer to sub-case $A(1)_1$ and $A(1)_2$ are approximately 185 and 181 m€ within the period of thirty years, respectively. The reason for this assessment is due to the potential widespread pollution in sub-case $A(1)_1$ against the potential detected pollution in a specific position inside the industrial field described in sub-case $A(1)_2$. Therefore, the potential additive negative cost, due to non-conformance ranging from 0% to 25%, to the total investment cost is approximately 4.7 and 0.577 million euros for sub-cases $A(1)_1$ and $A(1)_2$, respectively.

Comparing the assessed grade of sustainability in sub-case $A(1)_1$ with the opposite scenario A(0), it is observed that

- The LTP indicator is lower than the numerical value 1 by the 5th year of operation
- The ERR indicator is positive by the 5th year of operation
- The CBA (B/C) indicator is greater than the numerical value 1 by the 5th year of operation

The obvious point of efficiency, comparing sub-case A(1)1 with Case A(0), is assessed by the 5th year of operation, as shown in Figure 35. Despite this, the theoretical point of efficiency is assessed by the first eighteen days after the third working year as discussed in section 8.2.3.1.

Comparing the assessed grade of sustainability in sub-case $A(1)_2$ with the opposite scenario A(0), it is observed that

- The LTP indicator is lower than the numerical value 1 by the 4th year of operation

- The ERR indicator is positive by the 4th year of operation
- The CBA (B/C) indicator is greater than the numerical value 1 by the 5th year of operation

So, the apparent point of efficiency comparing sub-case $A(1)_2$ with Case A(0) is assessed by the 5th year of operation, as shown in Figure 39. Despite this, the theoretical point of efficiency is assessed by the first eighteen days after the third working year, as declared in section 8.2.3.2.

The total stochastic financial risks through SRA that refer to sub-case $A(1)_1$ and $A(1)_2$ are approximately 0.335 and 1.9 m€ within the period of thirty years, respectively. The grades of the stochastic financial risks through SRA are 0.04 and 0.29 for the sub-cases $A(1)_1$ and $A(1)_2$, respectively. The grades of the stochastic financial risks through SRA based on the corresponding at-risk costs (expressed by the theoretical Loss Functions) equal approximately 1 and 5.12 for sub-cases $A(1)_1$ and $A(1)_2$, respectively. The reason for these stochastic assessment results is that in SRA, the examination of financial risk is provided by applicable mathematical tools through an absolute theoretical point of view, emphasizing mainly their numerical values and not in actual conditions as in CBA.

Sub-cases $A(1)_{3\text{-}4}$

Sub-cases $A(1)_{3\text{-}4}$ focus on the exponential methodology, considering that despite investing in tailings management providing legal conformance at least 75%, there is an additive risk of non-conformance ranging between (0-25%) due to expectedly detected environmental pollution during the audit control in mining sites of a start-up mining company. The sub-case $A(1)_3$ examines the conditions of an expectedly detected environmental pollution in random locations inside the industrial field. Therefore, the negative targeting adopts stable numerical values during years of operation. The sub-case $A(1)_4$ examines the conditions of expectedly detected environmental pollution in a specific location inside the industrial field. Therefore, the negative targeting adopts unstable numerical values that increase during the operating years.

The total financial risks through CBA that refer to sub-case $A(1)_3$ and $A(1)_4$ are approximately 180.156 and 180.157 m€ within the period of thirty years, respectively. The reason for this low deviation of 0.001 m€ is due to the expected environmental pollution in both sub-cases. So, the spread of environmental pollution has not been mainly evaluated. However, its occurrence affects the total financial risk even if is widespread or localized. So, the expectedly additive negative cost, due to non-conformance ranging from 0-25%, to the total investment cost is approximately 0.0925 and 0.0936 m€ for sub-cases $A(1)_3$ and $A(1)_4$, respectively.

Comparing the assessed grade of sustainability in sub-case $A(1)_3$ with the opposite scenario A(0), it is observed that

- The LTP indicator is lower than numerical value 1 by the 5th year of operation
- The ERR indicator is positive by the 5th year of operation
- The CBA (B/C) indicator is greater than numerical value 1 by the 5th year of operation

So, the obvious point of efficiency comparing sub-case $A(1)_3$ with Case A(0) is assessed by the 5th year of operation, as shown in Figure 43. Despite this, the theoretical point of efficiency is assessed by the first eighteen days after the third working year as declared in chapter 8.2.3.3.

Comparing the assessed grade of sustainability in sub-case $A(1)_4$ with the opposite scenario A(0), it is observed that

- The LTP indicator is lower than the numerical value 1 by the 5th year of operation

- ➢ The ERR indicator is positive by the 5th year of operation
- ➢ The CBA (B/C) indicator is greater than the numerical value 1 by the 5th year of operation

So, the obvious point of efficiency comparing sub-case $A(1)_4$ with Case A(0) is assessed by the 5th year of operation, as shown in Figure 47. Despite this, the theoretical point of efficiency is assessed by the first eighteen days after the third working year, as presented in section 8.2.3.4.

The total stochastic financial risks through SRA, as referred to in sub-case A(1)3 and A(1)4, are approximately 4.2 and 25.7 million euros, respectively, within a thirty-year period. The grades of the stochastic financial risks through SRA are 0.54 and 3.94 for the sub-cases $A(1)_3$ and $A(1)_4$, respectively. The grades of the stochastic financial risks through SRA based on the corresponding at-risk costs (expressed by the theoretical Loss Functions) equal approximately 11 and 69 for sub-cases $A(1)_3$ and $A(1)_4$, respectively. The reason for these stochastic assessment results is that in SRA, the examination of financial risk is provided by applicable mathematical tools through an absolute theoretical point of view, emphasizing mainly their numerical values and not in actual conditions as in CBA. So, despite that sub-case $A(1)_4$ has no practical meaning because it could not occur in actual conditions as declared in 7.4.2 section, it is examined by SRA through a clear point of theoretical view.

Sub-case $A(1)_5$

Sub-case $A(1)_5$ focuses on the exponential methodology, considering that despite investing in tailings management providing legal conformance at least 75%, there is an additive risk of non-conformance ranging between (0-25%) due to expectedly detected environmental pollution during the audit control in mining sites of a start-up mining company. The sub-case $A(1)_5$ examines the conditions of an expectedly detected environmental pollution and adopts the axiom that the expected additive cost for non-conformance is proportional to the total investment cost that characterizes the efficiency of the tailings management system. Therefore, the negative targeting adopts a stable numerical value after the first year of operation. The sub-case $A(1)_5$ examines the conditions of expectedly detected environmental pollution that decreases annually while investments in maintenance and processing are made consecutively during the operating years.

The total financial risk through CBA that refers to sub-case $A(1)_5$ is approximately 182.2 m€ within the period of thirty years. The expected additive negative cost, due to non-conformance, ranging from 0 to 25%, to the total investment cost is approximately 2.2 m€ for the first year of operation, 798 € for the second year, and by the third year, there is no detection of it. The practical significance of this lies in the probability of expected environmental pollution, which is dramatically decreasing, while funding a closed system of industrial units to prevent the free disposal of hazardous waste, in line with environmental legal obligations.

Comparing the assessed grade of sustainability in sub-case $A(1)_5$ with the opposite scenario A(0), it is observed that

- ➢ The LTP indicator is lower than the numerical value 1 by the 5th year of operation
- ➢ The ERR indicator is positive by the 5th year of operation
- ➢ The CBA (B/C) indicator is greater than the numerical value 1 by the 5th year of operation

The obvious point of efficiency, comparing sub-case A(1)5 with Case A(0), is assessed by the 5th year of operation, as shown in Figure 51. Despite this, the theoretical point of efficiency is assessed on the twenty-first day after the third working year, as declared in section 8.2.3.5.

The total stochastic financial risk through SRA, as referred to in sub-case A(1)5, is approximately 5.2 million euros over a thirty-year period. The grade of the stochastic financial risks through SRA is 0.55. So, it is comparable with the corresponding grades of sub-cases $A(1)_{1-3}$. The SRA grade of sub-case $A(1)_5$ shows the highest numerical value among the sub-cases $A(1)_{1-3}$ that refer to actual conditions. Hence, based on the SRA methodology, sub-case $A(1)_5$ is characterized as the most risky sub-case scenario. Furthermore, the grade of the stochastic financial risk through SRA based on the corresponding at-risk cost (expressed by the theoretical Loss Function) equals approximately 5 for sub-case $A(1)_5$.

General Observations

As a general observation, it is essential to mention that the stochastic supplementary econometric indicator IRR of the B/C ratio, calculated by the CBA overall analysis, is expected to be extremely high for all sub-cases of the Case Scenario A(1) against the Case Scenario A(0). The main reason for this is the strict legal obligations regarding managing hazardous waste. Thus, the negative targeting for the case of consecutive non-conformance increases dramatically over the thirty-year forecast period. This leads to extremely high penalty costs in similar cases with Case Scenario A(0). Applied engineering, in line with the principles of sustainable development in the mining sector, necessitates consideration of relevant legislation, a comprehensive understanding of the technical parameters that impact the sustainability of similar projects, and an economic assessment of proposed mining waste management processes. The legal obligations for mining activities indirectly determine the necessity of tailings management to minimize potential environmental pollution hazards. The required technical parameters to achieve an efficient manner of tailings management, by cost savings due to fully legal conformance and utilizing the waste materials, are determined by the real industrial needs that refer to actual conditions (4) (23). The financial risk assessment continuously evaluates the sustainability of the suggested project to be funded (122) (54) (61).

The current PhD dissertation focuses on the financial risk assessment of a proposed project to manage 150 ktonnes of mining tailings annually. As mentioned in the 4.2 section, this tailings mass refers to the actual produced tailings mass. There are plenty of scientific results (as mentioned in section 3.2) ensuring a high rate of metal recovery efficiency (up to 95%) from mining tailings using the proportional concentration and mass of chemical reagents and catalysts, as in the proposed Case Scenario A(1), through a unique recovery step. Despite this, the proposed Case Scenario A(1) is scheduled by three major steps, each involving three separated stages of metal recovery. This guides to a cost overdesign that offers a high level of recovery efficiency assurance. As a result, there is a high safety factor in the cost assessment of the project.

Hence, regarding the financial risk assessment through CBA Methodology, the innovation of the current study concerns the implementation of the CBA module considering the corresponding legal obligations and the costs of estimated technical parameters (primary installation cost for the fully autonomous equipment to be used, annual operational cost, cost of maintenance, and potential benefit by the utilization of recovered metal material), and the deterministic approach of the Probability Density Function for each examined case.

Due to its ability to evaluate multiple parameters that may affect the total sustainability of each examined case and convert them into monetary terms, the indicative CBA appraisal tool provides an objectively accurate estimation of the total cost-effectiveness. The current scientific research gap focuses on the fact that, despite scientific results and suggested guidance regarding sustainable mining, implementing tailings management procedures, and the practical application of CBA to assess the efficiency point between comparable case scenarios is still limited. Therefore, the presented methodology of CBA demonstrates how to assess the efficiency point of all sub-cases $A(1)_{1-5}$ against Case Scenario A(0). It is very important to mention that sub-cases $A(1)_1$, $A(1)_3$, and $A(1)_4$ refer to theoretical estimations that showed approximately the same grades of financial risks as the most practical sub-cases $A(1)_2$ and $A(1)_5$.

The SRA methodology through Bayesian Analysis is applied to assess the cause of the estimated financial risks calculated by CBA that characterize each examined case. The risk indicators calculated through it demonstrate the rate of financial risk escalation and the degree of risk for non-conformance separately in each examined case. The SRA methodology is a well-known tool for risk estimation depending on each project's conditions, contributing positively to decision-making. Still, the SRA methodology has been applied in many industrial projects, even from the internal corporation management departments or from the side of investors (120) (121). However, the SRA approach to assess the degree of financial risks that refer to proportional cases as A(0) and A(1) is very early, as the CBA stage needs to be developed to offer financial risk assessment in the mining sector. Moreover, the innovation of SRA in evaluating CBA results focuses on its application in actual conditions, converted into countable numerical values.

The comparable results between CBA and SRA approaches, as shown in Table A20, demonstrate the necessity of SRA to offer a substantial financial risk assessment tool ultimately. It is observed that despite the financial risk assessment results from CBA showing approximately the same degree of financial riskfor all sub-cases of Case Scenario A(1), the SRA detects quite the same grades of stochastic financial risks for the sub-cases that refer to actual conditions, while showing deviation for the theoretical sub-cases of Case Scenario A(1). Therefore, the presented financial risk assessment tool, structured by CBA and SRA, could be applied in the sustainable mining sector to the corporate management department or adopted and developed by financial institutions that offer financial loans to mining companies.

It is essential to mention that, from a spherical point of view, it is impossible to detect widespread environmental pollution while investing in procedures that prevent it. So, the sub-cases $A(1)_1$ and $A(1)_3$ adopt a minor probability of occurrence. On the opposite side, the sub-case $A(1)_2$, which refers to the potential detection of environmental pollution in a specific location inside the industrial unit field that manages tailings mass, is more probable to exist. Moreover, the most probable sub-case is sub-case A(1)5, whereas the expected additive cost of non-conformance decreases year by year as investment in tailings management optimization increases. These logical observations are verified through the deviations from the comparison of risk grades that were calculated by both CBA and SRA for all examined cases, as shown in Table A20.

The financial estimation through CBA refers to the waste treatment and metal recovery by mining tailings whose annual mass is approximately 150 ktonnes. Therefore, Bayesian Analysis of SRA aligns the output of CBA's functions. The latter ensures the superiority of Case Scenario A(1) against Case Scenario A(0), as demonstrated in the current study.

One of the most important key factors for both approaches is the total probability of non-conformance $\theta_{(0)}$. This factor is accordingly defined in terms of the natural conditions that describe each case examined. Despite investing in establishing a closed system of industrial units to treat mining tailings in terms of 4Rs, there is the risk of potential non-full compliance with mandatory requirements due to the probability of identified environmental pollution. This probability, expressed by $\theta_{(3)}$, is the key factor for CBA and Stochastic Financial Risk Assessment of Case Scenario A(1).

All the probabilities $\theta_{(1)}$, $\theta_{(2)}$, $\theta_{(3)}$, and $\theta_{(4)}$ have a significant role on both CBA and SRA for case scenarios A(0) and A(1). Based on the legislation's requirements, the non-conformance does not refer only to the identified environmental pollution. Total non-compliance refers to each general corporative policy according to a) systematic disposal of huge volumes of hazardous waste in ground soil and b) non-metal recovery in terms of the Circular Economy.

In conclusion, considering the cost information gained by the final Economic Assessment presented in the Appendix for Case Scenario A(1) (Tables A6, A7), it is observed that the estimated annual Benefit Value (B.V.) from metal recovery equals approximately 46.7 % of the total investment cost (C.I.M. plus E.C. plus C.C.R.) for the first year of work. In addition, this estimated annual Benefit Value from metal recovery equals approximately 52.3 % of the total investment cost per year of work after the first one. This minor difference is due to the primary installation cost for the first year, while the corresponding cost for the following years is lower, expressed through the maintenance cost.

One of the most important axioms is that the value of metals, despite their potential fluctuation, is always high, especially for precious metals and rare earth elements, due to their high level of conductivity. Investment in metal recovery faces the challenge of future risks regarding the potential lack of a metal supply chain.

9. Conclusions

9.1. Summary of Research Findings

The key findings of this research highlight the techno-economic indicators that describe the level of economic sustainability for the examined cases, specifically regarding tailings management for a new start-up mining company. At the same time, the financial risk indicators calculated through the CBA methodology are cross-examined with the corresponding indicators calculated by the SRA approach.

The overall analysis focuses on identifying sustainable mining procedures that should be implemented to prevent environmental pollution hazards associated with mining activities. The criterion that characterizes the grade of sustainability of the environmentally friendly mining procedures is the financial risk indicators exported by both CBA-SRA.

To address the environmental challenges analyzed in Sections 1 and 2, which are caused by mining activities (described in Case Scenario A(0)), establishing a closed system of industrial units involving fully autonomous equipment with buffer tanks connected by a pipeline system is necessary. As described by Case Scenario A(1), this work layout prevents the free disposal of hazardous waste, implying the legal obligations that support the circular economy policy and environmental protection while offering benefits to a mining company that adopts its mechanism.

Mechanism of Case Scenario A(1) as described in section 5, considers the stereochemical engineering properties of metal cations to bind with non-metal acidic complex anions chemically. When this phenomenon occurs in the ground soil area (Case A(0)), it leads to the production of chemical salts, which are used to precipitate in the downstream area of the aquifer zone. On the other hand, when this phenomenon occurs under controlled conditions, within a closed system of industrial units as proposed in Case A(1), the produced chemical salts are purified using metal materials that sink to the bottom of buffer tanks.

Processing of Case Scenario A(1) refers to an expansion of existing practices that focus on tailings management, particularly metal recovery. The mass of chemical reagents, energy consumption, and dimensions of the equipment used are proportional to the estimated mass of tailings to be managed. All these technical parameters have a significant role in the techno-economic assessment of this project.

The primary techno-economic assessment, as presented in section 6, evaluates the estimated negative costs for both examined cases A(0) and A(1). The total negative cost of Case A(0) refers to the configured penalty costs that could be charged to a start-up mining company that does not conform to the mandatory legal obligations of the mining industry. The total negative cost of Case A(1) refers to the configured total cost of investment in conforming with the legal obligations by establishing a closed industrial unit system that could manage the produced tailings mass, accounting for the estimated benefit of the recovered metal mixture. Results of this primary economic analysis-assessment guide to an insufficient economic forecast for a future mining company's life cycle period. Therefore, the CBA methodology is applied following the terms and definitions of Circular Economy and environmental sustainability to estimate the total financial risk of both Cases A(0) and A(1).

The CBA methodology aims to assess the economic sustainability grade of each case scenario, providing an accurate comparative estimate among them. The legal obligations regarding non-conformance with environmental protection requirements determinably configure financial risk, which refers to Case Scenario A(0). However, financial risk, referring to Case Scenario A(1), is not absolutely configured by the determined investment cost. The main reason for this is that despite investing in establishing a closed industrial unit system to manage tailings, there are potential or expected risks of non-conformance that may be caused by equipment misshaping or inefficiency. Therefore, sub-cases $A(1)_{1\text{-}4}$ are examined through CBA to estimate the additive cost (based on the configuration of penalty costs according to the legal obligations) to the stable annual investment cost due to non-conformance, ranging from 0 to 25%. Moreover, sub-case $A(1)_5$ refers to the estimation of the expected additive cost due to non-conformance, ranging from 0-25%, which is proportional to the total investment cost that characterizes the quality of technical equipment.

The overall analysis provided by CBA showed that the financial risk for all sub-cases of Case A(1) is significantly lower than the corresponding financial risk of Case A(0) over a thirty-year period. Moreover, the point of efficiency for all the sub-cases of Case A(1) against Case A(0) is assessed by the fifth operational year. Comparing Case Scenario A(0) separately with each sub-case of Case Scenario A(1) shows that its LTP indicator increases annually in a consecutive manner. Furthermore, the CBA indicators (B/C, and ERR) are used to decrease and adopt very low numerical values, which verify the inefficiency of Case A(0) compared with Case A(1). Observing the results of CBA, while considering that the actual year of efficiency for case A(1) against case A(0) is indirectly defined by the legal obligations, it is practically assessed that the fifth working year refers to the actual efficiency point of all sub-cases of case A(1) against case A(0). Despite that, the theoretical efficiency point of Case A(1) to Case A(0) is assessed by the third year of work, as mentioned in section 8.2.3.

The financial engineering tool that decodes the grades of financial risk through CBA is SRA. The stochastic Risk Indicator R, calculated by SRA, is crucial for detecting the grade of sustainability risk for each similar project. The Risk Indicator of each case is calculated by the producing function of the separately defined Total Goal Function that describes each examined case multiplied by the Beta Function. In the current study, examining 30 years for both cases, the assessed ratio between grades of risks (R(A(0))/R(A(1))) ranges from 4.6 to 4.8. For the administration of a mining company, non-legal compliance with environmental legislation, as in case A(0), leads to an increase in risk grade of 4.6-4.8. This also corresponds to an increase in the risk of the sustainability grade of 4.6-4.8 times. This means that the R indicator verifies the estimated sustainability grade of the financial risk calculated by CBA, which describes case A(1) against case A(0), including low deviation. So, the increase in financial risk expressed through the Goal Function is due to the behavior of the Risk indicators $R(A(0),\theta_{(0)})$ and $R(A(1)_{2,5},\theta_{(0)})$.

Considering the Stochastic Financial Risk Indicators R(A(0),Y(i)) and $R(A(1)_{2,5},Y(i))$, which include the corresponding degree of risks $R(A(0),\theta_{(0)})$ and $R(A(1)_{2,5},\theta(0))$ and monetary variables, is assessed that the more sustainable business decision refers to the case A(1) against case A(0). Therefore, as an extension of this work, it is proposed to identify how the density of the Risk Indicator affects the escalation of the Financial Risk, as expressed through the Goal Function in each case separately.

From a practical mining engineering perspective, the study shows that case scenario A(1) is both economically and technically sustainable. In contrast, case scenario A(0) carries high risks and is not a sound business decision. Legal conformance guides to lower financial risks while indirectly offering a high benefit rate.

9.2. Contribution to the Advancement of Science and Industrial Applications

From a mining engineering perspective, the current study's practical application focuses on the financial risk assessment, supported by CBA-SRA, to assess the point of efficiency of the proposed case A(1) against case A(0) in actual industrial conditions. The practical application of CBA-SRA in the mining industry focuses on presenting the framework for assessing the degree of economic sustainability for the optimal business decision related to tailings management.

Each start-up mining company should objectively compare the potential scenarios of business growth, considering the total negative costs expressed by customized goal functions through CBA that affect the CBA indicators (LTP, ERR, and CBA index). Moreover, to identify the efficiency rate, the relevant risk indicators (R(A,Y), R(A,θ(0)), CBA Risk Ratios) are calculated using the SRA approach.

All mining companies should fully comply with the operational legal obligations, considering the corresponding investment cost, to eliminate potential environmental pollution risks. So, the role of financial risk engineering supported by CBA-SRA is valuable, mainly when this risk analysis assessment is applied before the on-site operation of the mining activities. The combination of CBA-SRA provides a financial engineering tool applicable to mining companies to assess potential investment plans and for financial institutions to determine their funding policy.

This financial engineering tool supports financial engineering solutions from both points of view: a) internal financial analysts/business consultants of a mining company and b) techno-economic analysts in bank institutions that aim to fund projects of sustainable development respecting the principles of Circular Economy.

Based on the axiom that as the applied science level rises in management, everything is evaluated economically in monetary terms, the presented financial engineering risk analysis tool, structured by CBA-SRA, could be implemented from both the internal corporation management sector of mining companies or funds that support them. Moreover, the innovative approach of SRA, verifying the deterministic CBA results showing extremely low deviation while separating the theoretical approaches from those that refer to actual conditions, should be developed to be practically applied to additional industrial sectors.

The development of SRA, considering its frequency of occurrence and its impact on the corresponding techno-economic assessments in other projects, is a crucial factor that needs further research. As a concept of teaching, especially in applied engineering sectors, project, and risk management by applying the engineering risk analysis tool of CBA-SRA is an indicative subject of science that offers professional education to new scientists (especially engineers and economists) to maximize their managerial skills.

9.3. Limitations of the Study

From the financial engineering perspective, the technical and physical parameters that refer to actual conditions are considered by CBA to provide the financial risk assessment. However, to cross-validate the CBA results, the SRA is applied. SRA calculates the stochastic risk indicators that describe each examined case, applying the methodology of Bayesian analysis. The R indicators describe the detection of risks. The stochastic financial risks describe the risk rates expressed in monetary terms. The stochastic risk grades express the grade that configures the stochastic financial risk rates. The CBA Risk Ratios indices configure the risk matrix correlated between all the examined conditions' total stochastic financial risk assessment.

As shown in section 7, the stochastic risk integral tools are implemented to calculate the stochastic risk indices. This mechanism detects the potential risk numerical values for the generating function of the separately defined goal functions about each examined condition, with the Beta distribution. The integral, including only the variable of $\theta_{(0)}$, detects the stochastic risk grade $R(A,\theta_{(0)})$. The stochastic financial risk is assessed in monetary terms by multiplying this grade with the corresponding negative cost. Based on the SRA, all sub-cases, except for $A(1)_4$, of Case A(1) show lower stochastic financial risk indices and grades of risk than Case A(0). As mentioned in section 8.5, this occurs because sub-case $A(1)_4$ could not refer to actual conditions. Investing annually in maintenance for the used equipment while expecting a misshaping or inefficient metal recovery process is impossible.

Considering the risk matrix results shown in Table A19 expressed by the CBA Risk Ratios between all examined conditions, it is observed that Case Scenario A(0) adopts the highest risk indices than all the other sub-cases of Case A(1), except $A(1)_4$. Sub-Case $A(1)_1$ adopts the minimum risk ratios of all the other sub-cases of Case A(1). Sub-Case $A(1)_2$ adopts the minimum risk ratios than sub-cases $A(1)_{3\text{-}5}$ of Case A(1) and Case A(0) while showing higher risk than sub-case $A(1)_1$. Sub-Case $A(1)_3$ adopts lower risk ratios than sub-cases $A(1)_{4\text{-}5}$ of Case A(1) and Case A(0) while showing higher risk than sub-case $A(1)_2$ and much higher than sub-case $A(1)_1$. Sub-case $A(1)_4$, shows the highest risk indicator of all the other conditions due to its clearly theoretical occurrence. Sub-case $A(1)_5$ adopts higher risk than sub-cases $A(1)_{1\text{-}3}$ and lower risk than sub-case $A(1)_4$ and Case A(0). The risk matrix from SRA is an adaptable tool that detects the sustainability grade for all examined conditions, accounting for their physical meaning in actual conditions.

Considering the CBA Risk Ratio matrix in Table A19 and the deviations between the results of the two approaches CBA-SRA, it is observed that :

- the estimated degree of risk for Case Scenario A(0) equals approximately 23-24.43
- the estimated degree of risk for the sub-case $A(1)_2$ equals approximately 5.12-5.68
- the estimated degree of risk for the sub-case $A(1)_5$ equals approximately 5-5.45
- despite sub-case $A(1)_2$ showing lower risk indicators than sub-case $A(1)_5$, the last one shows a higher CBA Risk ratio than the first. The reason for this focuses on the practical meaning of the axiom that the expected at-risk cost is proportional to the investment cost (much greater than the corresponding negative cost of penalties), which indirectly characterizes the quality and efficiency of the technical equipment.
- The more reliable sub-case of Case Scenario A(1) is sub-case $A(1)_5$ due to its lowest deviation of results among CBA-SRA.
- The second reliable sub-case of Case Scenario A(1) is sub-case $A(1)_2$ due to its lowest deviation of results among CBA-SRA.
- sub-cases $A(1)_1$ and $A(1)_3$ show a high level of deviation in the results by CBA-SRA. This verifies that these two sub-cases do not correspond to actual industrial conditions. These sub-cases refer to widespread environmental pollution risks, while a closed system of industrial units is established to prevent random pollution hazards at the mining site. Hence, they are evaluated clearly from a hypothetical point of view.
- Case Scenario A(1) risk equals approximately 5, while the opposite Case Scenario A(0) risk equals approximately 23.
- Cross-validation of CBA–SRA identifies the most reliable challenges by converting actual conditions into monetary or numerical values.

Therefore, the most probable sub-cases of Case Scenario A(1) are the sub-cases $A(1)_2$ and $A(1)_5$. The reason focuses on the impossibility of widespread detected environmental pollution in random locations when investing in a closed system of industrial units to optimize the tailings management procedures. Moreover, based on this, it is impossible to expect a consecutive rate of additive penalty costs as in sub-case $A(1)_4$.

On the opposite side, it is expected to potentially detect environmental pollution due to the misshaping of the equipment or any leachates in a specific location inside the established industrial units of tailings management as in sub-case $A(1)_2$. Furthermore, despite investing in tailings waste management, it is expected to detect any risks of environmental pollution that guide to non-conformance at a maximum range of 25% by the primary year of the newly established unit. While investing in its maintenance for consecutive years, the risk probability of expected environmental pollution is supposed to be decreased, as in sub-case $A(1)_5$.

Therefore, the alignment of CBA results by SRA, leads to a full compact assessment that detects the grade of sustainability of each examined case or sub-case. The CBA and SRA results for Case A(0) and sub-cases $A(1)_2$ and $A(1)_5$ show very low deviation. Hence, this verifies that Case Scenario A(0) adopts the highest risk grade, approximately equal to 23-24. At the same time, both sub-cases $A(1)_2$ and $A(1)_5$ adopt quite the same risk grade, approximately equal to 5.

9.4. Future Decisions in Mining Waste Management: Adhering to the Circular Economy Principles

The results show that investing in environmental safety procedures in tailings management (case A(1)) is financially more sustainable than paying penalties for non-compliance, as in cases of conventional tailings management (case A(0)). The study also demonstrates that metal recovery can be implemented on an industrial scale, offering a high grade of ERR and a low grade of LTP despite the proposed tailings reprocessing plan being costly and dimensionally overdesign. This results in a positive financial return for each mining company while adhering to Circular Economy Principles.

As entirely shown, following the legal obligations, it is more sustainable to invest in implementing the principles of 4Rs even if there is a financial risk of additional cost due to a potentially low rate of non-full conformance than accepting the penalty costs due to non-investing in waste management, adopting a high rate of non-conformance with environmental protective requirements. It is also observed that non-conformance terminology does not refer entirely to the detected environmental pollution. Non-conformance refers to all the obliged criteria, including the criterion of environmental pollution. The criteria that a mining company must comply with, in addition to hazard elimination of environmental pollution, include the following.

- Minimize their produced waste.
- Do not dispose of them in the ground soil layers. This is because even if they are geological fraction sublayers or geomembranes to the bottom of a tailings disposal site, there is an acidic leachate risk through them that causes acid drainage. At the same time, there is no chance for metal recovery.
- Implement recovery processes to recycle the beneficial materials before their final disposal.

This PhD dissertation proposed a costly and dimensionally overdesign tailings reprocessing project that was financially assessed and compared with the best-case conditions of conventional tailings management processes. The results of the combined assessment through the two approaches demonstrated that, although the innovative methodology of tailings reprocessing is characterized as high-cost, it remains more economically sustainable than the conventional method of tailings disposal for backfilling or stonewalling. This happens because of the incomplete chain of the Circular Economy's principles due to the lack of waste reduction, recovery, and recycling.

In conclusion, the mining industry should adopt the Circular Economy philosophy by calculating the cost of tailings management to recover contained metals when requesting financial loans to develop or primary establish. The physical phenomenon of acid drainage is caused by thickening at the tailings disposal site. To avoid this, each mining company should thicken the produced tailings from mining activities under controlled conditions through a closed system of unit plants, respecting the guidance of sustainable mining. This approach, similar to many other cases, can achieve a high rate of efficient metal recovery. Transitioning to sustainable mining will minimize supply risks in metals by 2050. The consecutive development of sustainable mining processes increases the life cycle of mining activities. Further research and development are needed to adopt these, focusing on applicable methodologies that optimize the tailings reprocessing. Simultaneously, proportional research and development to optimize the engineering risk analysis tool structured by CBA-SRA are needed to evaluate the sustainability grade for those optimized techniques between each other economically to select the most appropriate business decision.

References

1. Gayathri Naidu, Seongchul Ryu, Ramesh Thiruvenkatachari, Youngkwon Choi, Sanghyun Jeong, Saravanamuthu Vigneswaran a. 2019. A critical review on remediation, reuse, and resource recovery from acid mine drainage. Environmental Pollution. Volume 247, pages 1110-1124. DOI: https://doi.org/10.1016/j.envpol.2019.01.085 .

2. Adrianto, L. R. 2023. Environmental Perspectives on Emerging Resource Recovery Systems of Mine Tailings: A Life Cycle Consideration (Doctoral dissertation, ETH Zurich). DOI: https://doi.org/10.3929/ethz-b-000606842 .

3. Adewuyi, S. O., Anani, A., & Luxbacher, K. 2024. Advancing sustainable and circular mining through solid-liquid recovery of mine tailings. Process Safety and Environmental Protection. Volume 189, pages 31-46. DOI: https://doi.org/10.1016/j.psep.2024.06.086 .

4. Evan K. Paleologos, Abdel - Mohsen O.Mohamed. 2021. Risk Analysis and Management. Pollution Assessment for Sustainable Practices in Applied Sciences and Engineering. Pages 43-105. DOI: https://doi.org/10.1016/B978-0-12-809582-9.00002-5 .

5. United Nations Sustainable Development. 1992. United Nations Conference on EnvironmentaL and Development. Rio de Janeiro, Brazil. Agenda 21. Retrieved from https://sdgs.un.org/ .

6. Mineral Resources Law of the People's Republic of China. 1986. Ministry of Ecology and Environment the People's Republic of China. Retrieved from https://english.mee.gov.cn/Resources/laws/envir_elatedlaws/200710/t20071009_109919.shtml .

7. South Australian Legislation. 2020. Mining Regulations Version 11.7.2024. South Australia under the Mining ACT 1971. Retrieved from https://www.legislation.sa.gov.au/lz?path=%2FC%2FR%2FMining%20Regulations%202020 .

8. European Parliament. 2004. Directive 2004/35/CE of the European Parliament and of the Council. Retrieved from https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex%3A32004L0035 .

9. European Parliament. 2006. Directive 2006/21/EC of the European Parliamen and of the Council on the Management of Waste from Extractive Industries and Amending Directive 2004/35/EC. Retrieved from https://eur-lex.europa.eu/eli/dir/2006/21/oj/eng .

10. European Parliament. 2008. Directive 2008/98/EC of the European Parliament and of the Council on Waste and Repealing Certain Directives. Retrieved from https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex%3A32008L0098 .

11. Carmen Falagán, Barry M. Grail, D. Barrie Johnson. 2017. New approaches for extracting and recovering metals from mine tailings. Minerals Engineering. Volume 106, pages 71-78. DOI: https://doi.org/10.1016/j.mineng.2016.10.008 .

12. Adiansyah, J. S., Haque, N., Rosano, M., & Biswas, W. 2017. Application of a life cycle assessment to compare environmental performance in coal mine tailings management. Journal of environmental management. Volume 199, pages 181-191. DOI https://doi.org/10.1016/j.jenvman.2017.05.050 .

13. Assmuth, T., Simola, A., Pitkänen, T., Lyytimäki, J., Huttula, T. 2016. Integrated frameworks for assessing and managing health risks in the context of managed aquifer recharge with river water. Volume 12, Issue 1, pages 160-173. DOI: https://doi.org/10.1002/ieam.1660 .

14. Ahmadreza Golnaraghi Ghomi, Neda Asasian-Kolur, Seyedmehdi Sharifian, Alireza Golnaraghi. 2020. Biosorpion for sustainable recovery of precious metals from wastewater. Journal of Environmental Chemical Engineering. Volume 8, Issue 4. DOI: https://doi.org/10.1016/j.jece.2020.103996 .

15. Araujo, F. S., Taborda-Llano, I., Nunes, E. B., Santos, R. M. 2022.Recycling and Reuse of Mine Tailings: A Review of Advancements and Their Implications. Geosciences. Volume 12. DOI: https://doi.org/10.3390/geosciences12090319 .

16. Carneiro, A & Fourie, AB. 2018. A conceptual cost comparison of alternative tailings disposal strategies in Western Australia. Proceedings of the 21st International Seminar on Paste and Thickened Tailings. Australian Centre for Geomechanics, Perth, pages 439-454. DOI: https://doi.org/10.36487/ACG_rep/1805_36_Carneiro .

17. Kinnunen, P. H. M., & Kaksonen, A. H. 2019. Towards circular economy in mining: Opportunities and bottlenecks for tailings valorization. Journal of Cleaner Production. Volume 228, pages 153-160. DOI: https://doi.org/10.1016/j.jclepro.2019.04.171 .

18. Päivi Kauppila, Marja Liisa Räisänen, Sari Myllyoja. 2011. Best Environmental Practices in Metal Ore Mining. Finnish Environmental Institute. Retrieved from https://www.researchgate.net/publication/284046091_Best_environmental_practices_in_metal_ore_mining .

19. Kiwingim, W., Gormley, A., Douglas, R., & Arnscheidt, J. 2024. Environmental compliance assessment for the desulfurization of sulfide mine waste tailings: A case study of Ok Tedi Mine, Papua New Guinea. Environmental Challenges, Volume 15. DOI: https://doi.org/10.1016/j.envc.2024.100875 .

20. Bona Lim, Richard Diaz Alorro. 2021. Technospheric Mining of Mine Wastes: A Review of Applications and Challenges. Sustainable Chemistry. Volume 2, pages 686-706. DOI: https://doi.org/10.3390/suschem2040038 .

21. Garcia, F. F., Camilo Cotrim, C. F., Caramori, S. S., Bailão, E. F. L. C., Nabout, J. C., de Farias Neves Gitirana Junior, G., & Almeida, L. M. 2024. Mine tailings dams' failures: Serious environmental impacts, remote solutions. Environment Development Sustain. Volume 27, pages 18179-18201. DOI: https://doi.org/10.1007/s10668-024-04628-z .

22. Wencai Zhang, Rick Honaker. 2020. Process development for the recovery of rare earth elements and critical metals from an acid mine leachate. Minerals Engineering. Volume 153. DOI: https://doi.org/10.1016/j.mineng.2020.106382 .

23. Agnieszka Tubis, Sylwia Werbinska - Wojciechowska, Adam Wroblewski. 2020. Risk Assessment Methods in Mining Industry - A Systematic Review. Applied Sciences.Volume 10. DOI: https://doi.org/10.3390/app10155172 .

24. Kinnunen, P., Karhu, M., Yli-Rantala, E., Kivikytö-Reponen, P., Mäkinen, J. 2022. A review of circular economy strategies for mine tailings. Cleaner Engineering and Technology.Volume 8. DOI: https://doi.org/10.1016/j.clet.2022.100499 .

25. European Commission. 2009. Management of Tailings and Waste-Rock in Mining Activities. https://prtr-es.es/Data/images/Residuos-de-miner%C3%ADa.pdf .

26. Antony Jose, S., Calhoun, J., Renteria, O. B., Mercado, P., Nakajima, S., Hope, C. N., Mario Sotelo, Menezes, P. L. 2024. Promoting a Circular Economy in Mining Practices. Sustainability. Volume 16. DOI: https://doi.org/10.3390/su162411016 .

27. Lottermoser, B. 2010. Mine Wastes: Characterization, Treatment, and Environmental Impacts, 3rd ed.; Springer: Berlin/Heidelberg, Germany. Retrieved from https://download.e-bookshelf.de/download/0000/0113/54/L-G-0000011354-0002344546.pdf .

28. Mancini, S., Casale, M., Tazzini, A., & Dino, G. A. Mancini, S., Casale, M., Tazzini, A., & Dino, G. A. 2024. Use and Recovery of Extractive Waste and Tailings for Sustainable Raw Materials Supply. Mining. Volume 4, pages 149-167. DOI: https://doi.org/10.3390/mining4010010 .

29. European Commission. 2001. BRGM: Management of Mining, Quarrying and Ore-Processing Waste in the European Union; BRGM/RP-50319-FR; DG Environment, European Commission: Brussels, Belgium, 2001; p. 88. Retrieved from https://ec.europa.eu/environment/pdf/waste/studies/mining/0204finalreportbrgm.pdf .

30. European Commission: Directorate-General for Internal Market, Industry, Entrepreneurship and SMEs, Grohol, M. and Veeh, C., Study on the critical raw materials for the EU 2023 – Final report, Publications Office of the European Union, 2023. Retrieved from https://op.europa.eu/en/publication-detail/-/publication/57318397-fdd4-11ed-a05c-01aa75ed71a1 .

31. Hanna Tuovinen, Mila Pelkonen , Janne Lempinen , Esa Pohjolainen , David Read, Dina Solatie, Jukka Lehto. 2018. Behaviour of Metals during Bioheap Leaching at the Talvivaara Mine, Finland. Geosciences. Volume 8. DOI: https://doi.org/10.3390/geosciences8020066 .

32. Elkhansa Elbashier, Afnan Mussa, MhdAmmar Hafiz, Alaa H. Hawari. 2021. Recovery of rare earth elements from waste streams using membrane processes: An overview. Hydrometallurgy. Volume 204. DOI: https://doi.org/10.1016/j.hydromet.2021.105706 .

33. Ali Dawood Salman, Tatjána Juzsakova, Saja Mohsen, Thamer Adnan Abdullah, Phuoc-Cuong Le, Viktor Sebestyen, Brindusa Sluser, Igor Cretescu. 2022. Scandium Recovery Methods from Mining, Metallurgical Extractive Industries, and Industrial Wastes. Materials. Volume 15. DOI: https://doi.org/10.3390/ma15072376 .

34. Sandra Pavon, Nils Haneklaus, Kim Meerbach, Martin Bertau. 2022. Iron(III) removal and rare earth element recovery from a synthetic wet phosphoric acid solution using solvent extraction. Minerals Engineering. Volume 182. DOI: https://doi.org/10.1016/j.mineng.2022.107569 .

35. Bobba, S., Carrara, S., Huisman, J., Mathieux, F., Pavel, C. 2020. Critical Raw Materials for Strategic Technologies and Sectors in the EU. A Foresight Study. European Commission. DOI: https://www.researchgate.net/publication/344237416 .

36. ISO 31000:2018. Risk Management-Guidelines. Retrieved from https://www.iso.org/standard/65694.html .

37. Liu, C., Zhang, Q., Wang, H. 2020. Cost-benefit analysis of waste photovoltaic module recycling in China. Waste Management. Volume 118, pages 491-500. DOI: https://doi.org/10.1016/j.wasman.2020.08.052 .

38. Ronaldo S. Gallardo, Kenneth Jae T. Elevado. 2018. Cost-Benefit Analysis of concrete mixed with waste ceramic tiles and fly ash. Conference Paper. Retrieved from https://www.researchgate.net/publication/326439758_Cost-Benefit_Analysis_of_Concrete_Mixed_with_Waste_Ceramic_Tiles_and_Fly_Ash .

39. Shuqin Gao. 2024. An Exogenous Risk in Fiscal-Financial Sustainability: Dynamic Stochastic General Equilibrium Analysis of Climate Physical Risk and Adaptation Cost. Journal of Risk Financial Management. Volume 17. DOI: https://doi.org/10.3390/jrfm17060244 .

40. Silvia Gigli, Daniele Landi, Michele Germani. 2019. Cost-benefit analysis of a circular economy project: a study on a recycling system for end-of-life tyres. Journal of Cleaner Production. Volume 229, pages 680-694. DOI: https://doi.org/10.1016/j.jclepro.2019.03.223 .

41. Adator Stephanie Worlanyo, Li Jiangfeng. 2021. Evaluating the environmental and economic impact of mining for post-mined land restoration and land-use: A review . Journal of Environmental Management. Volume 279. DOI: https://doi.org/10.1016/j.jenvman.2020.111623 .

42. Agustín Hernández-Bastida, M. Pilar Fernández-Sánchez. 2012.A Sarmanov family with beta and gamma marginal distributions: an application to the Bayes premium in a collective risk model. Springer. Volume 21, pages 391-409. DOI: https://doi.org/10.1007/s10260-012-0194-3 .

43. Uchenna K. Kesieme, Hal Aral. 2015. Application of membrane distillation and solvent extraction for water and acid recovery from acidic mining waste and process solutions. Journal of Environmental Chemical Engineering. Volume 3, Issue 3, pages 2050-2056. DOI: https://doi.org/10.1016/j.jece.2015.07.008 .

44. Maria de Lurdes Dinis, António Fiúza, Aurora Futuro, Alexandre Leite, Diogo Martins, Janine Figueiredo, Joaquim Góis, Maria Cristina Vila. 2020. Characterization of a mine legacy site: an approach for environmental management and metals recovery. Environmental Science and Pollution Research. Volume 27, pages 10103-10114. DOI: https://doi.org/10.1007/s11356-019-06987-x .

45. Mónica Rodríguez-Galán, Francisco M. Baena-Moreno, Sara Vázquez, Fátima Arroyo-Torralvo, Luis F. Vilches, Zhien Zhang. 2019. Remediation of acid mine drainage. Environmental Chemistry Letters. Volume 17, pages 1529-1538. DOI: https://doi.org/10.1007/s10311-019-00894-w .

46. Samuel Famiyeh, Ebenezer Adaku, Laud Kissi-Mensah, Charles Teye Amoatey. 2014. Risk management for a tailings re-mining project in Ghana. Emerald Insight. Retrieved from https://www.researchgate.net/publication/27685755247 .

48. Neha Mehta, Giovanna Antonella Dino, Iride Passarella, Piergiorgio Rossetti, Domenico Antonio De Luca. 2020. Assessment of the Possible Reuse of Extractive Waste Coming from Abandoned Mine Sites: Case Study in Gorno, Italy. Sustainability. Volume 12. DOI: https://doi.org/10.3390/su12062471 .

49. Verónica García, Piia Häyrynen, Junkal Landaburu-Aguirre, Minna Pirilä, Riitta L. Keiski and Ane Urtiaga. 2013. Purification techniques for the recovery of valuable compounds from acid mine drainage and cyanide tailings: application of green engineering principles. Journal of Chemical Technology and Biotechnology. DOI: https://doi.org/10.1002/jctb.4328 .

50. Sartori, D., Catalano, G., Genco, M., Pancotti, C., Sirtori, E., Vignetti, S., & Bo, C. 2014 . Guide to cost-benefit analysis of investment projects. Economic appraisal tool for cohesion policy 2014-2020. Retrieved from https://ec.europa.eu/regional_policy/en/information/publications/guides/2014/guide-to-cost-benefit-analysis-of-investment-projects-for-cohesion-policy-2014-2020 .

51. David (Dave) G. Osborne, J. C. Fisher II, M. Barish, T. A. Toney. 2024. Increased Revenue and Cost Savings by Recovering Value from Plant Recirculation and Tailings Waste Streams. Journal of Energy for a Clean Environment. DOI: https://doi.org/10.1615/InterJEnerCleanEnv.2024051371 .

52. Pirbalouti, S. M., & Askari-Nasab, H. 2024 Stochastic Optimization and Deterministic Models in Strategic Mine Planning Problems: An Overview of Solution Methods and Comparisons. Technical Report. DOI: 10.13140/RG.2.2.23336.48647 .

53. P. Ortega-Jiménez, M.A. Sordo, A. Suárez-Llorens. 2021. Stochastic orders and multivariate measures of risk contagion. Insurance: Mathematics and Economics. Volume 96, pages 199-207. DOI: https://doi.org/10.1016/j.insmatheco.2020.11.008 .

54. Evan K. Paleologos, Abdel - Mohsen O.Mohamed, Devendra N. Singh, Brendan C. O'Kelly, Maisa El Gamal, Ameera Mohammad, Prithvendra Singh, Venkata S. N. S. Goli, Antonio J. Roque, Joy A. Oke, Hossam Aboel - Naga, Eng - Choon Leong. 2024. Sustainability Challenges of Clean - Energy Critical Minerals : Copper and Rare Earths. Environmental Geotechnics. Volume 12, pages 88-100. DOI: https://doi.org/10.1680/jenge.23.00062 .

55. M. Shrahili, I. Elbatal, Isyaku Muhammad, Mustapha Muhammad. 2021. Properties and applications of Beta Erlang-truncated exponential distribution. Journal of Mathematics and Computer Science. DOI: http://dx.doi.org/10.22436/jmcs.022.01.03 .

56. G.W. Peters and S.A. Sisson, 2006. Bayesian Inference, Monte Carlo Sampling and Operational Risk. The Journal of Operational Risk. Volume 1, pages 27-50. DOI: 10.21314/JOP.2006.014. DOI: https://doi.org/10.21314/JOP.2006.014 .

57. Kelly, D., & Smith, C. 2011. Bayesian inference for probabilistic risk assessment: A practitioner's guidebook. Springer Science & Business Media. DOI: 10.1007/978-1-84996-187-5 .

58. Fenton, N., & Neil, M. 2012. Risk assessment and decision analysis with Bayesian networks. (1st ed.). CRC Press. 1st Edition. DOI: https://doi.org/10.1201/9780367803018 .

59. Kabir, S. & Papadopoulos, Y. 2019. Applications of Bayesian networks and Petri nets in safety, reliability, and risk assessments: A review. Safety science. Journal of Computer and Communications. Volume 115, pages 154-175. DOI: https://doi.org/10.1016/j.ssci.2019.02.009 .

60. Lin, E. M., Sun, E. W., & Yu, M. T. 2020. Behavioral data-driven analysis with Bayesian method for risk management of financial services. International Journal of Production Economics. Volume 228. DOI: https://doi.org/10.1016/j.ijpe.2020.107737 .

61. Glaesser., Ian Lerche and Walter. 2006. Environmental Risk Assessment. Quantitative Measures, Anthropogenic Influeces, Human Impact. Springer. 1st Edition pages 82-85. Retreived from https://link.springer.com/book/10.1007/3-540-29709-X .

62. Natalia Araya, , Andrzej Kraslawski, , Luis A. Cisternas. 2020. Towards mine tailings valorization: Recovery of critical materials from Chilean mine tailings. Journal of Cleaner Production. Volume 263. DOI: https://doi.org/10.1016/j.jclepro.2020.121555 .

63. Machairas, E., Varouchakis, E.,A. 2023. Cost–Benefit Analysis and Risk Assessment for Mining Activities in Terms of Circular Economy and Their Environmental Impact. Geosciences. School of Mineral Resources Engineering, Technical University of Crete. DOI: https://doi.org/10.3390/geosciences13100318 .

64. Greek Government Gazette. 2022. Volume B of the Greek Parliament. Regulations for Compliance with the Environmental Legal Obligations during the Audit Controls by the National Environmental Protection Authorities. Retrieved from https://www.elinyae.gr/eth .

65. Yang, J., Yan,Y., Yang, J., Wang, L., Wan, X., Zheng, H., Liao, X., Lei, M., Chen, T. 2023. Dynamics of multiple stakeholders' benefits due to mining area environmental remediation based on risk reduction and ecosystem services. Journal of Cleaner Production. Volume 419. DOI: https://doi.org/10.1016/j.jclepro.2023.138338 .

66. Yeo, J., Chopra, S. S., von Eiff, D., Jeong, S., Zhang, L., & An, A. K. 2022. An integrated techno-economic analysis on wastewater reclamation in Hong Kong:A comprehensive cost-Benefit analysis with life cycle assessment. Journal of Cleaner Production. Volume 357. DOI: https://doi.org/10.1016/j.jclepro.2022.131838 .

67. Yang, W., D., Sun, Q., Ni, H., G. 2021. Cost-benefit analysis of metal recovery from e-waste:Implications for international policy. Waste Management. Volume 123, pages 42-47. DOI: https://doi.org/10.1016/j.wasman.2021.01.023 .

68. Varouchakis, E. A., Yetilmezsoy, K., & Karatzas, G. P. 2019. A decision-making framework for sustainable management of groundwater resources under uncertainty: combination of Bayesian risk approach and statistical tools. Water Policy. Volume 21, pages 602-622. DOI: https://doi.org/10.2166/wp.2019.128 .

69. Varouchakis, E. A., Palogos, I., & Karatzas, G. P. 2016. Application of Bayesian and cost benefit risk analysis in water resources management. Journal of Hydrology. Volume 534, pages 390-396. DOI: https://doi.org/10.1016/j.jhydrol.2016.01.007 .

70. Varouchakis, E., & Palogos, I. 2017. Application of risk analysis in water resourses management. Journal of Hydrology. DOI: https://doi.org/10.1016/j.jhydrol.2016.01.007 .

71. Luis Seguı, Oscar Alfranca, Joan Garcı´a. 2009. Techno-economical evaluation of water reuse for wetland restoration: a case study in a natural park in Catalonia, Northeastern Spain. Desalination. Volume 246, pages 179-189. DOI: https://doi.org/10.1016/j.desal.2008.03.051 .

72. Rubí Medina-Mijangos, Andrés De Andrés, Hilda Guerrero-Garcia-Rojas, Luis Seguí-Amórtegui. 2020. A methodology for the technical-economic analysis of municipal solid waste systems based on social cost-benefit analysis with a valuation of externalities. Environmental Science and Pollution. Springer Nature. Volume 28, pages 18807-18825. DOI: https://doi.org/10.1007/s11356-020-09606-2 .

73. Ming-Lang Tseng, Shiou-Yun Jeng and Chun-Wei Lin, Ming K. Lim. 2021. Recycled construction and demolition waste material: a cost– benefit analysis under uncertainty. Management of Environmental Quality: An International Journal. DOI: https://doi.org/10.1108/MEQ-08-2020-0175 .

74. Thomas R. Robbins, D. J. Medeiros, Terry P. Harrison. 2010. Evaluating the Erlang C and Erlang A Models for Call Center Modeling . Working Paper. East Carolina University, Greenville, NC. Retrieved from https://api.semanticscholar.org/CorpusID:21964417 .

75. Andrea Di Maria, Mikel Merchán, Muriel Marchand, David Eguizabal, Maider García De Cortázar, Karel Van Acker. 2022. Evaluating energy and resource efficiency for recovery of metallurgical residues using environmental and economic analysis. Journal of Cleaner Production. Volume 356. DOI: https://doi.org/10.1016/j.jclepro.2022.131790 .

76. Chinenye Chinwego, Hunter Wagner, Emily Giancola, Jonathan Jironvil, Adam Powell. 2022. Technoeconomic Analysis of Rare-Earth Metal Recycling Using Efficient Metal Distillation. Springer Nature. Volume 74, pages 1296–1305. DOI: https://doi.org/10.1007/s11837-021-05045-7 .

77. Hudson-Edwards, K. A., Kemp, D., Torres-Cruz, L. A., Macklin, M. G., Brewer, P. A., Owen, J. R., Thomas, C. J. 2024. Tailings storage facilities, failures and disaster risk. Nature Reviews Earth & Environment. Volume 5, pages 612-630. DOI: https://doi.org/10.1038/s43017-024-00576-4 .

78. Li, W., Deng, Y., Wang, H., Hu, Y., & Cheng, H. 2024. Potential risk, leaching behavior and mechanism of heavy metals from mine tailings under acid rain. Chemosphere. Volume 350. DOI: https://doi.org/10.1016/j.chemosphere.2023.140995 .

79. Salgado-Almeida, B., Briones-Escalante, A., Falquez-Torres, D., Filián-Haz, K., Guzmán-Martínez, F., Escobar-Segovia, K., Elizabeth Pena-Caprio & Jiménez-Oyola, S. 2024. Assessment of Environmental Pollution and Risks Associated with Tailing Dams in a Historical Gold Mining Area of Ecuador. Resources. Volume 13, page 105. DOI: https://doi.org/10.3390/resources13080105 .

80. Rowe, R. K., Joshi, P., Brachman, R. W. I., & McLeod, H. 2017. Leakage through holes in geomembranes below saturated tailings. Journal of Geotechnical and Geoenvironmental Engineering. Volume 143. DOI: https://doi.org/10.1061/(ASCE)GT.1943-5606.0001606 .

81. Rowe, R. K., & Fan, J. 2024. The Application of Geosynthetics in Tailings Storage Facilities: A General Review. Mining. Volume 4, pages 447-468. DOI: https://doi.org/10.3390/mining4020026 .

82. Cristian Tunsu, Yannick Menard, Dag Øistein Eriksen, Christian Ekberg. 2019. Recovery of critical materials from mine tailings: A comparative study of the solvent extraction of rare earths using acidic, solvating and mixed extractant systems. Journal of Cleaner Production. Volume 218, pages 425-437. DOI: https://doi.org/10.1016/j.jclepro.2019.01.312 .

83. Hamraoui, L.,Bergani, A., Ettoumi, M., Aboulaich, A., Taha, Y., Khalil, A., Neculita C.,M., Benzaazoua, M. 2024. Towards a Circular Economy in the Mining Industry: Possible Solutions for Water Recovery through Advanced Mineral Tailings Dewatering. Minerals. Volume 14, page 319. DOI: https://doi.org/10.3390/min14030319 .

84. Ettlinger, Verena. 2020. Cost-benefit analysis of historical copper slags and tailings : a case study from Boliden Rönnskär Smelter in Skelleftehamn. Second cycle, A2E. Uppsala: SLU, Dept. of Economics. Case Study. DOI: https://stud.epsilon.slu.se/15870/ .

85. Franco Muleya, Bodwin Mulenga, Sambo Lyson Zulu, Sunday Nwaubani, Chipozya Kosta Tembo, Henry Mushota. 2020. Investigating the suitability and cost-benefit of copper tailings as partial replacement of sand in concrete in Zambia: an exploratory study. Journal of Engineering, Design and Technology. DOI: https://doi.org/10.1108/JEDT-05-2020-0186 .

86. Wang, E., T., Chiang, S., Haneef, Z., Rao, V. R., Moss, R., & Vannucci, M. 2022. Bayesian Non-Homogeneous Hidden Markov Model with Variable Selection for Investigating Drivers of Seizure Risk Cycling. Applied Statistics. Volume 17, pages 333-356. DOI: https://doi.org/10.1214/22-AOAS1630 .

87. Veretennikov, A.,Y. 2010. On the Rate of Beta-Mixing and Convergence to a Stationary Distribution in Continuous-Time Erlang-Type Systems. Kharkevich Institute for Infromation Transmission Problems. Russian Academy of Sciences, Moscow School of Mathematics, University of Leeds, UK. Volume 46, pages 382-389. DOI: 10.1134/S0032946010040083 .

88. Awuah-Offei, K., & Adekpedjou, A. 2011. Application of life cycle assessment in the mining industry. The International Journal of Life Cycle Assessment. Volume 16, pages 82-89. DOI:10.1007/s11367-010-0246-6 .

89. Santero, N., & Hendry, J. 2016. Harmonization of LCA methodologies for the metal and mining industry. The international journal of life cycle assessment, Volume 21, pages 1543-1553. Springer Nature. DOI: https://doi.org/10.1007/s11367-015-1022-4 .

90. Boesch, M. E., Vadenbo, C., Saner, D., Huter, C., & Hellweg, S. 2014. An LCA model for waste incineration enhanced with new technologies for metal recovery and application to the case of Switzerland. Waste management, Volume 34, Issue 2, pages 378-389. DOI: https://doi.org/10.1016/j.wasman.2013.10.019 .

91. Cairncross, K., H. (2020). Life cycle assessment of process strategies for value recovery and environmental mitigation of mining waste (Doctoral dissertation, Stellebosch: Stellenbosch University).

92. Kouloumpis, V., & Yan, X. 2022. Life cycle assessment of a novel metal recovery method from co-processing of coal mine waste and low-grade printed circuit boards. Journal of Environmental Management, Volume 314. DOI: https://doi.org/10.1016/j.jenvman.2022.115074 .

93. Komnitsas, K. 2020. Social license to operate in mining: Present views and future trends. Resources, Volume 9, page79. DOI: https://doi.org/10.3390/resources9060079 .

94. Mariga, V. Sustainability analysis in the mining sector: a case study on new recycling technologies for sulphidic mine residues valorisation. Case Study. Retrieved from http://amslaurea.unibo.it/23270 .

95. Imig, A., Szabó, Z., Halytsia, O., Vrachioli, M., Kleinert, V., & Rein, A. 2021. Critical Review. A review on Risk Assessment in managed Aquifer Recharge. Integrated Environmental Assessment and Management Volume 18, pages 1513-1529. DOI: https://doi.org/10.1002/ieam.4584 .

96. Lúcia Helena Xavier, Ellen Cristine Giese, Ana Cristina, Ribeiro-Duthie, Fernando Antonio Freitas Lins. 2019. Sustainability and the circular economy: A theoretical approach focused on e-waste urban mining. Resources Policy. Volume 74. DOI: https://doi.org/10.1016/j.resourpol.2019.101467 .

97. S. Singh, L.B. Sukla, S.K. Goyal. 2020. Mine waste & circular economy. Materials Today: Proceedings.

98. Xiaomeng Geng, Nan Song, Youcai Zhao, Tao Zhou. 2022. Waste plastic resource recovery from landfilled refuse: A novel waterless cleaning method and its cost-benefit analysis . Journal of Environmental Management. Volume 306. DOI: https://doi.org/10.1016/j.jenvman.2022.114462 .

99. Wilgince Apollon, Iryna Rusyn, Nancy González-Gamboa, Tatiana Kuleshova, Alejandro Isabel Luna-Maldonado, Juan Antonio Vidales-Contreras, Sathish-Kumar Kamaraj. 2022. Improvement of zero waste sustainable recovery using microbial energy generation systems: A comprehensive review. Science of the Total Environment.Volume 817. DOI: https://doi.org/10.1016/j.scitotenv.2022.153055 .

100. Greek Government Gazette. 2009. Volume B of the Greek Parliament. Measures, Conditions and Restrictions for Waste Management of the Mining Industry, in Compliance with the Provisions of Directive 2006/21/EC regarding Waste Management. Retrieved from https://elinyae.gr/ethniki-nomothesia/ya-396242209e1032009-fek-2076b-2592009 .

101. Greek Government Gazette. 2011. Volume A of the Greek Parliament. Environmental Authorization and Classification of Projects and Industrial Activities. Retrieved from https://www.elinyae.gr/ethniki-nomothesia/n-40142011-fek-209a-2192011-0 .

102. Mónica Rodríguez-Galán, Francisco M. Baena-Moreno, Sara Vázquez, Fátima Arroyo-Torralvo, Luis F. Vilches, Zhien Zhang. 2019. Remediation of acid mine drainage. Environmental Chemistry Letters (2019). Volume 17, pages 1529–1538. DOI: https://doi.org/10.1007/s10311-019-00894-w .

103. Darío Bolanos Guerron, Jacqueline Capa, Luis Cumbal Flores. 2021. Retention of heavy metals from mine tailings using Technosols prepared with native soils and nanoparticles. Heliyon. Volume 7, Issue 7. DOI: https://doi.org/10.1016/j.heliyon.2021.e07631 .

104. Sre´cko Bevandic, Rosie Blannin, Jacqueline Vander Auwera, Nicolas Delmelle, David Caterina, Frederic Nguyen, Philippe Muchez. 2020. Geochemical and Mineralogical Characterisation of Historic Zn–Pb Mine Waste, Plombières, East Belgium. Minerals. DOI: https://doi.org/10.3390/min11010028 .

105. Hailong Tang, Xiaojing Wang, Weitao Shuai, Yangsheng Liu. 2016. Immobilization of rare earth elements of the mine tailings using phosphates and lime. Procedia Environmental Sciences. Volume 31, pages 255-263. DOI: https://doi.org/10.1016/j.proenv.2016.02.034 .

106. Bellenfant, G, Guezennec, AG, Bodenan, F, D'Hugues, P & Cassard, D. 2013. Reprocessing of mining waste: combining environmental management and metal recovery?, in M Tibbett, AB Fourie & C Digby (eds), Mine Closure 2013: Proceedings of the Eighth International Seminar on Mine Closure, Australian Centre for Geomechanics, Cornwall, pages 571-582. DOI: https://doi.org/10.36487/ACG_rep/1352_48_Bellenfant .

107. Adrien Rimélé, M., Roussos Dimitrakopoulos, Michel Gamache. 2018. A stochastic optimization method with in-pit waste and tailings disposal for open pit life-of-mine production planning. Resources Policy. Volume 57, pages 112-121. DOI: https://doi.org/10.1016/j.resourpol.2018.02.006 .

108. Karen Y. Macías-Macías, Agueda E. Ceniceros-Gómez, Margarita E. Gutiérrez-Ruiz, José Luz González-Chávez, L. Gerardo Martínez-Jardines. 2019. Extraction and recovery of the strategic element gallium from an iron mine tailing. Journal of Environmental Chemical Engineering. Volume 7, Issue 2. DOI: https://doi.org/10.1016/j.jece.2019.102964 .

109. Blengini G.A, Mathieux F., Mancini L., Nyberg M., Viegas H.M. 2019. Recovery of critical and other raw materials from mining waste and landfills. European Commision. JRC Science for Policy Report. State of play on existing practices. DOI: https://dx.doi.org/10.2760/600775 .

110. Yıldız, Tas¸kın Deniz. 2020. Waste management costs (WMC) of mining companies in Turkey: Can waste recovery help meeting these costs? Resources Policy. Volume 68. DOI: https://doi.org/10.1016/j.resourpol.2020.101706 .

111. Juanjuan Hao, Yishu Wang, Yufeng Wu, Fu Guo. 2020. Metal recovery from waste printed circuit boards: A review for current status and perspectives. Resources, Conservation & Recycling. Volume 157. DOI: https://doi.org/10.1016/j.resconrec.2020.104787 .

112. I.M.S.K. Ilankoon, Yousef Ghorbani, Meng Nan Chong, Gamini Herath, Thandazile Moyo, Jochen Petersen. 2018. E-waste in the international context – A review of trade flows, regulations, hazards, waste management strategies and technologies for value recovery. Waste Management. Volume 82, pages 258-275. DOI: https://doi.org/10.1016/j.wasman.2018.10.018 .

113. Sudabe Shahba, Reza Arjmandi, Masoud Monavari, Jamal Ghodusi. 2017. Application of multi-attribute decision-making methods in SWOT analysis of mine waste management (case study: Sirjan's Golgohar iron mine, Iran). Resources Policy. Volume 51, pages 67-76. DOI: https://doi.org/10.1016/j.resourpol.2016.11.002 .

114. Gorazd Žibret, Bruno Lemiere, Ana-Maria Mendez, Carlo Cormio, ,Danielle Sinnett, Peter Cleall, Katalin Szabó, M. Teresa Carvalho. 2020. National Mineral Waste Databases as an Information Source for Assessing Material Recovery Potential from Mine Waste, Tailings and Metallurgical Waste. Minerals. DOI: https://doi.org/10.3390/min10050446 .

115. Mengmeng Wang, Quanyin Tan, Joseph F. Chiang, Jinhui Li. 2017. Recovery of rare and precious metals from urban mines — A review. Front. Environ. Sci. Eng. 11, 1. DOI: https://doi.org/10.1007/s11783-017-0963-1 .

116. Balaram, V. 2019. Rare earth elements: A review of applications, occurrence, exploration, analysis, recycling, and environmental impact. Geoscience Frontiers. Volume 10, Issue 4, pages 1285-1303. DOI: https://doi.org/10.1016/j.gsf.2018.12.005 .

117. Evangelos Tzamos, Argyrios Papadopoulos, Giovanni Grieco, Stylianos Stoulos, Micol Bussolesi, Emmanouil Daftsis, Eleftheria Vgli, Dimitrios Dimitriadis, Athanasio Godelitsas. 2019. Investigation of Trace and Critical Elements (Including Actinides) in Flotation Sulphide Concentrates of Kassandra Mines (Chalkidiki, Greece). Geosciences. Volume 9. DOI: https://doi.org/10.3390/geosciences9040164 .

118. Ramírez Angel, Y. 2022. Water-Energy Nexus for Waste Minimisation in the Mining Industry.Acta Universitatis Lappeenrantaensis 1031. Retrieved from https://urn.fi/URN:ISBN:978-952-335-835-5 .

119. Gluzberg, V., E., Katz, Y., A. 2023. Impact of transient shocks to productivity on long-term social discounting. Journal of Climate Finance, Volume 5, 100027. DOI: https://doi.org/10.1016/j.jclimf.2023.100027 .

120. Nadeif, A., Taha, Y., Bouzahzah, H., Hakkou, R., Benzaazoua, M. 2019. Desulfurization of the old tailings at the Au-Ag-Cu tiouit mine (Anti-Atlas Morocco). Minerals, Volume 9. DOI: https://doi.org/10.3390/min9070401 .

121. Erika Machacek, Jessika Luth Richter, Ruth Lane. 2017. Governance and Risk–Value Constructions in Closing Loops of Rare Earth Elements in Global Value Chains. Resources. Volume 6. DOI: https://doi.org/10.3390/resources6040059 .

122. Amateis, Martin Silberberg and Patricia. 2017. McGraw Hill. Chemistry: The Molecular Nature of Matter and Change. Retrieved from https://chemistry.com.pk/books/chemistry-the-molecular-nature-of-matter-and-change-10ise/ .

123. Ayse Yuksekdag, Borte Kose-Mutlu, Beril Kaya, Mustafa Kumral, Mark R. Wiesner, Ismail Koyuncu a. 2021. Comprehensive characterization of secondary sources originating from Turkey in terms of rare earth elements and scandium. Science of the Total Environment. Volume 777. DOI: https://doi.org/10.1016/j.scitotenv.2021.146033 .

124. Remya P. Narayanan, Shajesh Palantavida, Marion H. Emmert, Nikolaos K. Kazantzis. 2021. A regiocentric economic sensitivity analysis for scandium recovery from red mud. Materialstoday:PROCEEDINGS. Volume 41, pages 577-582. DOI: https://doi.org/10.1016/j.matpr.2020.05.252 .

125. Mehdi Khorasanipour, Sara Rashidi. 2020. Geochemical fractionation pattern and environmental behaviour of rare earth elements (REEs) in mine wastes and mining contaminated sediments;Sarcheshmeh mine, SE of Iran. Journal of Geochemical Exploration. Volume 210. DOI: https://doi.org/10.1016/j.gexplo.2019.106450 .

126. George Blankson Abaka-Wood, Kathy Ehrig, Jonas Addai-Mensah, William Skinner. 2022. Recovery of Rare Earth Elements Minerals from Iron-Oxide-Silicate-Rich Tailings: Research Review. Minerals Engineering. Volume 136, pages 50-61. DOI: https://doi.org/10.1016/j.mineng.2019.02.026 .

127. Oscar A. Marín, Andrzej Kraslawski, Luis A. Cisternas. 2022. Estimating processing cost for the recovery of valuable elements from mine tailings using dimensional analysis . Minerals Engineering. Volume 184. DOI: https://doi.org/10.1016/j.mineng.2022.107629 .

128. Vasileios Stratiotou Efstratiadis and Nikolaos Michailidis. 2022. Sustainable Recovery, Recycle of Critical Metals and Rare Earth Elements from Waste Electric and Electronic Equipment (Circuits, Solar, Wind) and Their Reusability in Additive Manufacturing Applications: A Review. Metals. Volume 12. DOI: https://doi.org/10.3390/met12050794 .

129. Laurel A. Schaider, David B. Senn, Emily R. Estes, Daniel J. Brabander, James P. Shine. 2014. Sources and fates of heavy metals in a mining-impacted stream:Temporal variability and the role of iron oxides. Science of the Total Environment. Volume 490, pages 456-466. DOI: https://doi.org/10.1016/j.scitotenv.2014.04.126 .

130. S. Foucher, F. Battaglia-Brunet, I. Ignatiadis, D. Morin. 2001. Treatment by sulfate-reducing bacteria of Chessy acid-mine drainage and metals recovery. Chemical Engineering Science. Volume 56, Issue 4, pages 1639-1645. DOI: https://doi.org/10.1016/S0009-2509(00)00392-4 .

131. Leandro Augusto Gouvêa de Godoi, Eugenio Foresti, Márcia Helena Rissato Zamariolli Damianovic. 2017. Down-flow fixed-structured bed reactor: An innovative reactor configuration applied to acid mine drainage treatment and metal recovery. Journal of Environmental Management. Volume 197, pages 597-604. DOI: https://doi.org/10.1016/j.jenvman.2017.04.027 .

132. M. Hermassi, M. Granados , C. Valderrama , C. Ayora , J.L. Cortina. 2022. Recovery of rare earth elements from acidic mine waters: An unknown secondary resource. Science of the Total Environment. Volume 810. DOI: https://doi.org/10.1016/j.scitotenv.2021.152258 .

133. Herib Blanco, Wouter Nijs, Johannes Ruf, André Faaij. 2018. Potential of Power-to-Methane in the EU energy transition to a low carbon system using cost optimization. Applied Energy. Volume 232, pages 323-340. DOI: https://doi.org/10.1016/j.apenergy.2018.08.027 .

134. Yue Hu, Luobin Zhang, Hui Lv, Shijian Lu, Gang Xu, Cheng Xu. 2021. A low energy consumption de-carbonization natural gas combined cycle power generation system based on LiBr/H2O absorption heat transformer . International Journal of Greenhouse Gas Control. Volume 109. DOI: https://doi.org/10.1016/j.ijggc.2021.103389 .

135. Shuronjit Kumar Sarker, Nawshad Haque, Muhammed Bhuiyan, Warren Bruckard. 2022. Recovery of strategically important critical minerals from mine tailings . Journal of Environmental Chemical Engineering. Volume 10, Issue 3. DOI: https://doi.org/10.1016/j.jece.2022.107622 .

136. Rudolf Suppes, Soraya Heuss-Aßbichler. 2021. Resource potential of mine wastes: A conventional and sustainable perspective on a case study tailings mining project. Journal of Cleaner Production. Volume 297. DOI: https://doi.org/10.1016/j.jclepro.2021.126446 .

137. HellasGold Reviews & Reports. 2024. Retrieved from https://www.hellas-gold.com/en/sustainability/corporate-social-responsibility/reviews-reports .

138. Tulay Yilmaz, Amine Yucel, Yakup Cakmak, Sinan Uyanik, Adem Yurtsever, Deniz Ucar. 2019. Treatment of acidic mine drainage in up-flow sulfidogenic reactor: Metal recovery and the pH neutralization. Journal of Water Process Engineering. Volume 32. DOI: https://doi.org/10.1016/j.jwpe.2019.100916 .

139. The Buisiness Research Company. Retrieved from https://www.thebusinessresearchcompany.com/report/mining-chemicals-global-market-report .

140. ManisChemicals Retrieved from https://manischemicals.com/en/raw-material-chemically/1081-nitric-acid-of-industrial-origin.htm l.

141. ReAgent. Chemical Suppliers. Retrieved from https://www.chemicals.co.uk .

142. Eric L. Crump, P.E. 2000. U.S. Environmental Protection Agency Office of Air Quality Planning and Standards Innovative Strategies and Economics Group MD-15, Research Triangle Park, NC 27711 . Retrieved from https://www.epa.gov/ .

143. Shanmuganathan, R., Brindhadevi, K. 2019. Comparative analysis of various types of multipliers for effective low power. Microelectronic Engineering. Volume 214, pages 28-37. DOI: https://doi.org/10.1016/j.mee.2019.04.015 .

144. Xochitl Dominguez-Benetton, Jeet Chandrakant Varia, Guillermo Pozo, Oskar Modin, Annemiek Ter Heijne, Jan Fransaer, Korneel Rabaey. 2018. Metal recovery by microbial electro-metallurgy. Progress in Materials Science. Volume 94, pages 435-461. DOI: https://doi.org/10.1016/j.pmatsci.2018.01.007 .

145. Jinyan Yang, Ya Tang, Kai Yang, Ashaki A. Rouff, Evert J. Elzinga, Jen-How Huang. 2014. Leaching characteristics of vanadium in mine tailings and soils near a vanadium titanomagnetite mining site. Journal of Hazardous Materials. Volume 264, pages 498-504. DOI: http://dx.doi.org/10.1016/j.jhazmat.2013.09.063 .

146. J.G. Fantidis, D.V. Bandekas, C. Potolias, N. Vordos. 2013. Cost of PV electricity – Case study of Greece. Solar Energy. Volume 91, pages 120-130. DOI: https://doi.org/10.1016/j.solener.2013.02.001.

147. Daan Hulshof, Jan-Pieter van der Maat, Machiel Mulder. 2016. Market fundamentals, competition and natural-gas prices. Energy Policy. Volume 94, pages 480-491. DOI: https://doi.org/10.1016/j.enpol.2015.12.016.

148. Independent Power Transmission Operator, 2022. Retrieved from https://www.admie.gr/en/market/reports/weighted-average-market-price .

149. Waste Energy and Water Regulatory Authority, 2022. Retrieved from https://www.rae.gr/psysikoaerio/agora/mesostathmiki-timi-eisagogis/ .

150. Kamyar Raoufi, Karl R. Haapala, Tom Etheridge, Sriram Manoharan, Brian K. Paul. 2022. Cost and environmental impact assessment of stainless steel microscale chemical reactor components using conventional and additive manufacturing processes . Journal of Manufacturing Systems .Volume 62, pages 202-217. Retrieved from https://doi.org/10.1016/j.jmsy.2021.11.017.

151. D. Humbird, R. Davis, J.D.McMillan. 2017. Aeration costs in stirred-tank and bubble column bioreactors. Biochemical Engineering Journal. Volume 127, pages 161-166. DOI: https://doi.org/10.1016/j.bej.2017.08.006 .

152. Sidi Deng, Denis Prodius, Ikenna C. Nlebedim, Aihua Huang, Yuehwern Yih, John W. Sutherland. 2021. A dynamic price model based on supply and demand with application to techno-economic assessments of rare earth element recovery technologies. Sustainable Production and Consumption Volume 27, pages 1718-1727. DOI: https://doi.org/10.1016/j.spc.2021.04.013 .

153. Favot, M., Massarutto, A. 2019. Rare-earth elements in the circular economy: The case of yttrium. Journal of Environmental Management. Volume 240, pages 504-510. DOI: https://doi.org/10.1016/j.jenvman.2019.04.002 .

154. Seiler, Volker. 2021.China-to-FOB price transmission in the rare earth elements market and the end of Chinese export restrictions. Energy Economics. Volume 102. DOI: https://doi.org/10.1016/j.eneco.2021.105485 .

155. Zhou, C., Gong, Z., Hu, J., Cao, A., & Liang, H. 2015. A cost-benefit analysis of landfill mining and material recycling in China. Waste Management. Volume 35, pages 191-198. DOI: https://doi.org/10.1016/j.wasman.2014.09.029 .

156. Cox, B., Innis, S., Steen, J., & Kunz, N. 2023. The environmental and economic case for valuing water recovery and its relationship with tailings storage conservation. Minerals Engineering. Volume 201. DOI: https://doi.org/10.1016/j.mineng.2023.108157 .

157. Gordon E. Willmot, X. Sheldon Lin. 2011. Risk modelling with the mixed Erlang distribution. Applied Stochastic Models in Business and Industry. DOI: https://doi.org/10.1002/asmb.838 .

158. Alducci, B., Hynes, S. 2023. Protecting coastal infrastructure in the face of increasing climate related pressures: A cost-benefit analysis of maintaining a tidal kayaking feature in Ireland. Marine Policy. Volume 150. DOI: https://doi.org/10.1016/j.marpol.2023.105550 .

159. Kazmi, S., M., A., Aslam, M. 2022. Bayesian Shrinkage Estimation for the Erlang Distribution using Type-II Progressive Censored Data. Research Square. DOI: https://doi.org/10.21203/rs.3.rs-1970259/v1 .

160. Li, M., Ye, J. 2022. Design and Implementation of Demand Side Response Based on Binomial Distribution. Energies.DOI: https://doi.org/10.3390/en15228431 .

161. Schippers, A., Glombitza, F., & Sand, W. 2014. Geobiotechnology (pp. 1-47). Berlin Heidelberg: Springer. Advances in Biochemical Engineering/biotechnology. Volume 141. DOI: https://doi.org/10.1007/978-3-642-54710-2 .

162. Jean-Luc Mukaba, Chuks Paul Eze, Omoniyi Pereao, Leslie Felicia Petrik. 2021. Rare Earths' Recovery from Phosphogypsum: An Overview on Direct and Indirect Leaching Techniques. Minerals. Volume 11. DOI: https://doi.org/10.3390/min11101051 .

163. Shanmuganathan, R., Brindhadevi, K. 2019. Comparative analysis of various types of multipliers for effective low power. Microelectronic Engineering. Volume 214, pages 28-37. DOI: https://doi.org/10.1016/j.mee.2019.04.015 .

164. Evangelos Machairas, Emmanouil A. Varouchakis, Manos Sfakianakis, Stelios Rozakis, Muhammet Deveci, Michail Galetakis. 2025. Cost-benefit analysis through stochastic risk assessment on mining waste management considering the circular economy's requirement. Journal of Cleaner Production. Volume 489. DOI: https://doi.org/10.1016/j.jclepro.2024.144388 .

Appendix

Table A 1 Configuration of the Penalty Cost for Non-Compliance during the five first consecutive operating years considering the Greek Legislations (64) that are implied with the legal Obligations declared by the European Directives (9) (10)

Years	Annual Y(i) Factor		Base of Penalty Cost	ΣΕΑΕ	ΣΒ	Σεεπ 1 History of Non-Compliance	Σεεπ 2 Cooperation during Audit Control	Σεεπ 3 Cooperation after Audit Control	Σεεπ 4 Intention on immediate financial gains	Σεεπ 5 Total Compliance behavior after Audit Control (Implementation of alternative procedures)	Σεεπ 6 Public Profitability	Σεεπ 7 Social Profitability	Total Σεεπ		EP or K(i)	SUM EP or K(i)
A/A	Y(i)		B	ΣΕΑΕ	ΣΒ	Σεεπ1	Σεεπ2	Σεεπ3	Σεεπ4	Σεεπ5	Σεεπ6	Σεεπ7	0.4≤Σεεπ≤1.5 (M(i)≤4)	Total Σεεπ (1^{st}-5^{th})	€	€
1	$\frac{Σεεπ(1^{st})}{Σεεπ(1^{st})}$	1	12,000	3	26	0.8	0.90	0.60	1.10	0.90	0.90	0.90	0.35	0.40	374,400	374,400
2	$\frac{Σεεπ(2^{nd})}{Σεεπ(1^{st})}$	3.27	12,000	3	26	1.2	0.90	1	1.10	1.10	1.00	1.00	1.31	1.31	1,223,164	1,597,564
3	$\frac{Σεεπ(3^{rd})}{Σεεπ(1^{st})}$	3.27	12,000	3	26	1.2	0.90	1	.1.10	1.10	1.00	1.00	1.31	1.31	1,223,164	2,820,729
4	$\frac{Σεεπ(4^{th})}{Σεεπ(1^{st})}$	3.27	12,000	3	26	1.2	0.90	1	1.10	1.10	1.00	1.00	1.31	1.31	1,223,164	4,043,894
5	$\frac{Σεεπ(5^{th})}{Σεεπ(1^{st})}$	8.17	12,000	3	26	3	0.90	1	1.10	1.10	1.00	1.00	3.27	3.27	3,057,912	7,101,806

Table A 2 Economic Assessment through CBA in Case Scenario A(0). Period of 30 Years

Years	Y(i) Index per Year	Theoretical Estimated Numerical Value for Y(i) Index	Numerical Value for Probability Density Function	Number of Minimum Audit Control Sites per Year	Number of Sum Audit Control Sites	Minimum Probability of Non-Conformance with Environmental Protection Requirements	$C=B*\Sigma B*\Sigma_{EAE}$	Total Σεεπ	Σεεπi	Primary Negative Cost K(i)	Configuration of the Loss Function $K(1)*[Y(i)]^2$	Configuration of the Goal Function $K(1)*[Y(i)]^2*f(Y)$ €	Sum of Penalty Cost for Non-Compliance Scenario A(0)
(i)	Y(i)	E[Y]	f(Y)	Nj	Ni	$\theta_{(0)}$	C	Total Σεεπ	Σεεπi Ratio	K(i) €	L(A(0).Y) €	$G(A(0).\theta_{(0)})$ €	SUM $G(A(0).\theta_{(0)})$ €
1	1.00	3.21	0.0247	4	4	0.75	936,000	0.40	1.00	374,400.00	374,400.00	9,258.53	9,258.53
2	3.27	6.42	1.5714	4	8		"	1.31	3.27	1,223,164	3,996,079	6,063,793	6,073,052.20
3	"	9.63	"	4	12	"	"	"	"	"	"	"	12,136,845.87
4	"	12.84	"	4	16	"	"	"	"	"	"	"	18,200,639.55
5	8.17	16.05	10.88	4	20	"	"	3.27	8.17	3,057,912	24,975,496	271,948,168	290,148,808.13
6	"	19.26	"	4	24	"	"	"	"	"	"	"	562,096,976.72
7	"	22.47	"	4	28	"	"	"	"	"	"	"	834,045,145.31
8	"	25.68	"	4	32	"	"	"	"	"	"	"	1,105,993,313.90
9	"	28.89	"	4	36	"	"	"	"	"	"	"	1,377,941,482.49
10	"	32.1	"	4	40	"	"	"	"	"	"	"	1,649,889,651.08
11	"	35.31	"	4	44	"	"	"	"	"	"	"	1,921,837,819.67
12	"	38.52	"	4	48	"	"	"	"	"	"	"	2,193,785,988.26
13	"	41.73	"	4	52	"	"	"	"	"	"	"	2,465,734,156.85
14	"	44.94	"	4	56	"	"	"	"	"	"	"	2,737,682,325.44
15	"	48.15	"	4	60	"	"	"	"	"	"	"	3,009,630,494.02
16	"	51.36	"	4	64	"	"	"	"	"	"	"	3,281,578,662.61
17	"	54.57	"	4	68	"	"	"	"	"	"	"	3,553,526,831.20
18	"	57.78	"	4	72	"	"	"	"	"	"	"	3,825,474,999.79
19	"	60.99	"	4	76	"	"	"	"	"	"	"	4,097,423,168.38
20	"	64.2	"	4	80	"	"	"	"	"	"	"	4,369,371,336.97
21	"	67.41	"	4	84	"	"	"	"	"	"	"	4,641,319,505.56
22	"	70.62	"	4	88	"	"	"	"	"	"	"	4,913,267,674.15
23	"	73.83	"	4	92	"	"	"	"	"	"	"	5,185,215,842.74
24	"	77.04	"	4	96	"	"	"	"	"	"	"	5,457,164,011.32
25	"	80.25	"	4	100		"	"	"	"	"	"	5,729,112,179.91

Years	Y(i) Index per Year	Theoretical Estimated Numerical Value for Y(i) Index	Numerical Value for Probability Density Function	Number of Minimum Audit Control Sites per Year	Number of Sum Audit Control Sites	Minimum Probability of Non-Conformance with Environmental Protection Requirements	$C=B^{*}\Sigma B^{*}\Sigma_{EAE}$	Total Σεεπ	Σεεπi	Primary Negative Cost K(i)	Configuration of the Loss Function $K(1)^{*}[Y(i)]^{2}$	Configuration of the Goal Function $K(1)^{*}[Y(i)]^{2}{*}f(Y)$ €	Sum of Penalty Cost for Non-Compliance Scenario A(0)
(i)	Y(i)	E[Y]	f(Y)	Nj	Ni	$\theta_{(0)}$	C	Total Σεεπ	Σεεπi Ratio	K(i) €	L(A(0).Y) €	$G(A(0).\theta_{(0)})$ €	SUM $G(A(0).\theta_{(0)})$ €
26	"	83.46	"	4	104	"	"	"	"	"	"	"	6,001,060,348.50
27	"	86.67	"	4	108	"	"	"	"	"	"	"	6,273,008,517.09
28	"	89.88	"	4	112	"	"	"	"	"	"	"	6,544,956,685.68
29	"	93.09	"	4	116	"	"	"	"	"	"	"	6,816,904,854.27
30	"	96.3	"	4	120	" "	"	"	"	"	"	"	7,088,853,022.86

Table A 3 Estimated Metal Mass into the Tailings and their Price Values' Fluctuation (118) (153)

	Years		2021		2022		2023		2024		2025		2026	
			Base Value											
		Annual Metals Mass (kg)	$/ Kg	$	$/ kg (+50%)	$	$/ kg (+33%)	$	$/ kg (+67%)	$	$/ kg (+0%)	$	$/ kg (+83%)	$
Base & Precious Metals	Ba	-	0	0	0	0	0	0	0	0	0	0	0	0
	Cd	-	0	0	0	0	0	0	0	0	0	0	0	0
	Cu	**58**	6	348	9	522	8	470	10	581	6	348	11	637

Years		2021		2022		2023		2024		2025		2026	
		Base Value											
Annual Metals	Mass (kg)	$/ Kg	$	$/ kg (+50%)	$	$/ kg (+33%)	$	$/ kg (+67%)	$	$/ kg (+0%)	$	$/ kg (+83%)	$
Mo	**30**	32	960	48	1,440	43	1,296	53	1,603	32	960	59	1,757
Ni	**25**	14	350	21	525	19	473	23	585	14	350	26	641
Pb	**32**	3	96	5	144	4	130	5	160	3	96	5	176
Sb	**192**	13	2,496	20	3,744	18	3,370	22	4,168	13	2,496	24	4,568
Se	**3**	50	150	75	225	68	203	84	251	50	150	92	275
Zn	**578**	3	1,734	5	2,601	4	2,341	5	2,896	3	1,734	5	3,173
Ag	**35**	808	28,281	1,212	42,422	1,091	38,179	1,349	47,229	808	28,281	1,479	51,754
Au	**4**	65,000	260,000	97,500	390,000	87,750	351,000	108,550	434,200	65,000	260,000	118,950	475,800
Pd	**30**	48,000	1,440,000	72,000	2,160,000	64,800	1,944,000	80,160	2,404,800	48,000	1,440,000	87,840	2,635,200
Pt	**32**	35,000	1,120,000	52,500	1,680,000	47,250	1,512,000	58,450	1,870,400	35,000	1,120,000	64,050	2,049,600
REEs La	**36**	1	36	2	54	1	49	2	60	1	36	2	66
REEs Ce	**152**	6	912	9	1,368	8	1,231	10	1,523	6	912	11	1,669

Years		2021		2022		2023		2024		2025		2026	
		Base Value											
Annual Metals	Mass (kg)	$/ Kg	$	$/ kg (+50%)	$	$/ kg (+33%)	$	$/ kg (+67%)	$	$/ kg (+0%)	$	$/ kg (+83%)	$
Pr	**37**	55	2,035	83	3,053	74	2,747	92	3,398	55	2,035	101	3,724
Nd	**301**	77	23,177	116	34,766	104	31,289	129	38,706	77	23,177	141	42,414
Sm	**9**	1,771	15,939	2,657	23,909	2,391	21,518	2,958	26,618	1,771	15,939	3,241	29,168
Eu	**288**	33	9,504	50	14,256	45	12,830	55	15,872	33	9,504	60	17,392
Gd	**290**	27	7,830	41	11,745	36	10,571	45	13,076	27	7,830	49	14,329
Ho	**3**	59	177	89	266	80	239	99	296	59	177	108	324
Y	**0**	31	0	47	0	42	0	52	0	31	0	57	0
Er	**5**	25	125	38	188	34	169	42	209	25	125	46	229
Tm	**2**	1	2	2	3	1	3	2	3	1	2	2	4
Yb	**8**	16	128	24	192	22	173	27	214	16	128	29	234
Lu	**2**	610	1,220	915	1,830	824	1,647	1,019	2,037	610	1,220	1,116	2,233

Years	2021		2022		2023		2024		2025		2026	
	Base Value											
Annual Metals Mass (kg)	$/ Kg	$	$/ kg (+50%)	$	$/ kg (+33%)	$	$/ kg (+67%)	$	$/ kg (+0%)	$	$/ kg (+83%)	$
Annual Benefit Value ($)	**2,915,500**		**4,373,250**		**3,935,925**		**4,868,885**		**2,915,500**		**5,335,365**	

Table A 4 Estimated Metal Mass into the Tailings and their Price Values' Fluctuation (118) (153)

	Years		2021 Base Value		2027		2028		2029		2030		2031		2032	
		Annual Metals Mass (kg)	$/ Kg	$	$/ kg (+66%)	$	$/ kg (+100%)	$	$/ kg (130%)	$	$/ kg (130%)	$	$/ kg (+16,6%)	$	$/ kg (+150%)	$
Base & Precious Metals	Ba	**-**	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	Cd	**-**	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	Cu	**58**	6	348	10	578	12	696	14	800	14	800	7	406	15	870
	Mo	**30**	32	960	53	1,594	64	1,920	74	2,208	74	2,208	37	1,119	80	2,400
	Ni	**25**	14	350	23	581	28	700	32	805	32	805	16	408	35	875
	Pb	**32**	3	96	5	159	6	192	7	221	7	221	3	112	8	240
	Sb	**192**	13	2,496	22	4,143	26	4,992	30	5,741	30	5,741	15	2,910	33	6,240
	Se	**3**	50	150	83	249	100	300	115	345	115	345	58	175	125	375
	Zn	**578**	3	1,734	5	2,878	6	3,468	7	3,988	7	3,988	3	2,022	8	4,335

	Years		2021		2027		2028		2029		2030		2031		2032	
			Base Value													
		Annual Metals Mass (kg)	$/ Kg	$	$/ kg (+66%)	$	$/ kg (+100%)	$	$/ kg (130%)	$	$/ kg (130%)	$	$/ kg (+16,6%)	$	$/ kg (+150%)	$
	Ag	**35**	808	28,281	1,341	46,947	1,616	56,562	1,858	65,046	1,858	65,046	942	32,976	2,020	70,703
	Au	**4**	65,000	260,000	107,900	431,600	130,000	520,000	149,500	598,000	149,500	598,000	75,790	303,160	162,500	650,000
	Pd	**30**	48,000	1,440,000	79,680	2,390,400	96,000	2,880,000	110,400	3,312,000	110,400	3,312,000	55,968	1,679,040	120,000	3,600,000
	Pt	**32**	35,000	1,120,000	58,100	1,859,200	70,000	2,240,000	80,500	2,576,000	80,500	2,576,000	40,810	1,305,920	87,500	2,800,000
REEs	La	**36**	1	36	2	60	2	72	2	83	2	83	1	42	3	90
	Ce	**152**	6	912	10	1,514	12	1,824	14	2,098	14	2,098	7	1,063	15	2,280
	Pr	**37**	55	2,035	91	3,378	110	4,070	127	4,681	127	4,681	64	2,373	138	5,088
	Nd	**301**	77	23,177	128	38,474	154	46,354	177	53,307	177	53,307	90	27,024	193	57,943
	Sm	**9**	1,771	15,939	2,940	26,459	3,542	31,878	4,073	36,660	4,073	36,660	2,065	18,585	4,428	39,848

Years		2021		2027		2028		2029		2030		2031		2032	
		Base Value													
	Annual Metals Mass (kg)	$/ Kg	$	$/ kg (+66%)	$	$/ kg (+100%)	$	$/ kg (130%)	$	$/ kg (130%)	$	$/ kg (+16,6%)	$	$/ kg (+150%)	$
Eu	**288**	33	9,504	55	15,777	66	19,008	76	21,859	76	21,859	38	11,082	83	23,760
Gd	**290**	27	7,830	45	12,998	54	15,660	62	18,009	62	18,009	31	9,130	68	19,575
Ho	**3**	59	177	98	294	118	354	136	407	136	407	69	206	148	443
Y	**0**	31	0	51	0	62	0	71	0	71	0	36	0	78	0
Er	**5**	25	125	42	208	50	250	58	288	58	288	29	146	63	313
Tm	**2**	1	2	2	3	2	4	2	5	2	5	1	2	3	5
Yb	**8**	16	128	27	212	32	256	37	294	37	294	19	149	40	320
Lu	**2**	610	1,220	1,013	2,025	1,220	2,440	1,403	2,806	1,403	2,806	711	1,423	1,525	3,050

Years	2021		2027		2028		2029		2030		2031		2032	
	Base Value													
Annual Metals Mass (kg)	$/ Kg	$	$/ kg (+66%)	$	$/ kg (+100%)	$	$/ kg (130%)	$	$/ kg (130%)	$	$/ kg (+16,6%)	$	$/ kg (+150%)	$
Annual Benefit Value ($)		**2,915,500**		**4,839,730**		**5,831,000**		**6,705,650**		**6,705,650**		**3,399,473**		**7.288.750**

Table A 5 Estimated Benefit gained through the Utilization of Recovered Metal Mass by the Tailings and their Sum of Price Values' Fluctuation (118) (153)

Sum of Annual Benefit Values ($)-10 Years	59,114,679
Sum of Annual Benefit Values (€)-10 Years	54,383,329
Annual Benefit Value (€/Yr)	5,438,333

Table A 6 Configuration of the Real Goal Function (or the actual Loss Function) in Case Scenario A(1) for the waste management of $150*10^6$ kg tailing mass annually

Years	Total Cost Ratio K(i)/K(1) per Year (i)	Cost of Primary Installation/ Maintenance per Year I/M Cost	Cost of Energy per Year E.C.	Cost of Chemical Reagents per Year C.R.C.	Beneficial Values B.V.	K(i)=[I/M] + [E] + [C.R.] – B.V. (Cost per Year)	Real Goal Function Calibrated K(i) L(A(1)_SF,Y(i)) or $G(A(1),Y(i))_r$ €
(i)	Y(i) or $Y(i)_a$	€	€	€	€	€	Plus 20% L(A(1),SF)
1	1	1,530,000	1,330,000	8,763,300	5,438,333	6,184,967	7,421,960
2	0,802	306,000	1,330,000	8,763,300	5,438,333	4,960,967	5,953,160
…n	0,802	306,000	1,330,000	8,763,300	5,438,333	4,960,967	5,953,160

Table A 7 Results of the actual Loss and Real Goal Functions L(A(1)_SF,Y(i)) and $G(A(1),Y(i))_r$ that show the annual stable investment cost in metal recovery for all the sub-cases in Case Scenario A(1)

Time	Cost Ratio	Probability Density Function	Total Cost of Investment			Revenue	Loss Function or Real Goal Function	Loss Function or Real Goal Function, Including SF (20%)	Configuration of Real Goal Function	SUM of Real Goal Function
Years	Total Cost Ratio K(i)/K(1) per Year (i)	Erlang Probability Density Function	Cost of Primary Installation/ Maintenance per Year I/M Cost	Cost of Energy per Year Cost	Cost of Chemical Reagents per Year C.R.C.	Beneficial Values B.V.	K(i)=[C.I.M] + [C.E. + [C.C.R.] – [B.V.] (Cost per Year)	Real Goal Function Calibrated K(i) SF	K(i)	SUM $G(A(1),Y(i))_r$
(i)	Y(i) or $Y(i)_a$	f(Y(i))	C.I.M. €	E.C. €	C.C.R. €	B.V. €	K(i) €	Plus 20% L(A(1)_SF,Y(i)) €	$G(A(1),Y(i))_r$ or L(A(1)_SF,Y(i)) €	SUM L(A(1)_SF,Y(i)) €
1	1	0,298	1,530,000	1,330,000	8,763,300	5,438,333	6,184,967	7,421,960	7,421,960	7,421,960
2	0.802	0	306,000	1,330,000	8,763,300	5,438,333	4,960,967	5,953,160	5,953,160	13,375,120
3										19,328,281
4										25,281,441

Time	Cost Ratio	Probability Density Function	Total Cost of Investment			Revenue	Loss Function or Real Goal Function	Loss Function or Real Goal Function, Including SF (20%)	Configuration of Real Goal Function	SUM of Real Goal Function
Years	Total Cost Ratio K(i)/K(1) per Year (i)	Erlang Probability Density Function	Cost of Primary Installation/ Maintenance per Year I/M Cost	Cost of Energy per Year Cost	Cost of Chemical Reagents per Year C.R.C.	Beneficial Values B.V.	K(i)=[C.I.M] + [C.E. + [C.C.R.] – [B.V.] (Cost per Year)	Real Goal Function Calibrated K(i) SF	K(i)	SUM $G(A(1),Y(i))_r$
(i)	Y(i) or $Y(i)_a$	f(Y(i))	C.I.M. €	E.C. €	C.C.R. €	B.V. €	K(i) €	Plus 20% L(A(1)_SF,Y(i)) €	$G(A(1),Y(i))_r$ or L(A(1)_SF,Y(i)) €	SUM L(A(1)_SF,Y(i)) €
5										31,234,602
6										37,187,762
7										43,140,922
8										49,094,083
9										55,047,243
10										61,000,404
11										66,953,564
12										72,906,724
13										78,859,885
14										84,813,045
15										90,766,206
16	0.802	0	306,000	1,330,000	8,763,300	5,438,333	4,960,967	5,953,160	5,953,160	96,719,366
17										102,672,526
18										108,625,687
19										114,578,847
20										120,532,008
21										126,485,168
22										132,438,328

Time	Cost Ratio	Probability Density Function	Total Cost of Investment			Revenue	Loss Function or Real Goal Function	Loss Function or Real Goal Function, Including SF (20%)	Configuration of Real Goal Function	SUM of Real Goal Function
Years	Total Cost Ratio K(i)/K(1) per Year (i)	Erlang Probability Density Function	Cost of Primary Installation/ Maintenance per Year I/M Cost	Cost of Energy per Year Cost	Cost of Chemical Reagents per Year C.R.C.	Beneficial Values B.V.	K(i)=[C.I.M] + [C.E. + [C.C.R.] – [B.V.] (Cost per Year)	Real Goal Function Calibrated K(i) SF	K(i)	SUM $G(A(1),Y(i))_r$
(i)	Y(i) or $Y(i)_a$	f(Y(i))	C.I.M. €	E.C. €	C.C.R. €	B.V. €	K(i) €	Plus 20% L(A(1)_SF,Y(i)) €	$G(A(1),Y(i))_r$ or L(A(1)_SF,Y(i)) €	SUM L(A(1)_SF,Y(i)) €
23										138,391,489
24										144,344,649
25										150,297,810
26										156,250,970
27										162,204,130
28										168,157,291
29										174,110,451
30										180,063,612

Table A 8 Results of the Financial Risk Assessment through CBA in the sub-case $A(1)_1$

Years	Negative Targeting ($\Sigma\varepsilon\varepsilon\pi_i/\Sigma\varepsilon\varepsilon\pi_1$)	Probability Density Function	Stable Number of Audit Control Sites per Year	Additive Number of Audit Control Sites	Maximum Probability of Non-Conformance with Environmental Protection Requirements	Stable Factor of Penalty Cost (€) $C=B^*\Sigma B^*\Sigma_{EAE}$ (as shown in Equation 1)	$\Sigma\varepsilon\varepsilon\pi_i$	Penalty Cost (€) K(i) (C*Total $\Sigma\varepsilon\varepsilon\pi_i$)	Loss Function	SUM of the At-Risk Loss Function	At-Risk Cost	SUM of the At-Risk Cost	SUM of the Real Cost of Investment (Shown in Table A7)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment
(i)	Σεεπi Ratio $Y(i)$ or $Y(i)_t$	PDF(i) or f(Y)	Nj	Ni	θ(0)	C	Total Σεεπi	K(i)	$L(A(0),Y(i))$ (€) $K(1)^*[Y(i)]^2$	SUM $L(A(0),Y(i))$ (€) SUM $[K(1)^*[Y(i)]^2]$	$G(A(1)_1,Y(i))_t$ (€) $K(1)^*[Y(i)]^{2*}f(Y)$	SUM $G(A(1)_1,Y(i))_t$ (€) SUM $G(A(1),Y(i))_t$	SUM $G(A(1)_1,Y(i))_r$ (€)	SUM $G(A(1)_1,Y(i))_{Total}$ (€)
1				4						374,400		157,950	7,421,960	7,579,910
2				8						748,800		315,900	13,375,121	13,691,021
3				12						1,123,200		473,850	19,328,281	19,802,131
4				16						1,497,600		631,800	25,281,442	25,913,242
5				20						1,872,000		789,750	31,234,602	32,024,352
6	1.00	0.422	4	24	0.25	936,000	0.4	374,400	374,400	2,246,400	157,950	947,700	37,187,762	38,135,462
7				28						2,620,800		1,105,650	43,140,923	44,246,573
8				32						2,995,200		1,263,600	49,094,083	50,357,683
9				36						3,369,600		1,421,550	55,047,244	56,468,794
10				40						3,744,000		1,579,500	61,000,404	62,579,904
11				44						4,118,400		1,737,450	66,953,564	68,691,014
12				48						4,492,800		1,895,400	72,906,725	74,802,125
13				52						4,867,200		2,053,350	78,859,885	80,913,235
14				56						5,241,600		2,211,300	84,813,046	87,024,346

Years	Negative Targeting ($\Sigma\varepsilon\varepsilon\pi_i/\Sigma\varepsilon\varepsilon\pi_1$)	Probability Density Function	Stable Number of Audit Control Sites per Year	Additive Number of Audit Control Sites	Maximum Probability of Non-Conformance with Environmental Protection Requirements	Stable Factor of Penalty Cost (€) $C=B*\Sigma B*\Sigma_{EAE}$ (as shown in Equation 1)	$\Sigma\varepsilon\varepsilon\pi_i$	Penalty Cost (€) K(i) (C*Total $\Sigma\varepsilon\varepsilon\pi_i$)	Loss Function	SUM of the At-Risk Loss Function	At-Risk Cost	SUM of the At-Risk Cost	SUM of the Real Cost of Investment (Shown in Table A7)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment
(i)	Σεεπi Ratio Y(i) or $Y(i)_t$	PDF(i) or f(Y)	Nj	Ni	θ(0)	C	Total Σεεπi	K(i)	L(A(0),Y(i)) (€) $K(1)*[Y(i)]^2$	SUM L(A(0),Y(i)) (€) SUM $[K(1)*[Y(i)]^2]$	$G(A(1)_1,Y(i))_t$ (€) $K(1)*[Y(i)]^2*f(Y)$	SUM $G(A(1)_1,Y(i))_t$ (€) SUM $G(A(1),Y(i))_t$	SUM $G(A(1)_1,Y(i))_r$ (€)	SUM $G(A(1)_1,Y(i))_{Total}$ (€)
15				60						5,616,000		2,369,250	90,766,206	93,135,456
16				64						5,990,400		2,527,200	96,719,366	99,246,566
17				68						6,364,800		2,685,150	102,672,527	105,357,677
18				72						6,739,200		2,843,100	108,625,687	111,468,787
19				76						7,113,600		3,001,050	114,578,848	117,579,898
20				80						7,488,000		3,159,000	120,532,008	123,691,008
21	1.00	0.422	4	84	0.25	936,000	0.4	374,400	374,400	7,862,400	157,950	3,316,950	126,485,168	129,802,118
22				88						8,236,800		3,474,900	132,438,329	135,913,229
23				92						8,611,200		3,632,850	138,391,489	142,024,339
24				96						8,985,600		3,790,800	144,344,650	148,135,450
25				100						9,360,000		3,948,750	150,297,810	154,246,560
26				104						9,734,400		4,106,700	156,250,970	160,357,670
27				108						10,108,800		4,264,650	162,204,131	166,468,781
28				112						10,483,200		4,422,600	168,157,291	172,579,891
29				116						10,857,600		4,580,550	174,110,452	178,691,002
30				120						11,232,000		4,738,500	180,063,612	184,802,112

Table A 9 Results of the Financial Risk Assessment through CBA in the sub-case $A(1)_2$

Years	Negative Targeting ($\Sigma\varepsilon\varepsilon\pi_i/\Sigma\varepsilon\varepsilon\pi_1$)	Probability Density Function	Stable Number of Audit Control Sites per Year	Additive Number of Audit Control Sites	Maximum Probability of Non-Conformance with Environmental Protection Requirements	Stable Factor of Penalty Cost (€) $C=B*\Sigma B*\Sigma_{EAE}$ (as shown in Equation 1)	$\Sigma\varepsilon\varepsilon\pi_i$ Indicator	Penalty Cost (€) K(i) (C*Total $\Sigma\varepsilon\varepsilon\pi_i$)	Loss Function	SUM of the At-Risk Loss Function	At-Risk Cost	SUM of the At-Risk Cost	SUM of the Real Cost of Investment (Shown in Table A7)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment
(i)	Σεεπi Ratio Y(i) or $Y(i)_t$	PDF(i) or f(Y)	Nj	Ni	θ(0)	C	Total Σεεπi	K(i)	L(A(0),Y(i)) (€) $K(1)*[Y(i)]^2$	SUM L(A(0),Y(i)) (€) SUM $[K(1)*[Y(i)]^2]$	$G(A(1)_1,Y(i))_t$ (€) $K(1)*[Y(i)]^2*f(Y)$	SUM $G(A(1)_1,Y(i))_t$ (€) SUM $G(A(1),Y(i))_t$	SUM $G(A(1)_1,Y(i))_r$ (€)	SUM $G(A(1)_1,Y(i))_{Total}$ (€)
1	1.00	0.422	4	4	0.25	936,000	0.40	374,400	374,400	374,400	157,950	157,950	7,421,960	7,579,910
2	3.27	0.035	4	8	0.25	936,000	1.31	1,223,165	3,996,079	4,370,479	139,696	297,646	13,375,121	13,672,767
3	3.27	0.035	4	12	0.25	936,000	1.31	1,223,165	3,996,079	8,366,559	139,696	437,342	19,328,281	19,765,623
4	3.27	0.035	4	16	0.25	936,000	1.31	1,223,165	3,996,079	12,362,638	139,696	577,038	25,281,442	25,858,480
5	8.17	0.000	4	20	0.25	936,000	3.27	3,057,912	24,975,496	37,338,134	0	577,038	31,234,602	31,811,640
6	8.17	0.000	4	24	0.25	936,000	3.27	3,057,912	24,975,496	62,313,631	0	577,038	37,187,762	37,764,800
7	8.17	0.000	4	28	0.25	936,000	3.27	3,057,912	24,975,496	87,289,127	0	577,038	43,140,923	43,717,961
8	8.17	0.000	4	32	0.25	936,000	3.27	3,057,912	24,975,496	112,264,623	0	577,038	49,094,083	49,671,121
9	8.17	0.000	4	36	0.25	936,000	3.27	3,057,912	24,975,496	137,240,120	0	577,038	55,047,244	55,624,282
10	8.17	0.000	4	40	0.25	936,000	3.27	3,057,912	24,975,496	162,215,616	0	577,038	61,000,404	61,577,442
11	8.17	0.000	4	44	0.25	936,000	3.27	3,057,912	24,975,496	187,191,112	0	577,038	66,953,564	67,530,602

Years	Negative Targeting $(Σεεπ_i/Σεεπ_1)$	Probability Density Function	Stable Number of Audit Control Sites per Year	Additive Number of Audit Control Sites	Maximum Probability of Non-Conformance with Environmental Protection Requirements	Stable Factor of Penalty Cost (€) $C=B*ΣB*Σ_{EAE}$ (as shown in Equation 1)	$Σεεπ_i$ Indicator	Penalty Cost (€) K(i) (C*Total $Σεεπ_i$)	Loss Function	SUM of the At-Risk Loss Function	At-Risk Cost	SUM of the At-Risk Cost	SUM of the Real Cost of Investment (Shown in Table A7)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment
(i)	Σεεπi Ratio $Y(i)$ or $Y(i)_t$	PDF(i) or f(Y)	Nj	Ni	θ(0)	C	Total Σεεπi	K(i)	L(A(0),Y(i)) (€) $K(1)*[Y(i)]^2$	SUM L(A(0),Y(i)) (€) SUM $[K(1)*[Y(i)]^2]$	$G(A(1)_1,Y(i))_t$ (€) $K(1)*[Y(i)]^2*f(Y)$	SUM $G(A(1)_1,Y(i))_t$ (€) SUM $G(A(1),Y(i))_t$	SUM $G(A(1)_1,Y(i))_r$ (€)	SUM $G(A(1)_1,Y(i))_{Total}$ (€)
12	8.17	0.000	4	48	0.25	936,000	3.27	3,057,912	24,975,496	212,166,608	0	577,038	72,906,725	73,483,763
13	8.17	0.000	4	52	0.25	936,000	3.27	3,057,912	24,975,496	237,142,105	0	577,038	78,859,885	79,436,923
14	8.17	0.000	4	56	0.25	936,000	3.27	3,057,912	24,975,496	262,117,601	0	577,038	84,813,046	85,390,084
15	8.17	0.000	4	60	0.25	936,000	3.27	3,057,912	24,975,496	287,093,097	0	577,038	90,766,206	91,343,244
16	8.17	0.000	4	64	0.25	936,000	3.27	3,057,912	24,975,496	312,068,593	0	577,038	96,719,366	97,296,404
17	8.17	0.000	4	68	0.25	936,000	3.27	3,057,912	24,975,496	337,044,090	0	577,038	102,672,527	103,249,565
18	8.17	0.000	4	72	0.25	936,000	3.27	3,057,912	24,975,496	362,019,586	0	577,038	108,625,687	109,202,725
19	8.17	0.000	4	76	0.25	936,000	3.27	3,057,912	24,975,496	386,995,082	0	577,038	114,578,848	115,155,886
20	8.17	0.000	4	80	0.25	936,000	3.27	3,057,912	24,975,496	411,970,578	0	577,038	120,532,008	121,109,046
21	8.17	0.000	4	84	0.25	936,000	3.27	3,057,912	24,975,496	436,946,075	0	577,038	126,485,168	127,062,206
22	8.17	0.000	4	88	0.25	936,000	3.27	3,057,912	24,975,496	461,921,571	0	577,038	132,438,329	133,015,367
23	8.17	0.000	4	92	0.25	936,000	3.27	3,057,912	24,975,496	486,897,067	0	577,038	138,391,489	138,968,527
24	8.17	0.000	4	96	0.25	936,000	3.27	3,057,912	24,975,496	511,872,563	0	577,038	144,344,650	144,921,688
25	8.17	0.000	4	100	0.25	936,000	3.27	3,057,912	24,975,496	536,848,060	0	577,038	150,297,810	150,874,848
26	8.17	0.000	4	104	0.25	936,000	3.27	3,057,912	24,975,496	561,823,556	0	577,038	156,250,970	156,828,008
27	8.17	0.000	4	108	0.25	936,000	3.27	3,057,912	24,975,496	586,799,052	0	577,038	162,204,131	162,781,169

Years	Negative Targeting ($\Sigma\varepsilon\varepsilon\pi_i/\Sigma\varepsilon\varepsilon\pi_1$)	Probability Density Function	Stable Number of Audit Control Sites per Year	Additive Number of Audit Control Sites	Maximum Probability of Non-Conformance with Environmental Protection Requirements	Stable Factor of Penalty Cost (€) C=B*ΣB*Σ_{EAE} (as shown in Equation 1)	$\Sigma\varepsilon\varepsilon\pi_i$ Indicator	Penalty Cost (€) K(i) (C*Total $\Sigma\varepsilon\varepsilon\pi_i$)	Loss Function	SUM of the At-Risk Loss Function	At-Risk Cost	SUM of the At-Risk Cost	SUM of the Real Cost of Investment (Shown in Table A7)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment
(i)	Σεεπi Ratio Y(i) or $Y(i)_t$	PDF(i) or f(Y)	Nj	Ni	θ(0)	C	Total Σεεπi	K(i)	L(A(0),Y(i)) (€) $K(1)*[Y(i)]^2$	SUM L(A(0),Y(i)) (€) SUM $[K(1)*[Y(i)]^2]$	$G(A(1)_1,Y(i))_t$ (€) $K(1)*[Y(i)]^2*f(Y)$	SUM $G(A(1)_1,Y(i))_t$ (€) SUM $G(A(1),Y(i))_t$	SUM $G(A(1)_1,Y(i))_r$ (€)	SUM $G(A(1)_1,Y(i))_{Total}$ (€)
28	8.17	0.000	4	112	0.25	936,000	3.27	3,057,912	24,975,496	611,774,548	0	577,038	168,157,291	168,734,329
29	8.17	0.000	4	116	0.25	936,000	3.27	3,057,912	24,975,496	636,750,045	0	577,038	174,110,452	174,687,490
30	8.17	0.000	4	120	0.25	936,000	3.27	3,057,912	24,975,496	661,725,541	0	577,038	180,063,612	180,640,650

Table A 10 Results of the Financial Risk Assessment through CBA in the sub-case $A(1)_3$

Years (i)	Negative Targeting ($\Sigma\varepsilon\varepsilon\pi_i/\Sigma\varepsilon\varepsilon\pi_1$) Σεεπi Ratio $Y(i)$ or $Y(i)_t$	Probability Density Function PDF(i) or f(Y)	Stable Number of Audit Control Sites per Year Nj	Additive Number of Audit Control Sites Ni	Maximum Probability of Non-Conformance with Environmental Protection Requirements θ(0)	Stable Factor of Penalty Cost (€) $C=B*\Sigma B*\Sigma_{EAE}$ (as shown in Equation 1) C	$\Sigma\varepsilon\varepsilon\pi_i$ Indicator Total Σεεπi	Penalty Cost (€) K(i) (C*Total $\Sigma\varepsilon\varepsilon\pi_i$) K(i)	Loss Function L(A(0),Y(i)) (€) K(1)*[Y(i)]	SUM of the At-Risk Loss Function SUM L(A(0),Y(i)) (€) SUM [K(1)*[Y(i)]]	At-Risk Cost $G(A(1)_1,Y(i))_t$ (€) K(1)*[Y(i)]*f(Y)	SUM of the At-Risk Cost SUM $G(A(1)_1,Y(i))_t$ (€) SUM $G(A(1),Y(i))_t$	SUM of the Real Cost of Investment (Shown in Table A7) SUM $G(A(1)_1,Y(i))_r$ (€)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment SUM $G(A(1)_1,Y(i))_{Total}$ (€)
1	1.00	2.47E-01	4	4	0.25	936,000	0.40	374,400	374,400	374,400	92,444	92,444	7,421,960	7,514,405
2	1.00	2.94E-04	4	8	0.25	936,000	0.40	374,400	374,400	748,800	110	92,554	13,375,121	13,467,675
3	1.00	3.71E-08	4	12	0.25	936,000	0.40	374,400	374,400	1,123,200	0	92,555	19,328,281	19,420,836
4	1.00	1.13E-12	4	16	0.25	936,000	0.40	374,400	374,400	1,497,600	0	92,555	25,281,442	25,373,996
5	1.00	1.22E-17	4	20	0.25	936,000	0.40	374,400	374,400	1,872,000	0	92,555	31,234,602	31,327,157
6	1.00	5.73E-23	4	24	0.25	936,000	0.40	374,400	374,400	2,246,400	0	92,555	37,187,762	37,280,317
7	1.00	1.36E-28	4	28	0.25	936,000	0.40	374,400	374,400	2,620,800	0	92,555	43,140,923	43,233,477
8	1.00	1.80E-34	4	32	0.25	936,000	0.40	374,400	374,400	2,995,200	0	92,555	49,094,083	49,186,638
9	1.00	1.43E-40	4	36	0.25	936,000	0.40	374,400	374,400	3,369,600	0	92,555	55,047,244	55,139,798
10	1.00	7.26E-47	4	40	0.25	936,000	0.40	374,400	374,400	3,744,000	0	92,555	61,000,404	61,092,959
11	1.00	2.45E-53	4	44	0.25	936,000	0.40	374,400	374,400	4,118,400	0	92,555	66,953,564	67,046,119
12	1.00	5.73E-60	4	48	0.25	936,000	0.40	374,400	374,400	4,492,800	0	92,555	72,906,725	72,999,279
13	1.00	9.55E-67	4	52	0.25	936,000	0.40	374,400	374,400	4,867,200	0	92,555	78,859,885	78,952,440
14	1.00	1.17E-73	4	56	0.25	936,000	0.40	374,400	374,400	5,241,600	0	92,555	84,813,046	84,905,600

Years	Negative Targeting (Σεεπ$_i$/Σεεπ$_1$)	Probability Density Function	Stable Number of Audit Control Sites per Year	Additive Number of Audit Control Sites	Maximum Probability of Non-Conformance with Environmental Protection Requirements	Stable Factor of Penalty Cost (€) C=B*ΣB*Σ$_{EAE}$ (as shown in Equation 1)	Σεεπ$_i$ Indicator	Penalty Cost (€) K(i) (C*Total Σεεπ$_i$)	Loss Function	SUM of the At-Risk Loss Function	At-Risk Cost	SUM of the At-Risk Cost	SUM of the Real Cost of Investment (Shown in Table A7)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment
(i)	Σεεπi Ratio Y(i) or Y(i)$_t$	PDF(i) or f(Y)	Nj	Ni	θ(0)	C	Total Σεεπi	K(i)	L(A(0),Y(i)) (€) K(1)*[Y(i)]	SUM L(A(0),Y(i)) (€) SUM [K(1)*[Y(i)]]	G(A(1)$_1$,Y(i))$_t$ (€) K(1)*[Y(i)]*f(Y)	SUM G(A(1)$_1$,Y(i))$_t$ (€) SUM G(A(1),Y(i))$_t$	SUM G(A(1)$_1$,Y(i))$_r$ (€)	SUM G(A(1)$_1$,Y(i))$_{Total}$ (€)
15	1.00	1.07E-80	4	60	0.25	936,000	0.40	374,400	374,400	5,616,000	0	92,555	90,766,206	90,858,761
16	1.00	7.47E-88	4	64	0.25	936,000	0.40	374,400	374,400	5,990,400	0	92,555	96,719,366	96,811,921
17	1.00	4.06E-95	4	68	0.25	936,000	0.40	374,400	374,400	6,364,800	0	92,555	102,672,527	102,765,081
18	1.00	1.74E-102	4	72	0.25	936,000	0.40	374,400	374,400	6,739,200	0	92,555	108,625,687	108,718,242
19	1.00	5.97E-110	4	76	0.25	936,000	0.40	374,400	374,400	7,113,600	0	92,555	114,578,848	114,671,402
20	1.00	1.66E-117	4	80	0.25	936,000	0.40	374,400	374,400	7,488,000	0	92,555	120,532,008	120,624,563
21	1.00	3.75E-125	4	84	0.25	936,000	0.40	374,400	374,400	7,862,400	0	92,555	126,485,168	126,577,723
22	1.00	7.03E-133	4	88	0.25	936,000	0.40	374,400	374,400	8,236,800	0	92,555	132,438,329	132,530,883
23	1.00	1.10E-140	4	92	0.25	936,000	0.40	374,400	374,400	8,611,200	0	92,555	138,391,489	138,484,044
24	1.00	1.43E-148	4	96	0.25	936,000	0.40	374,400	374,400	8,985,600	0	92,555	144,344,650	144,437,204
25	1.00	1.59E-156	4	100	0.25	936,000	0.40	374,400	374,400	9,360,000	0	92,555	150,297,810	150,390,365
26	1.00	1.50E-164	4	104	0.25	936,000	0.40	374,400	374,400	9,734,400	0	92,555	156,250,970	156,343,525
27	1.00	1.21E-172	4	108	0.25	936,000	0.40	374,400	374,400	10,108,800	0	92,555	162,204,131	162,296,685
28	1.00	8.40E-181	4	112	0.25	936,000	0.40	374,400	374,400	10,483,200	0	92,555	168,157,291	168,249,846

Years	Negative Targeting (Σεεπ$_i$/Σεεπ$_1$)	Probability Density Function	Stable Number of Audit Control Sites per Year	Additive Number of Audit Control Sites	Maximum Probability of Non-Conformance with Environmental Protection Requirements	Stable Factor of Penalty Cost (€) C=B*ΣB*Σ$_{EAE}$ (as shown in Equation 1)	Σεεπ$_i$ Indicator	Penalty Cost (€) K(i) (C*Total Σεεπ$_i$)	Loss Function	SUM of the At-Risk Loss Function	At-Risk Cost	SUM of the At-Risk Cost	SUM of the Real Cost of Investment (Shown in Table A7)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment
(i)	Σεεπi Ratio Y(i) or Y(i)$_t$	PDF(i) or f(Y)	Nj	Ni	θ(0)	C	Total Σεεπi	K(i)	L(A(0),Y(i)) (€) K(1)*[Y(i)]	SUM L(A(0),Y(i)) (€) SUM [K(1)*[Y(i)]]	G(A(1)$_1$,Y(i))$_t$ (€) K(1)*[Y(i)]*f(Y)	SUM G(A(1)$_1$,Y(i))$_t$ (€) SUM G(A(1),Y(i))$_t$	SUM G(A(1)$_1$,Y(i))$_r$ (€)	SUM G(A(1)$_1$,Y(i))$_{Total}$ (€)
29	1.00	5.06E-189	4	116	0.25	936,000	0.40	374,400	374,400	10,857,600	0	92,555	174,110,452	174,203,006
30	1.00	2.66E-197	4	120	0.25	936,000	0.40	374,400	374,400	11,232,000	0	92,555	180,063,612	180,156,167

Table A 11 Results of the Financial Risk Assessment through CBA in the sub-case $A(1)_4$

Years	Negative Targeting ($\Sigma\varepsilon\varepsilon\pi_i/\Sigma\varepsilon\varepsilon\pi_1$)	Probability Density Function	Stable Number of Audit Control Sites per Year	Additive Number of Audit Control Sites	Maximum Probability of Non-Conformance with Environmental Protection Requirements	Stable Factor of Penalty Cost (€) $C=B*\Sigma B*\Sigma_{EAE}$ (as shown in Equation 1)	$\Sigma\varepsilon\varepsilon\pi_i$ Indicator	Penalty Cost (€) K(i) (C*Total $\Sigma\varepsilon\varepsilon\pi_i$)	Loss Function	SUM of the At-Risk Loss Function	At-Risk Cost	SUM of the At-Risk Cost	SUM of the Real Cost of Investment (Shown in Table A7)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment
(i)	Σεεπi Ratio $Y(i)$ or $Y(i)_t$	PDF(i) or f(Y)	Nj	Ni	θ(0)	C	Total Σεεπi	K(i)	L(A(0),Y(i)) (€) K(1)*[Y(i)]	SUM L(A(0),Y(i)) (€) SUM [K(1)*[Y(i)]]	$G(A(1)_1,Y(i))_t$ (€) K(1)*[Y(i)]*f(Y)	SUM $G(A(1)_1,Y(i))_t$ (€) SUM $G(A(1),Y(i))_t$	SUM $G(A(1)_1,Y(i))_r$ (€)	SUM $G(A(1)_1,Y(i))_{Total}$ (€)
1	1.00	2.47E-01	4	4	0.25	936,000	0.40	374,400	374,400	374,400	92,444	92,444	7,421,960	7,514,405
2	3.27	2.94E-04	4	8	0.25	936,000	1.31	1,223,165	3,996,079	4,370,479	1,175	93,619	13,375,121	13,468,740
3	3.27	3.71E-08	4	12	0.25	936,000	1.31	1,223,165	3,996,079	8,366,559	0	93,619	19,328,281	19,421,900
4	3.27	1.13E-12	4	16	0.25	936,000	1.31	1,223,165	3,996,079	12,362,638	0	93,619	25,281,442	25,375,061
5	8.17	1.22E-17	4	20	0.25	936,000	3.27	3,057,912	24,975,496	37,338,134	0	93,619	31,234,602	31,328,221
6	8.17	5.73E-23	4	24	0.25	936,000	3.27	3,057,912	24,975,496	62,313,631	0	93,619	37,187,762	37,281,382
7	8.17	1.36E-28	4	28	0.25	936,000	3.27	3,057,912	24,975,496	87,289,127	0	93,619	43,140,923	43,234,542
8	8.17	1.80E-34	4	32	0.25	936,000	3.27	3,057,912	24,975,496	112,264,623	0	93,619	49,094,083	49,187,702
9	8.17	1.43E-40	4	36	0.25	936,000	3.27	3,057,912	24,975,496	137,240,120	0	93,619	55,047,244	55,140,863
10	8.17	7.26E-47	4	40	0.25	936,000	3.27	3,057,912	24,975,496	162,215,616	0	93,619	61,000,404	61,094,023
11	8.17	2.45E-53	4	44	0.25	936,000	3.27	3,057,912	24,975,496	187,191,112	0	93,619	66,953,564	67,047,184
12	8.17	5.73E-60	4	48	0.25	936,000	3.27	3,057,912	24,975,496	212,166,608	0	93,619	72,906,725	73,000,344
13	8.17	9.55E-67	4	52	0.25	936,000	3.27	3,057,912	24,975,496	237,142,105	0	93,619	78,859,885	78,953,504
14	8.17	1.17E-73	4	56	0.25	936,000	3.27	3,057,912	24,975,496	262,117,601	0	93,619	84,813,046	84,906,665

Years	Negative Targeting ($\Sigma\varepsilon\varepsilon\pi_i/\Sigma\varepsilon\varepsilon\pi_1$)	Probability Density Function	Stable Number of Audit Control Sites per Year	Additive Number of Audit Control Sites	Maximum Probability of Non-Conformance with Environmental Protection Requirements	Stable Factor of Penalty Cost (€) $C=B*\Sigma B*\Sigma_{EAE}$ (as shown in Equation 1)	$\Sigma\varepsilon\varepsilon\pi_i$ Indicator	Penalty Cost (€) K(i) (C*Total $\Sigma\varepsilon\varepsilon\pi_i$)	Loss Function	SUM of the At-Risk Loss Function	At-Risk Cost	SUM of the At-Risk Cost	SUM of the Real Cost of Investment (Shown in Table A7)	SUM of the Total Cost including At-Risk Cost and Real Cost of Investment
(i)	Σεεπi Ratio Y(i) or $Y(i)_t$	PDF(i) or f(Y)	Nj	Ni	θ(0)	C	Total Σεεπi	K(i)	L(A(0),Y(i)) (€) K(1)*[Y(i)]	SUM L(A(0),Y(i)) (€) SUM [K(1)*[Y(i)]]	$G(A(1)_1,Y(i))_t$ (€) K(1)*[Y(i)]*f(Y)	SUM $G(A(1)_1,Y(i))_t$ (€) SUM $G(A(1),Y(i))_t$	SUM $G(A(1)_1,Y(i))_r$ (€)	SUM $G(A(1)_1,Y(i))_{Total}$ (€)
15	8.17	1.07E-80	4	60	0.25	936,000	3.27	3,057,912	24,975,496	287,093,097	0	93,619	90,766,206	90,859,825
16	8.17	7.47E-88	4	64	0.25	936,000	3.27	3,057,912	24,975,496	312,068,593	0	93,619	96,719,366	96,812,986
17	8.17	4.06E-95	4	68	0.25	936,000	3.27	3,057,912	24,975,496	337,044,090	0	93,619	102,672,527	102,766,146
18	8.17	1.74E-102	4	72	0.25	936,000	3.27	3,057,912	24,975,496	362,019,586	0	93,619	108,625,687	108,719,306
19	8.17	5.97E-110	4	76	0.25	936,000	3.27	3,057,912	24,975,496	386,995,082	0	93,619	114,578,848	114,672,467
20	8.17	1.66E-117	4	80	0.25	936,000	3.27	3,057,912	24,975,496	411,970,578	0	93,619	120,532,008	120,625,627
21	8.17	3.75E-125	4	84	0.25	936,000	3.27	3,057,912	24,975,496	436,946,075	0	93,619	126,485,168	126,578,788
22	8.17	7.03E-133	4	88	0.25	936,000	3.27	3,057,912	24,975,496	461,921,571	0	93,619	132,438,329	132,531,948
23	8.17	1.10E-140	4	92	0.25	936,000	3.27	3,057,912	24,975,496	486,897,067	0	93,619	138,391,489	138,485,108
24	8.17	1.43E-148	4	96	0.25	936,000	3.27	3,057,912	24,975,496	511,872,563	0	93,619	144,344,650	144,438,269
25	8.17	1.59E-156	4	100	0.25	936,000	3.27	3,057,912	24,975,496	536,848,060	0	93,619	150,297,810	150,391,429
26	8.17	1.50E-164	4	104	0.25	936,000	3.27	3,057,912	24,975,496	561,823,556	0	93,619	156,250,970	156,344,590
27	8.17	1.21E-172	4	108	0.25	936,000	3.27	3,057,912	24,975,496	586,799,052	0	93,619	162,204,131	162,297,750
28	8.17	8.40E-181	4	112	0.25	936,000	3.27	3,057,912	24,975,496	611,774,548	0	93,619	168,157,291	168,250,910
29	8.17	5.06E-189	4	116	0.25	936,000	3.27	3,057,912	24,975,496	636,750,045	0	93,619	174,110,452	174,204,071
30	8.17	2.66E-197	4	120	0.25	936,000	3.27	3,057,912	24,975,496	661,725,541	0	93,619	180,063,612	180,157,231

Table A 12 Results of the Financial Risk Assessment through CBA in the sub-case $A(1)_5$

Time	Cost Ratio	Probability Density Function	Total Cost of Investment			Revenue	Loss Function or Reliable Goal Function	Loss Function or Reliable Goal Function, Including SF (20%)	Configuration of Reliable Goal Function	SUM of Reliable Goal Function	Theoretical Goal Function	SUM of Theoretical Goal Function	Total SUM of Investment Cost, Including Reliable and Theoretical Goal Functions
Years	Total Cost Ratio K(i)/K(1) per Year (i)	Erlang Probability Density Function	Cost of Primary Installation/ Maintenance per Year I/M Cost	Cost of Energy per Year Cost	Cost of Chemical Reagents per Year C.R.C.	Beneficial Values B.V.	K(i)=[C.I.M] + [C.E. + [C.C.R.] – [B.V.] (Cost per Year)	Reliable Goal Function Calibrated K(i) SF	K(i)*Y(i)	SUM $G(A(1),Y(i))_r$	At Risk Negative Cost $K(i)_t$ [K(i)*Y(i)*f(Y(i))]	SUM $G(A(1),Y(i))_t$	Total SUM G(A(1),Y(i))
(i)	$Y(i)_a$ or $Y(i)_t$	PDF(i) or f(Y(i))	C.I.M. €	E.C. €	C.C.R. €	B.V. €	K(i) €	Plus 20% L(A(1),SF) €	$G(A(1),Y(i))_r$ or L(A(1),SF) €	SUM L(A(1),SF) €	$G(A(1),Y(i))_t$ €	$K(i)_t$ or SUM $G(A(1),Y(i))_t$ €	SUM G(A(1),SF) €
1	1	0,298	1,530,000	1,330,000	8,763,300	5,438,333	6,184,967	7,421,960	7,421,960	7,421,960	2,209,007	2,209,007	9,630,967
2	0.802	0	306,000	1,330,000	8,763,300	5,438,333	4,960,967	5,953,160	5,953,160	13,375,120	791	2,209,798	15,584,127
3										19,328,281	0.04		21,537,288
4										25,281,441	0.00		27,490,448
5										31,234,602	0.00		33,443,609
6										37,187,762	0.00		39,396,769
7										43,140,922	0.00		45,349,929
8										49,094,083	0.00		51,303,090
9										55,047,243	0.00		57,256,250
10										61,000,404	0.00		63,209,411
11										66,953,564	0.00		69,162,571
12										72,906,724	0.00		75,115,731
13										78,859,885	0.00		81,068,892
14										84,813,045	0.00		87,022,052

Time	Cost Ratio	Probability Density Function	Total Cost of Investment			Revenue	Loss Function or Reliable Goal Function	Loss Function or Reliable Goal Function, Including SF (20%)	Configuration of Reliable Goal Function	SUM of Reliable Goal Function	Theoretical Goal Function	SUM of Theoretical Goal Function	Total SUM of Investment Cost, Including Reliable and Theoretical Goal Functions
Years	Total Cost Ratio K(i)/K(1) per Year (i)	Erlang Probability Density Function	Cost of Primary Installation/ Maintenance per Year I/M Cost	Cost of Energy per Year Cost	Cost of Chemical Reagents per Year C.R.C.	Beneficial Values B.V.	K(i)=[C.I.M] + [C.E. + [C.C.R.] – [B.V.] (Cost per Year)	Reliable Goal Function Calibrated K(i) SF	K(i)*Y(i)	SUM $G(A(1),Y(i))_r$	At Risk Negative Cost $K(i)_t$ [K(i)*Y(i)*f(Y(i))]	SUM $G(A(1),Y(i))_t$	Total SUM G(A(1),Y(i))
(i)	$Y(i)_a$ or $Y(i)_t$	PDF(i) or f(Y(i))	C.I.M. €	E.C. €	C.C.R. €	B.V. €	K(i) €	Plus 20% L(A(1),SF) €	$G(A(1),Y(i))_r$ or L(A(1),SF) €	SUM L(A(1),SF) €	$G(A(1),Y(i))_t$ €	$K(i)_t$ or SUM $G(A(1),Y(i))_t$ €	SUM G(A(1),SF) €
15										90,766,206	0.00		92,975,213
16										96,719,366	0.00		98,928,373
17	0.802	0	306,000	1,330,000	8,763,300	5,438,333	4,960,967	5,953,160	5,953,160	102,672,526	0.00	2,209,798	104,881,533
18										108,625,687	0.00		110,834,694
19										114,578,847	0.00		116,787,854
20										120,532,008	0.00		122,741,015
21										126,485,168	0.00		128,694,175
22										132,438,328	0.00		134,647,335
23										138,391,489	0.00		140,600,496
24										144,344,649	0.00		146,553,656
25										150,297,810	0.00		152,506,817
26										156,250,970	0.00		158,459,977
27										162,204,130	0.00		164,413,137
28										168,157,291	0.00		170,366,298
29										174,110,451	0.00		176,319,458
30										180,063,612	0.00		182,272,619

Table A 13 Partial Factors inside Financial Risk Integral through Bayesian Analysis in Case Scenario A(0), to identify the Financial Stochastic Risk

E[Y2]	$\mathbf{E[Y(1)^2] = [m(\theta_0)]^2 + \sigma^2 = [m(\theta_0)]^2 + 4\,\theta_0(1-\theta_0)}$ $\mathbf{E[Y(2,3,4)^2] = [m(\theta_0)]^2 + \sigma^2 = [m(\theta_0)]^2 + 16\,\theta_0(1-\theta_0)}$ $\mathbf{E[Y(5)^2] = [m(\theta_0)]^2 + \sigma^2 = [m(\theta_0)]^2 + 20\,\theta_0(1-\theta_0)}$
L(A(0),Y(i))	$\mathbf{L\big(A(0), Y(1)\big) = K1\ E\{[Y(i)]^2\}}$ $\mathbf{Y(i) = Y(1) = 0.000174 + 4 * \theta_0 * (1-\theta_0), 0 \le i \le 1}$ $\mathbf{L\big(A(0), Y(2,3,4)\big) = K1\ E\{[Y(i)]^2\}}$ $\mathbf{Y(i) = Y(2,3,4) = 0,000174 + 16\,\theta_0(1-\theta_0), 1 \le i \le 4}$ $\mathbf{L\big(A(0), Y(5)\big) = K1\ E\{[Y(i)]^2\}}$ $\mathbf{Y(i) = Y(5) = 0.000174 + 20\,\theta_0(1-\theta_0), 4 \le i}$
G(A(0),Y(i))	$\mathbf{G\big(A(0), Y(1)\big) = K1\ f\big(Y(i)\big)\ E\{[Y(i)]^2\}}$ $\mathbf{Y(i) = Y(1) = 0.000174 + 4\,\theta_0\ (1-\theta_0), 0 \le i \le 1}$ $\mathbf{G\big(A(0), Y(2,3,4)\big) = K1\ f\big(Y(i)\big)\ E\{[Y(i)]^2\}}$ $\mathbf{Y(i) = Y(2,3,4) = 0.000174 + 16\,\theta_0(1-\theta_0), 1 \le i \le 4}$ $\mathbf{G\big(A(0), Y(5)\big) = K1\ f\big(Y(i)\big)\ E\{[Y(i)]^2\}}$ $\mathbf{Y(i) = Y(5) = 0.000174 + 20\,\theta_0(1-\theta_0), 4 \le i}$

Table A 14 Partial Factors inside Financial Risk Integral through Bayesian Analysis in sub-cases $A(1)_1$ and $A(1)_2$ of the Case Scenario A(0), to identify the Financial Stochastic Risk

$E[Y^2]$	$E[Y(1)^2] = [m(\theta_0)]^2 + \sigma^2 = [m(\theta_0)]^2 + 4\,\theta_0(1-\theta_0)$ $E[Y(2,3,4)^2] = [m(\theta_0)]^2 + \sigma^2 = [m(\theta_0)]^2 + 16\,\theta_0(1-\theta_0)$ $E[Y(5)^2] = [m(\theta_0)]^2 + \sigma^2 = [m(\theta_0)]^2 + 20\,\theta_0(1-\theta_0)$
$L(A(1)_{1,2}, Y(i))_{Total}$	$L(A(1)_{1,2}, Y(1)) = K1\,E\{[Y(1)]^2\},$ $Y(i) = Y(1) = 0.000174 + 4\,\theta_0\,(1-\theta_0), 0 \le i \le 1$ $L(A(1)_{1,2}, Y(2,3,4)) = K1\,E\{[Y(2,3,4)]^2\}$ $Y(i) = Y(2,3,4) = 0.000174 + 16\,\theta_0\,(1-\theta_0), 1 < i \le 4$ $L(A(1)_{1,2}, Y(5)) = K1\,E\{[Y(5)]^2\}$ $Y(i) = Y(5) = 0.000174 + 20\,\theta_0\,(1-\theta_0), 4 < i$
$E[G(A(1)_{1,2}, Y(i))]$	$G(A(1)_{1,2}, Y(1)) = K1\,f(Y(i))\,E\{[Y(i)]^2\}$ $E[Y(i)] = Y(1) = 0.000174 + 4\,\theta_0\,(1-\theta_0), 0 \le i \le 1$ $G(A(1)_{1,2}, Y(2,3,4)) = K1\,f(Y(i))\,E\{Y(\iota)\}^2$ $E[Y(i)] = Y(2,3,4) = 0.000174 + 16\,\theta_0(1-\theta_0), 1 < i \le 4$ $G(A(1)_{1,2}, Y(5)) = K1\,f(Y(i))\,E\{[Y(i)]^2\}$ $E[Y(i)] = Y(5) = 0.000174 + 20\,\theta_0(1-\theta_0), 4 < i$

Table A 15 Partial Factors inside Financial Risk Integral through Bayesian Analysis in sub-cases $A(1)_3$ to $A(1)_5$ of the Case Scenario A(1), to identify the Financial Stochastic Risk

E[Y]	$\mathbf{E[Y(1)] = 4\theta_0}$ $\mathbf{E[Y(2,3,\dots n)] = (8,12,..n)\theta_0}$
$L(A(1)_{3,4,5}, Y(i))_{Total}$	$\mathbf{L(A(1)_{3,4,5}, Y(1)) = K1\ E[Y(1)],\quad 0 \le i \le 1}$ $\mathbf{L(A(1)_{3,4,5}, Y(2,3,4,\dots n)) = K1\ E[Y(2,3,..n)],\quad 1 \le i \le n}$
$E[G(A(1)_{3,4,5}, Y(i))]$	$\mathbf{G(A(1)_{3,4,5}, Y(1)) = K1\ f(Y(1))E\{[Y(1)]\},\quad 0 \le i \le 1}$ $\mathbf{G(A(1)_{3,4,5}, Y(2,3,4,\dots n)) = K1\ f(Y(2,3,..n))\ E\{[Y(2,3,..n)]\}, 1 \le i \le n}$

Table A 16 Comparative Evaluation of the Financial Risk Indicator – LTP for all sub-cases of the Case Scenario A(1) to the opposite Case Scenario A(0)

Financial Risk Indicator -LTP										
Years	LTP									
(i)	A(0) - $A(1)_1$	$A(1)_1$ - A(0)	A(0) - $A(1)_2$	$A(1)_2$ - A(0)	A(0) - $A(1)_3$	$A(1)_3$ - A(0)	A(0) - $A(1)_4$	$A(1)_4$ - A(0)	A(0) - $A(1)_5$	$A(1)_5$ - A(0)
1	0.001	1637.389	0.001	1637.389	0.001	1623.239	0.001	1623.239	0.000	2080.452
2	0.222	4.509	0.401	2.496	0.225	4.435	0.225	4.436	0.195	5.132
3	0.306	3.263	0.801	1.249	0.312	3.200	0.312	3.200	0.282	3.549
4	0.351	2.848	1.201	0.833	0.359	2.788	0.359	2.788	0.331	3.021
5	4.530	0.221	19.139	0.052	4.631	0.216	4.631	0.216	4.338	0.231
6	7.370	0.136	37.078	0.027	7.539	0.133	7.539	0.133	7.134	0.140
7	9.425	0.106	55.017	0.018	9.646	0.104	9.646	0.104	9.196	0.109
8	10.981	0.091	72.956	0.014	11.243	0.089	11.243	0.089	10.779	0.093
9	12.201	0.082	90.894	0.011	12.495	0.080	12.495	0.080	12.033	0.083
10	13.182	0.076	108.833	0.009	13.503	0.074	13.503	0.074	13.051	0.077
11	13.989	0.071	126.772	0.008	14.332	0.070	14.332	0.070	13.894	0.072
12	14.664	0.068	144.711	0.007	15.026	0.067	15.026	0.067	14.603	0.068
13	15.237	0.066	162.649	0.006	15.615	0.064	15.615	0.064	15.208	0.066
14	15.729	0.064	180.588	0.006	16.122	0.062	16.122	0.062	15.730	0.064
15	16.157	0.062	198.527	0.005	16.562	0.060	16.562	0.060	16.185	0.062
16	16.532	0.060	216.466	0.005	16.948	0.059	16.948	0.059	16.586	0.060
17	16.864	0.059	234.404	0.004	17.290	0.058	17.289	0.058	16.941	0.059
18	17.159	0.058	252.343	0.004	17.594	0.057	17.593	0.057	17.258	0.058
19	17.424	0.057	270.282	0.004	17.866	0.056	17.866	0.056	17.542	0.057
20	17.662	0.057	288.221	0.003	18.111	0.055	18.111	0.055	17.799	0.056
21	17.878	0.056	306.159	0.003	18.334	0.055	18.334	0.055	18.032	0.055
22	18.075	0.055	324.098	0.003	18.536	0.054	18.536	0.054	18.245	0.055
23	18.255	0.055	342.037	0.003	18.721	0.053	18.721	0.053	18.440	0.054
24	18.420	0.054	359.975	0.003	18.891	0.053	18.891	0.053	18.618	0.054
25	18.571	0.054	377.914	0.003	19.047	0.053	19.047	0.053	18.783	0.053
26	18.711	0.053	395.853	0.003	19.192	0.052	19.192	0.052	18.936	0.053
27	18.841	0.053	413.792	0.002	19.326	0.052	19.326	0.052	19.077	0.052
28	18.962	0.053	431.730	0.002	19.450	0.051	19.450	0.051	19.208	0.052
29	19.075	0.052	449.669	0.002	19.566	0.051	19.566	0.051	19.331	0.052
30	19.180	0.052	467.608	0.002	19.674	0.051	19.674	0.051	19.446	0.051

Table A 17 Comparative Evaluation of the Financial Risk Indicator – ERR for all sub-cases of the Case Scenario A(1) to the opposite Case Scenario A(0)

	Financial Risk Indicator - ERR									
Years	ERR									
(i)	A(0) - $A(1)_1$	$A(1)_1$ - A(0)	A(0) - $A(1)_2$	$A(1)_2$ - A(0)	A(0) - $A(1)_3$	$A(1)_3$ - A(0)	A(0) - $A(1)_4$	$A(1)_4$ - A(0)	A(0) - $A(1)_5$	$A(1)_5$ - A(0)
1	0.999	-1636.389	0.999	-1636.389	0.999	-1622.239	0.999	-1622.239	1.000	-2079.452
2	0.778	-3.509	0.599	-1.496	0.775	-3.435	0.775	-3.436	0.805	-4.132
3	0.694	-2.263	0.199	-0.249	0.688	-2.200	0.688	-2.200	0.718	-2.549
4	0.649	-1.848	-0.201	0.167	0.641	-1.788	0.641	-1.788	0.669	-2.021
5	-3.530	0.779	-18.139	0.948	-3.631	0.784	-3.631	0.784	-3.338	0.769
6	-6.370	0.864	-36.078	0.973	-6.539	0.867	-6.539	0.867	-6.134	0.860
7	-8.425	0.894	-54.017	0.982	-8.646	0.896	-8.646	0.896	-8.196	0.891
8	-9.981	0.909	-71.956	0.986	-10.243	0.911	-10.243	0.911	-9.779	0.907
9	-11.201	0.918	-89.894	0.989	-11.495	0.920	-11.495	0.920	-11.033	0.917
10	-12.182	0.924	-107.833	0.991	-12.503	0.926	-12.503	0.926	-12.051	0.923
11	-12.989	0.929	-125.772	0.992	-13.332	0.930	-13.332	0.930	-12.894	0.928
12	-13.664	0.932	-143.711	0.993	-14.026	0.933	-14.026	0.933	-13.603	0.932
13	-14.237	0.934	-161.649	0.994	-14.615	0.936	-14.615	0.936	-14.208	0.934
14	-14.729	0.936	-179.588	0.994	-15.122	0.938	-15.122	0.938	-14.730	0.936
15	-15.157	0.938	-197.527	0.995	-15.562	0.940	-15.562	0.940	-15.185	0.938
16	-15.532	0.940	-215.466	0.995	-15.948	0.941	-15.948	0.941	-15.586	0.940
17	-15.864	0.941	-233.404	0.996	-16.290	0.942	-16.289	0.942	-15.941	0.941
18	-16.159	0.942	-251.343	0.996	-16.594	0.943	-16.593	0.943	-16.258	0.942
19	-16.424	0.943	-269.282	0.996	-16.866	0.944	-16.866	0.944	-16.542	0.943
20	-16.662	0.943	-287.221	0.997	-17.111	0.945	-17.111	0.945	-16.799	0.944
21	-16.878	0.944	-305.159	0.997	-17.334	0.945	-17.334	0.945	-17.032	0.945
22	-17.075	0.945	-323.098	0.997	-17.536	0.946	-17.536	0.946	-17.245	0.945
23	-17.255	0.945	-341.037	0.997	-17.721	0.947	-17.721	0.947	-17.440	0.946
24	-17.420	0.946	-358.975	0.997	-17.891	0.947	-17.891	0.947	-17.618	0.946
25	-17.571	0.946	-376.914	0.997	-18.047	0.947	-18.047	0.947	-17.783	0.947
26	-17.711	0.947	-394.853	0.997	-18.192	0.948	-18.192	0.948	-17.936	0.947
27	-17.841	0.947	-412.792	0.998	-18.326	0.948	-18.326	0.948	-18.077	0.948
28	-17.962	0.947	-430.730	0.998	-18.450	0.949	-18.450	0.949	-18.208	0.948
29	-18.075	0.948	-448.669	0.998	-18.566	0.949	-18.566	0.949	-18.331	0.948
30	-18.180	0.948	-466.608	0.998	-18.674	0.949	-18.674	0.949	-18.446	0.949

Table A 18 Comparative Evaluation of the Financial Risk Indicators CBA and IRR for all sub-cases of the Case Scenario A(1) to the opposite Case Scenario A(0)

Financial Risk Indicator – CBA (B/C)										
Years	CBA									
(i)	$A(0) - A(1)_1$	$A(1)_1 - A(0)$	$A(0) - A(1)_2$	$A(1)_2 - A(0)$	$A(0) - A(1)_3$	$A(1)_3 - A(0)$	$A(0) - A(1)_4$	$A(1)_4 - A(0)$	$A(0) - A(1)_5$	$A(1)_5 - A(0)$
1	817.695	0.999	817.695	0.999	810.619	0.999	810.619	0.999	1039.226	0.999
2	1.254	0.556	0.248	0.110	1.218	0.549	1.218	0.549	1.566	0.610
3	0.632	0.387	0.375	0.231	0.600	0.375	0.600	0.375	0.775	0.436
4	0.424	0.298	0.584	0.411	0.394	0.283	0.394	0.283	0.510	0.338
5	0.890	8.060	0.974	8.883	0.892	8.262	0.892	8.262	0.885	7.676
6	0.932	13.739	0.987	14.683	0.934	14.078	0.934	14.077	0.930	13.268
7	0.947	17.850	0.991	18.904	0.948	18.292	0.948	18.291	0.946	17.391
8	0.954	20.963	0.993	22.114	0.956	21.486	0.956	21.485	0.954	20.558
9	0.959	23.402	0.994	24.636	0.960	23.990	0.960	23.989	0.958	23.066
10	0.962	25.365	0.995	26.671	0.963	26.006	0.963	26.006	0.962	25.102
11	0.964	26.978	0.996	28.347	0.965	27.664	0.965	27.664	0.964	26.787
12	0.966	28.328	0.997	29.751	0.967	29.052	0.967	29.052	0.966	28.205
13	0.967	29.474	0.997	30.945	0.968	30.231	0.968	30.230	0.967	29.415
14	0.968	30.459	0.997	31.972	0.969	31.244	0.969	31.243	0.968	30.460
15	0.969	31.315	0.997	32.866	0.970	32.124	0.970	32.124	0.969	31.370
16	0.970	32.065	0.998	33.650	0.970	32.896	0.970	32.896	0.970	32.171
17	0.970	32.728	0.998	34.343	0.971	33.579	0.971	33.579	0.970	32.881
18	0.971	33.319	0.998	34.962	0.972	34.187	0.972	34.187	0.971	33.515
19	0.971	33.848	0.998	35.516	0.972	34.732	0.972	34.732	0.971	34.084
20	0.972	34.325	0.998	36.015	0.972	35.223	0.972	35.223	0.972	34.598
21	0.972	34.757	0.998	36.468	0.973	35.668	0.973	35.667	0.972	35.065
22	0.972	35.150	0.998	36.881	0.973	36.073	0.973	36.072	0.973	35.490
23	0.973	35.509	0.999	37.258	0.973	36.443	0.973	36.442	0.973	35.879
24	0.973	35.839	0.999	37.604	0.974	36.782	0.974	36.782	0.973	36.237
25	0.973	36.143	0.999	37.922	0.974	37.095	0.974	37.095	0.973	36.566
26	0.973	36.423	0.999	38.217	0.974	37.384	0.974	37.384	0.974	36.871
27	0.973	36.683	0.999	38.490	0.974	37.651	0.974	37.651	0.974	37.154
28	0.974	36.924	0.999	38.744	0.974	37.900	0.974	37.900	0.974	37.417
29	0.974	37.149	0.999	38.980	0.974	38.132	0.974	38.132	0.974	37.662
30	0.974	37.359	0.999	39.201	0.975	38.348	0.975	38.348	0.974	37.891

Internal Rate of Return Indicator										
Years	IRR %									
(i)	Differential Investment € $A(1)_1$ - A(0)	IRR $A(1)_1$-A(0)	Differential Investment € $A(1)_2$ - A(0)	IRR $A(1)_2$-A(0)	Differential Investment € $A(1)_3$ - A(0)	IRR $A(1)_3$-A(0)	Differential Investment € $A(1)_4$ - A(0)	IRR $A(1)_4$-A(0)	Differential Investment € A(0) - $A(1)_5$	IRR $A(1)_5$-A(0)
1	-7,570,651		-7,570,651		-7,505,146		-7,505,146		-7,412,702	
2	-47,316		-29,062		110,524		109,459		110,633	
3	-47,316		-29,062		110,634		110,633		110,633	
4	-47,316		-29,062		110,634		110,634		110,633	
5	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
6	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
7	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
8	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
9	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
10	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
11	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
12	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
13	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
14	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
15	265,837,059	173	265,995,009	173	265,995,009	174	265,995,009	174	266,987,202	175
16	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
17	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
18	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
19	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
20	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
21	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
22	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
23	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
24	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
25	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
26	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
27	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
28	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
29	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	
30	265,837,059		265,995,009		265,995,009		265,995,009		266,987,202	

Table A 19 Stochastic Risk Assessment Results. Indicators $R(A(0)$ or $A(1)_{1\text{-}5},Y(i))$, $R(A(0)$ or $A(1)_{1\text{-}5},\theta(0))$, and Stochastic Risk Matrix of the Comparable CBA Risk Ratios for all examined Cases describing Degree of Risk

Grade of the Stochastic Risk Assessment based on the Corresponding Total Goal Function	Case Scenario A(0)	Case Scenario sub-case $A(1)_1$	Case Scenario sub-case $A(1)_2$	Case Scenario sub-case $A(1)_3$	Case Scenario sub-case $A(1)_4$	Case Scenario sub-case $A(1)_5$
Period	$R(A(0),\theta_{(0)})$	$R(A(1)_1,\theta_{(0)})$	$R(A(1)_2,\theta_{(0)})$	$R(A(1)_3,\theta_{(0)})$	$R(A(1)_4,\theta_{(0)})$	$R(A(1)_5,\theta_{(0)})$
1st Class (1st Year)	5		0.043		0.54	0.54
2nd Class (2nd – 4th Year)	6	0.043	0.129	0.54	1.6	0.011
3rd Class (over the 5th Year)	12		0.121		1.8	
$\sum R(\theta_{(0)})$	**23**	**0.043**	**0.29**	**0.54**	**3.94**	**0.551**
Total Grade of the Stochastic Financial Risk Assessment (€)	**Case Scenario A(0)**	**Case Scenario sub-case $A(1)_1$**	**Case Scenario sub-case $A(1)_2$**	**Case Scenario sub-case $A(1)_3$**	**Case Scenario sub-case $A(1)_4$**	**Case Scenario sub-case $A(1)_5$**
Period	$R(A(0),Y(i))$	$R(A(1)_1,Y(i))$	$R(A(1)_2,Y(i))$	$R(A(1)_3,Y(i))$	$R(A(1)_4,Y(i))$	$R(A(1)_5,Y(i))$
1st Class (1st Year)	1,872,000		335,243		4,210,034	5,200,722
2nd Class (2nd – 4th Year)	2,246,400	335,243	816,255	4,210,034	10,124,096	89,787
3rd Class (over the 5th Year)	4,492,800		765,635		11,389,608	
$\sum R(A(0)$ or $A(1)_{1\text{-}5}, Y(i))$	**8,611,200**	**335,243**	**1,917,133**	**4,210,034**	**25,723,738**	**5,290,509**
CBA Risk Ratios $[A(0)-A(1)_{1\text{-}5}]$ and $[A(1)_{1\text{-}5}-A(0)]$ of the Total Stochastic Financial Risk Assessment	**Case Scenario A(0)**	**Case Scenario sub-case $A(1)_1$**	**Case Scenario sub-case $A(1)_2$**	**Case Scenario sub-case $A(1)_3$**	**Case Scenario sub-case $A(1)_4$**	**Case Scenario sub-case $A(1)_5$**
Case Scenario A(0)	$\frac{\sum R(A(0),Y(i))}{\sum R(A(0),Y(i))}=1$	$\frac{\sum R(A(1)_1,Y(i))}{\sum R(A(0),Y(i))}=0.039$	$\frac{\sum R(A(1)_2,Y(i))}{\sum R(A(0),Y(i))}=0.223$	$\frac{\sum R(A(1)_3,Y(i))}{\sum R(A(0),Y(i))}=0.489$	$\frac{\sum R(A(1)_4,Y(i))}{\sum R(A(0),Y(i))}=2.987$	$\frac{\sum R(A(1)_5,Y(i))}{\sum R(A(0),Y(i))}=0.614$
Case Scenario A(1) sub-case $A(1)_1$	$\frac{\sum R(A(0),Y(i))}{\sum R(A(1)_1,Y(i))}=25.686$	$\frac{\sum R(A(1)_1,Y(i))}{\sum R(A(1)_1,Y(i))}=1$	$\frac{\sum R(A(1)_2,Y(i))}{\sum R(A(1)_1,Y(i))}=5.719$	$\frac{\sum R(A(1)_3,Y(i))}{\sum R(A(1)_1,Y(i))}=12.558$	$\frac{\sum R(A(1)_4,Y(i))}{\sum R(A(1)_1,Y(i))}=76.732$	$\frac{\sum R(A(1)_5,Y(i))}{\sum R(A(1)_1,Y(i))}=15.781$
Case Scenario A(1) sub-case $A(1)_2$	$\frac{\sum R(A(0),Y(i))}{\sum R(A(1)_2,Y(i))}=4.492$	$\frac{\sum R(A(1)_1,Y(i))}{\sum R(A(1)_2,Y(i))}=0.175$	$\frac{\sum R(A(1)_2,Y(i))}{\sum R(A(1)_2,Y(i))}=1$	$\frac{\sum R(A(1)_3,Y(i))}{\sum R(A(1)_2,Y(i))}=2.196$	$\frac{\sum R(A(1)_4,Y(i))}{\sum R(A(1)_2,Y(i))}=13.418$	$\frac{\sum R(A(1)_5,Y(i))}{\sum R(A(1)_2,Y(i))}=2.760$
Case Scenario A(1) sub-case $A(1)_3$	$\frac{\sum R(A(0),Y(i))}{\sum R(A(1)_3,Y(i))}=2.045$	$\frac{\sum R(A(1)_1,Y(i))}{\sum R(A(1)_3,Y(i))}=0.08$	$\frac{\sum R(A(1)_2,Y(i))}{\sum R(A(1)_3,Y(i))}=0.455$	$\frac{\sum R(A(1)_3,Y(i))}{\sum R(A(1)_3,Y(i))}=1$	$\frac{\sum R(A(1)_4,Y(i))}{\sum R(A(1)_3,Y(i))}=6.110$	$\frac{\sum R(A(1)_5,Y(i))}{\sum R(A(1)_3,Y(i))}=1.257$
Case Scenario A(1) sub-case $A(1)_4$	$\frac{\sum R(A(0),Y(i))}{\sum R(A(1)_4,Y(i))}=0.335$	$\frac{\sum R(A(1)_1,Y(i))}{\sum R(A(1)_4,Y(i))}=0.013$	$\frac{\sum R(A(1)_2,Y(i))}{\sum R(A(1)_4,Y(i))}=0.075$	$\frac{\sum R(A(1)_3,Y(i))}{\sum R(A(1)_4,Y(i))}=0.164$	$\frac{\sum R(A(1)_4,Y(i))}{\sum R(A(1)_4,Y(i))}=1$	$\frac{\sum R(A(1)_5,Y(i))}{\sum R(A(1)_4,Y(i))}=0.206$
Case Scenario A(1) sub-case $A(1)_5$	$\frac{\sum R(A(0),Y(i))}{\sum R(A(1)_5,Y(i))}=1.628$	$\frac{\sum R(A(1)_1,Y(i))}{\sum R(A(1)_5,Y(i))}=0.063$	$\frac{\sum R(A(1)_2,Y(i))}{\sum R(A(1)_5,Y(i))}=0.362$	$\frac{\sum R(A(1)_3,Y(i))}{\sum R(A(1)_5,Y(i))}=0.796$	$\frac{\sum R(A(1)_4,Y(i))}{\sum R(A(1)_5,Y(i))}=4.862$	$\frac{\sum R(A(1)_5,Y(i))}{\sum R(A(1)_5,Y(i))}=1$

Table A 20 Grades of Risks of Case A(0) and sub-cases $A(1)_{1-5}$ of the Case Scenario A(1), calculated by CBA and SRA

CBA Approach-Grades of Financial Risks						SRA Approach-Grades of Risk based on the corresponding Theoretical Loss Functions					
A(0)	$A(1)_1$	$A(1)_2$	$A(1)_3$	$A(1)_4$	$A(1)_5$	A(0)	$A(1)_1$	$A(1)_2$	$A(1)_3$	$A(1)_4$	$A(1)_5$
$\frac{G(A(0),Y(30))_{Total}}{G(A(0),Y(5))_{Total}}$	$\frac{G(A(1)_1,Y(30))_{Total}}{G(A(1)_1,Y(5))_{Total}}$	$\frac{G(A(1)_2,Y(30))_{Total}}{G(A(1)_2,Y(5))_{Total}}$	$\frac{G(A(1)_3,Y(30))_{Total}}{G(A(1)_3,Y(5))_{Total}}$	$\frac{G(A(1)_4,Y(30))_{Total}}{G(A(1)_4,Y(5))_{Total}}$	$\frac{G(A(1)_5,Y(30))_{Total}}{G(A(1)_5,Y(5))_{Total}}$	$\frac{\sum R(A(0),Y(i))}{L(A(0),Y(1))}$	$\frac{\sum R(A(1)_1,Y(i))}{L(A(0),Y(1))}$	$\frac{\sum R(A(1)_2,Y(i))}{L(A(0),Y(1))}$	$\frac{\sum R(A(1)_3,Y(i))}{L(A(0),Y(1))}$	$\frac{\sum R(A(1)_4,Y(i))}{L(A(0),Y(1))}$	$\frac{\sum R(A(1)_5,Y(i))}{G(A(1)_5,Y(1))_t}$
24.43	5.77	5.68	5.75	5.75	5.45	23	0.9	5.12	11.2	68.7	5

Deviations % among CBA - SRA					
A(0)	$A(1)_1$	$A(1)_2$	$A(1)_3$	$A(1)_4$	$A(1)_5$
6.21	84.4	9.8	94.7	1094.8	8.2

Table A 21 Calculated Stochastic Risk Grade through Stochastic Risk Integrals for the Case Scenario A(0) using MathCad Software Application_Version_10.0.1.0

Stochastic Risk Assessment Grade	Case Scenario A(0)
Period	$R(A(0),\theta_{(0)})$
1st Class (1st Year)	$\int_{\frac{75}{100}}^{\frac{100}{100}} 4\cdot\left(\left(\frac{1}{300}\right)^{2}+4\,u\cdot(1-u)\right)\,\mathrm{pbinom}(1,1,u)\cdot\mathrm{pbeta}(300,4,u)\,du=5$
2nd Class (2nd – 4th Year)	$\int_{\frac{75}{100}}^{\frac{100}{100}} 16\cdot\left(\left(\frac{1}{300}\right)^{2}+4\,u\cdot(1-u)\right)\,\mathrm{pbinom}(3,4,u)\cdot\mathrm{pbeta}(300,4,u)\,du=6$
3rd Class (> 5th Year)	$\int_{\frac{75}{100}}^{\frac{100}{100}} 20\cdot\left(\left(\frac{1}{300}\right)^{2}+4\,u\cdot(1-u)\right)\,\mathrm{pbinom}(8,5,u)\cdot\mathrm{pbeta}(300,4,u)\,du=12$
$\sum R(\theta_{(0)})$	23

Table A 22 Calculated Stochastic Risk Grade through Stochastic Risk Integral for the sub-case $A(1)_1$ of the Case Scenario A(1) using MathCad Software Application_Version_10.0.1.0

Stochastic Risk Assessment Grade	Case sub-case $A(1)_1$ Scenario
Period	$R(A(1)_1, \theta_{(0)})$
1st Class (>1st Year)	$$\int_{\frac{0}{100}}^{\frac{25}{100}} 4\cdot\left(\left(\frac{1}{300}\right)^2+4\,u\cdot(1-u)\right)\,\mathrm{pbinom}(1,1,u)\cdot\mathrm{pbeta}(300,4,u)\,du=0.0439$$
$\sum R(\theta_{(0)})$	0.0439

Table A 23 Calculated Stochastic Risk Grade through Stochastic Risk Integrals for the sub-case $A(1)_2$ of the Case Scenario A(1) using MathCad Software Application_Version_10.0.1.0

Stochastic Risk Assessment Grade	Case sub-case $A(1)_2$ Scenario
Period	$R(A(1)_2, \theta_{(0)})$
1st Class (1st Year)	$\int_{\frac{0}{100}}^{\frac{25}{100}} 4\cdot\left(\left(\frac{1}{300}\right)^2+4\,u\cdot(1-u)\right)\,\text{pbinom}(1,1,u)\cdot\text{pbeta}(300,4,u)\,du=0.0439$
2nd Class (2nd – 4th Year)	$\int_{\frac{0}{100}}^{\frac{25}{100}} 16\cdot\left(\left(\frac{1}{300}\right)^2+4\,u\cdot(1-u)\right)\,\text{pbinom}(3,4,u)\cdot\text{pbeta}(300,4,u)\,du=0.129$
3rd Class (> 5th Year)	$\int_{\frac{0}{100}}^{\frac{25}{100}} 20\cdot\left(\left(\frac{1}{300}\right)^2+4\,u\cdot(1-u)\right)\,\text{pbinom}(8,5,u)\cdot\text{pbeta}(300,4,u)\,du=0.121$
$\sum R(\theta_{(0)})$	0.293

Table A 24 Calculated Stochastic Risk Grade through Stochastic Risk Integral for the sub-case $A(1)_3$ of the Case Scenario A(1) using MathCad Software Application_Version_10.0.1.0

Stochastic Risk Assessment Grade	Case Scenario sub-case $A(1)_3$
Period	$R(A(1)_3, \theta_{(0)})$
1^{st} Class ($>1^{st}$ Year)	$\int_{\frac{0}{100}}^{\frac{25}{100}} 4 \cdot u \cdot \text{pexp}(1, u)\ \text{pbeta}(300, 4, u)\ du = 0.54$
$\sum R(\theta_{(0)})$	0.54

Table A 25 Calculated Stochastic Risk Grade through Stochastic Risk Integrals for the sub-case $A(1)_4$ of the Case Scenario A(1) using MathCad Software Application_Version_10.0.1.0

Stochastic Risk Assessment Grade	Case sub-case $A(1)_4$ Scenario
Period	$R(A(1)_4, \theta_{(0)})$
1^{st} Class (1^{st} Year)	$\int_{\frac{0}{100}}^{\frac{25}{100}} 4 \cdot u \cdot \mathrm{pexp}(1, u)\ \mathrm{pbeta}(300, 4, u)\, du = 0.54$
2^{nd} Class (2^{nd} – 4^{th} Year)	$\int_{\frac{0}{100}}^{\frac{25}{100}} 16 \cdot u \cdot \mathrm{pexp}(4, u)\ \mathrm{pbeta}(300, 4, u)\, du = 1.6$
3^{rd} Class (> 5^{th} Year)	$\int_{\frac{0}{100}}^{\frac{25}{100}} 20 \cdot u \cdot \mathrm{pexp}(5, u)\ \mathrm{pbeta}(300, 4, u)\, du = 1.8$
$\sum R(\theta_{(0)})$	3.94

Table A 26 Calculated Stochastic Risk Grade through Stochastic Risk Integrals for the sub-case $A(1)_5$ of the Case Scenario A(1) using MathCad Software Application_Version_10.0.1.0

Stochastic Risk Assessment Grade	Case sub-case $A(1)_5$ Scenario
Period	$R(A(1)_5, \theta_{(0)})$
1st Class (1st Year)	$\int_{\frac{0}{100}}^{\frac{25}{100}} 4 \cdot u \cdot pexp(1,u)\ pbeta(300,4,u)\, du = 0.54$
2nd Class (>2nd Year)	$\int_{\frac{0}{100}}^{\frac{25}{100}} 4 \cdot u \cdot pexp(1,u)\ pbeta(300,4,u)\, du = 0.019$
$\sum R(\theta_{(0)})$	0.559