Event-Triggered Control of Freeway Traffic Flow with Connected and Automated Vehicles

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Abstract:
We propose an event-triggered control (ETC) strategy for the Aw-Rascle-Zhang (ARZ) traffic model under congested conditions. The considered ARZ-type model, governed by first-order hyperbolic partial differential equations (PDEs), captures traffic dynamics involving both Adaptive Cruise Control (ACC) and human-driven vehicles. Control actions adjust the time gap for ACC vehicles and are updated based on a suitable triggering rule. We conduct the stability analysis on a linearized and transformed system (a 2×2 linear hyperbolic system with in-domain control), and we asses input-to-state stability (ISS) with respect to actuation errors. A small-gain approach guides the design of appropriate triggering rule (small-gain event based triggering condition), ensuring exponential stability while preventing the Zeno phenomenon. Numerical simulations demonstrate the effectiveness of the proposed control strategy in stabilizing traffic flow.

Keywords: Event-triggered control, in-domain control, ARZ traffic model, hyperbolic PDEs, input-to-state stability, small-gain condition.

1. INTRODUCTION

Over recent decades, controlling highway traffic flow has been a major research focus, leading to significant advances in dynamic freeway modeling Papageorgiou (1980), integrated traffic corridor control Papageorgiou (1995), and conservation law-based approaches for traffic systems Bayen et al. (2022). Among various traffic flow models, including the Payne-Whitham (PW) model Payne (1971), the Lighthill–Whitham–Richards (LWR) model Whitham (1974), the ARZ framework, which combines the Aw-Rascle model Aw and Rascle (2000) with Zhang's non-equilibrium dynamics Zhang (2002), has gained particular attention due to its ability to capture the complex interplay between vehicle density and velocity Yu and Krstic (2019). Strategies such as coordinated ramp metering Goatin et al. (2016) and downstream boundary control Zhang et al. (2024) have been proposed to mitigate stop-and-go traffic oscillations and improve traffic stability.

The integration of ACC Yu and Wang (2022) and connected/automated vehicles Li et al. (2023) introduces additional challenges in mixed traffic environments involving human-driven and ACC-equipped vehicles. Recent efforts, such as delay-compensated traffic control for connected/automated vehicles Qi et al. (2023) and mitigating phantom traffic jams in hybrid systems Molnar and Orosz (2024), have shown promise, but traditional methods are often constrained by high communication and computational demands, limiting their practicality in complex traffic scenarios. ETC strategy has emerged as an efficient solution by reducing control update frequency while maintaining system stability Tabuada (2007) and has proved to be relevant in PDE control for both *1D parabolic PDEs* (Selivanov and Fridman (2016); Espitia et al. (2021); Rathnayake and Diagne (2024); Koudohode et al. (2024); Rathnayake et al. (2024)) and *1D Hyperbolic PDEs* (Wang and Krstic (2021); Somathilake and Rathnayake (2024)), with potential applications to traffic congestion control Espitia et al. (2020); Zhang et al. (2024).

Building upon Bekiaris-Liberis and Delis (2021), which propose an ARZ-type traffic flow model integrating both human-driven and ACC-equipped vehicles, along with an in-domain control approach, we extend this framework by introducing a ETC scheme to stabilize traffic flow governed by a mixed-traffic ARZ model, while updating control actions – adjusting the time gap for ACC vehicles – based on a suitable small-gain based triggering rule. We rely on Input-to-state stability (ISS) and small-gain arguments Karafyllis and Krstic (2018); Espitia et al. (2021) to assess the stability of the closed-loop system. Indeed, we employ ISS estimates with respect to actuation deviation, a small-gain condition that drives the selection of the triggering parameters, and small-gain arguments to obtain estimates of the closed-loop system. We state the avoidance of the

Zeno phenomenon, that allows to conclude the exponential stability result.

The paper is organized as follows: Section 2 introduces the ARZ model and problem formulation. Section 3 presents the emulation of the nominal control towards an event-triggered control scheme. Section 4 introduces the small-gain based event-triggered control strategy and the main results. Section 5 illustrates the results with numerical simulations, and Section 6 concludes with future research directions.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

The ARZ model is a widely used traffic flow model based on PDEs, capable of describing dynamic changes in traffic density and velocity. It captures critical phenomena such as "stop-and-go" waves and sudden traffic slowdowns, making it particularly effective for analyzing and controlling congestion.

2.1 ARZ type Traffic Model for Mixed Traffic

The ARZ model can be extended to describe mixed traffic environments containing both human-driven and ACC-equipped vehicles. In this scenario, a mixed speed function V_{mix} is introduced to account for the combined behavior of both types of vehicles, where the control variable h_{acc} represents the time gap for ACC vehicles, calculated as the ratio of spacing to velocity. The modified ARZ model, adapted from Bekiaris-Liberis and Delis (2021), is given as

$$\rho_t(t,\bar{x}) = -\rho_{\bar{x}}(t,\bar{x})v(t,\bar{x}) - \rho(t,\bar{x})v_{\bar{x}}(t,\bar{x}), \tag{1}$$

$$\begin{aligned} v_t(t,\bar{x}) =& -\rho(t,\bar{x})\frac{\partial V_{\text{mix}}\left(\rho(t,\bar{x}),h_{\text{acc}}(t,\bar{x})\right)}{\partial\rho}v_{\bar{x}}(t,\bar{x}) \\ &- v(t,\bar{x})v_{\bar{x}}(t,\bar{x}) \\ &+ \frac{V_{\text{mix}}\left(\rho(t,\bar{x}),h_{\text{acc}}(t,\bar{x})\right) - v(t,\bar{x})}{\tau_{\text{mix}}}, \end{aligned} \tag{2}$$

$$\rho(t,0) = q_{\text{in}}/v(t,0), \tag{3}$$

$$v_t(t,L) = \frac{V_{\text{mix}}\left(\rho(t,L),h_{\text{acc}}(t,L)\right) - v(t,L)}{\tau_{\text{mix}}}. \tag{4}$$

Here, $\rho(t,\bar{x})$ represents traffic density, $v(t,\bar{x})$ represents traffic velocity where $(t,\bar{x}) \in \mathbb{R}^+ \times [0,L]$, q_{in} is the external inflow. The parameters are listed in Table 1. The mixed speed V_{mix} and effective time-gap h_{mix} for the mixed traffic are expressed as $V_{\text{mix}}(\rho, h_{\text{acc}}) = \frac{1}{h_{\text{mix}}(h_{\text{acc}})}\left(\frac{1}{\rho} - l\right)$, $h_{\text{mix}}(h_{\text{acc}}) = \frac{\alpha + (1-\alpha)\frac{\tau_{\text{acc}}}{\tau_{\text{m}}}}{\alpha + (1-\alpha)\frac{\tau_{\text{acc}}}{\tau_{\text{m}}}\frac{h_{\text{acc}}}{h_{\text{m}}}}h_{\text{acc}}$.

2.2 Linearization and Diagonalization

The steady-state equilibria of the system (1)-(4) are determined by a constant inflow rate q_{in} and a fixed steady-state time gap for ACC vehicles, $\bar{h}_{\text{acc}}$. This results in a steady-state mixed time gap, given by $\bar{h}_{\text{mix}} = \frac{\alpha + (1-\alpha)\frac{\tau_{\text{acc}}}{\tau_{\text{m}}}}{\alpha + (1-\alpha)\frac{\tau_{\text{acc}}}{\tau_{\text{m}}}\frac{\bar{h}_{\text{acc}}}{h_{\text{m}}}}\bar{h}_{\text{acc}}$. To simplify the analysis, the system is linearized around a steady-state ($\bar{\rho}$, $\bar{v}$) that satisfies the conditions $\bar{v} = \frac{q_{\text{in}}}{\bar{\rho}}, \quad \frac{1}{\bar{\rho}} - l = \bar{h}_{\text{mix}}\bar{v}$.

The error variables are defined as $\tilde{\rho}(t,\bar{x}) = \rho(t,\bar{x}) - \bar{\rho}, \quad \tilde{v}(t,\bar{x}) = v(t,\bar{x}) - \bar{v}$.

Table 1. System parameters

Parameter	Description
$\rho_{\min}$	Minimum traffic density
L	Length of highway segment
l	Average vehicle length
q_{in}	External inflow rate
τ_{mix}	Time constant for mixed traffic
τ_{acc}	Time constant for ACC vehicles
τ_{m}	Time constant for manual vehicles
α	Proportion of ACC vehicles
$\bar{h}_{\text{acc}}$	Steady-state time gap for ACC vehicles
$\bar{h}_{\text{mix}}$	Steady-state time gap for mixed vehicles
h_{m}	Time gap for manual vehicles
$h_{\text{acc}}(\bar{x},t)$	Distributed time gap for ACC vehicles

The linearized system of (1)-(4) becomes

$$\tilde{\rho}_t(t,\bar{x}) + \bar{v}\tilde{\rho}_{\bar{x}}(t,\bar{x}) = -\bar{\rho}\tilde{v}_{\bar{x}}(t,\bar{x}), \tag{5}$$

$$\begin{aligned} \tilde{v}_t(t,\bar{x}) - \tfrac{l}{\bar{h}_{\text{mix}}}\tilde{v}_{\bar{x}}(t,\bar{x}) =& -\tfrac{1}{\bar{\rho}^2\tau_{\text{mix}}\bar{h}_{\text{mix}}}\tilde{\rho}(t,\bar{x}) - \tfrac{1}{\tau_{\text{mix}}}\tilde{v}(t,\bar{x}) \\ &- \tfrac{\alpha}{\tau_{\text{acc}}\bar{h}_{\text{acc}}^2}(\tfrac{1}{\bar{\rho}} - l)h_{\text{acc}}(t,\bar{x}), \end{aligned} \tag{6}$$

$$\tilde{\rho}(t,0) = -\tfrac{\bar{\rho}}{\bar{v}}\tilde{v}(t,0), \tag{7}$$

$$\begin{aligned} \tilde{v}_t(t,L) =& -\tfrac{1}{\bar{\rho}^2\tau_{\text{mix}}\bar{h}_{\text{mix}}}\tilde{\rho}(t,L) - \tfrac{1}{\tau_{\text{mix}}}\tilde{v}(t,L) \\ &- \tfrac{\alpha}{\tau_{\text{acc}}\bar{h}_{\text{acc}}^2}(\tfrac{1}{\bar{\rho}} - l)h_{\text{acc}}(t,L). \end{aligned} \tag{8}$$

Using the following coordinate transformation

$$z(t,x) = \frac{\bar{v}}{\bar{\rho}}\left(\tilde{\rho}(t,Lx) + \bar{h}_{\text{mix}}\bar{\rho}^2\tilde{v}(t,Lx)\right), v(t,x) = \tilde{v}(t,Lx),$$

where $x \in [0,1]$, the system is diagonalized from (5)-(8)

$$\begin{aligned} z_t(t,x) + \tfrac{\bar{v}}{L}z_x(t,x) =& -\tfrac{1}{\tau_{\text{mix}}}z(t,x) \\ &- \bar{h}_{\text{mix}}\bar{\rho}\bar{v}\tfrac{\alpha}{\tau_{\text{acc}}\bar{h}_{\text{acc}}^2}(\tfrac{1}{\bar{\rho}} - l)U(t,x), \end{aligned} \tag{9}$$

$$\begin{aligned} v_t(t,x) - \tfrac{l}{\bar{h}_{\text{mix}}L}v_x(t,x) =& -\tfrac{1}{\bar{\rho}\bar{v}\tau_{\text{mix}}\bar{h}_{\text{mix}}}z(t,x) \\ &- \tfrac{\alpha}{\tau_{\text{acc}}\bar{h}_{\text{acc}}^2}(\tfrac{1}{\bar{\rho}} - l)U(t,x), \end{aligned} \tag{10}$$

$$z(t,0) = -l\bar{\rho}v(t,0), \tag{11}$$

$$\begin{aligned} v_t(t,1) =& -\tfrac{1}{\bar{\rho}\bar{v}\tau_{\text{mix}}\bar{h}_{\text{mix}}}z(t,1) \\ &- \tfrac{\alpha}{\tau_{\text{acc}}\bar{h}_{\text{acc}}^2}(\tfrac{1}{\bar{\rho}} - l)U(t,1), \end{aligned} \tag{12}$$

with initial conditions

$$z(0,x) = \tfrac{\bar{v}}{\bar{\rho}}\left(\tilde{\rho}(0,Lx) + \bar{h}_{\text{mix}}\bar{\rho}^2\tilde{v}(0,Lx)\right), \tag{13}$$

$$v(0,x) = \tilde{v}_0(Lx), \tag{14}$$

where $U(t,x) = h_{\text{acc}}(t,Lx)$, $x \in [0,1]$. Both z and v share the same units. This diagonalized form simplifies the stability and control analysis, setting the stage for event-triggered control design. The parameters are detailed in Table 1.

3. NOMINAL FEEDBACK CONTROL LAW AND EMULATION

3.1 Feedback Control Law (Nominal)

To stabilize system (9)-(12) exponentially, we adopt the feedback control law from Bekiaris-Liberis and Delis (2021)

$$U(t,x) = \tfrac{\tau_{\text{acc}}\bar{h}_{\text{acc}}^2}{\alpha((1/\bar{\rho})-l)}\left(-\tfrac{1}{\bar{\rho}\bar{v}\tau_{\text{mix}}\bar{h}_{\text{mix}}}z(t,x) + \kappa v(t,x)\right), \tag{15}$$

where $\kappa > 0$. This control eliminates the instability-inducing source term in (10), ensuring exponential stability (refer to Proposition 1 in Bekiaris-Liberis and Delis (2021)).

3.2 Emulation of the Control Law

The control law is applied in an event-triggered manner, updating only at discrete time instants t_j, $j \in \mathbb{N}$. Between updates, the control remains constant. The modified control law is

$$U_d(t,x) = \frac{\tau_{\rm acc}\,\bar{h}^2_{\rm acc}}{\alpha((1/\bar{\rho})-l)}\Big(-\frac{1}{\bar{\rho}\bar{v}\tau_{\rm mix}\,\bar{h}_{\rm mix}}z(t_j,x)+\kappa v(t_j,x)\Big), \tag{16}$$

for $x \in [0,1]$ and $t \in [t_j, t_{j+1})$. Then we work with the following closed-loop system for all $t \in [t_j, t_{j+1})$

$$\begin{aligned} z_t(t,x) + \tfrac{\bar{v}}{L}z_x(t,x) &= -\tfrac{1}{\tau_{\rm mix}}z(t,x) \\ &\quad -\bar{h}_{\rm mix}\,\bar{\rho}\bar{v}\tfrac{\alpha}{\tau_{\rm acc}\,\bar{h}^2_{\rm acc}}(\tfrac{1}{\bar{\rho}}-l)U_d(t,x), \quad (17)\\ v_t(t,x) - \tfrac{l}{\bar{h}_{\rm mix}\,L}v_x(t,x) &= -\tfrac{1}{\bar{\rho}\bar{v}\tau_{\rm mix}\,\bar{h}_{\rm mix}}z(t,x) \\ &\quad -\tfrac{\alpha}{\tau_{\rm acc}\,\bar{h}^2_{\rm acc}}(\tfrac{1}{\bar{\rho}}-l)U_d(t,x), \quad (18)\\ z(t,0) &= -l\bar{\rho}v(t,0), \quad (19)\\ v_t(t,1) &= -\tfrac{1}{\bar{\rho}\bar{v}\tau_{\rm mix}\,\bar{h}_{\rm mix}}z(t,1) \\ &\quad -\tfrac{\alpha}{\tau_{\rm acc}\,\bar{h}^2_{\rm acc}}(\tfrac{1}{\bar{\rho}}-l)U_d(t,1), \quad (20) \end{aligned}$$

The control law $U_d(t,x)$ is rewritten as $U_d(t,x) = U(t,x) + d^*(t,x)$ where $d^*(t,x)$ represents the actuation error

$$\begin{aligned} d^*(t,x) = &\frac{\tau_{\rm acc}\,\bar{h}^2_{\rm acc}}{\alpha((1/\bar{\rho})-l)}\Big(-\frac{1}{\bar{\rho}\bar{v}\tau_{\rm mix}\,\bar{h}_{\rm mix}}\left(z(t_j,x)-z(t,x)\right) \\ &+\kappa\left(v(t_j,x)-v(t,x)\right)\Big). \end{aligned} \tag{21}$$

The closed-loop system becomes

$$\begin{aligned} z_t(t,x)+\lambda_1 z_x(t,x) &= -\kappa a v(t,x) - b_1 d^*(t,x), \quad (22)\\ v_t(t,x)-\lambda_2 v_x(t,x) &= -\kappa v(t,x) - b_2 d^*(t,x), \quad (23)\\ z(t,0) &= -r v(t,0), \quad (24)\\ v(t,1) &= \eta(t), \quad (25)\\ \dot{\eta}(t) &= -\kappa\eta(t) - b_2 d^*(t,1), \quad (26) \end{aligned}$$

where $\eta(t) \in \mathbb{R}$ is an auxiliary variable to reformulate the PDE boundary condition at $x = 1$ into a dynamic boundary condition governed by an ODE, other parameters are defined as $\lambda_1 = \frac{\bar{v}}{L}$, $\lambda_2 = \frac{l}{\bar{h}_{\rm mix}\,L}$, $a = \bar{\rho}\bar{v}\bar{h}_{\rm mix}$, $r = l\bar{\rho}$, $b_1 = \bar{h}_{\rm mix}\,\bar{\rho}\bar{v}\frac{\alpha}{\tau_{\rm acc}\,\bar{h}^2_{\rm acc}}(\frac{1}{\bar{\rho}}-l)$, $b_2 = \frac{\alpha}{\tau_{\rm acc}\,\bar{h}^2_{\rm acc}}(\frac{1}{\bar{\rho}}-l)$.

3.3 Well-posedness aspects

The hyperbolic PDE system (22)-(26) is affected by a dynamic boundary condition which is an ODE-solution being absolutely continuous, and by source terms that are reset to zero at triggering time instants. Therefore, as in Prieur et al. (2014) - where the hyperbolic PDE is affected by source terms that are subject to switching - we adopt the notion of solution along the characteristics for all $t \in [t_j, t_{j+1})$; hence concluding that for any initial data $(z(t_j,x), v(t_j,x)) \in (L^\infty(0,1))^2$, the closed-loop system has a unique solution $(z,v) \in \mathcal{C}^0\left([t_j,t_{j+1}];(L^\infty(0,1))^2\right)$. Then by the step-by-step method, we can construct the solution for all $t \in [0, \lim_{j\to\infty}(t_j))$.

4. EVENT-TRIGGERED CONTROL STRATEGY AND MAIN RESULTS

This section outlines the event-triggered in-domain control strategy and presents the main results, including the existence of a positive lower bound for the time interval between consecutive updates and the exponential stability of the closed-loop system under event-triggered control.

Definition 1. *Let $\beta_1, \beta_2 > 0$ be design parameters. The event-triggered control scheme includes*

1. Triggering Mechanism: Event times $t_j \geq 0$ ($t_0 = 0$) are determined by:

a) if $\Big\{t \in \mathbb{R}^+ | t > t_j \wedge \|d^*(t,\cdot)\|_\infty \geq \beta_1\|z(t,\cdot)\|_\infty + \beta_2\|v(t,\cdot)\|_\infty\Big\} = \emptyset$ then the set of the times of the events is $\{t_0,...,t_j\}$,

b) otherwise, the next event time is:

$$\begin{aligned} t_{j+1} = \inf\Big\{&t \in \mathbb{R}^+ | t > t_j \wedge \|d^*(t,\cdot)\|_\infty \\ &\geq \beta_1\|z(t,\cdot)\|_\infty + \beta_2\|v(t,\cdot)\|_\infty\Big\}, \end{aligned} \tag{27}$$

where the actuation deviation $d^*(t,\cdot)$ is given by (21) for all $t \in [t_j, t_{j+1})$.

2. The control action: The feedback control law is defined by (16) for all $t \in [t_j, t_{j+1})$.

4.1 Stability Analysis

The stability of the closed-loop system (22)–(26) is analyzed next, and requires ISS estimates for (22)-(26), as given in the following lemma.

Lemma 1. *For the closed-loop system with initial conditions $z(0,x) = z_0$ and $v(0,x) = v_0$, where $(z_0,v_0) \in (L^\infty(0,1))^2$, the following estimates hold for $\varsigma \in (0,\kappa)$ and $t \in [0, \lim_{j\to\infty} t_j)$:*

$$\begin{aligned} \|z(t,\cdot)\|_\infty \leq\; &e^{-\varsigma t}e^{\frac{\varsigma}{\lambda_1}}\|z(0,\cdot)\|_\infty + \frac{2|b_1|}{\lambda_1}e^{\left(\frac{\kappa|a|+|b_1|}{|b_1|}+\frac{\varsigma}{\lambda_1}\right)} \\ &\times \max_{s\in[t-\frac{1}{\lambda_1},t]}\Big((\|d^*(s,\cdot)\|_\infty + \|v(s,\cdot)\|_\infty)e^{-\varsigma(t-s)}\Big) \\ &+ e^{\frac{\varsigma}{\lambda_1}}|r|\max_{s\in[t-\frac{1}{\lambda_1},t]}\Big(|v(s,0)|e^{-\varsigma(t-s)}\Big), \end{aligned} \tag{28}$$

$$\begin{aligned} \|v(t,\cdot)\|_\infty \leq\; &e^{-\varsigma t}e^{\frac{\varsigma}{\lambda_2}}\|v(0,\cdot)\|_\infty + \frac{|b_2|}{\lambda_2}e^{\left(1-\frac{\kappa}{\lambda_2}+\frac{\varsigma}{\lambda_2}\right)} \\ &\times \max_{s\in[t-\frac{1}{\lambda_2},t]}\Big(\|d^*(s,\cdot)\|_\infty e^{-\varsigma(t-s)}\Big) \\ &+ e^{\frac{\varsigma}{\lambda_2}}\max_{s\in[t-\frac{1}{\lambda_2},t]}\Big(|\eta(s)|e^{-\varsigma(t-s)}\Big), \end{aligned} \tag{29}$$

$$|\eta(t)| \leq e^{-\varsigma t}|\eta(0)| + \frac{|b_2|}{\kappa-\varsigma}\max_{s\in[0,t]}\Big(|d^*(s,1)|e^{-\varsigma(t-s)}\Big). \tag{30}$$

The proof of this lemma can be found in the extended version (Wang et al. (2025)) and is omitted here due to space constraints.

The following small-gain condition specifies the selection of β_1 and β_2, which are the parameters associated with the triggering condition (27)

$$\beta_1\left(\frac{|b_2||r|}{\kappa}+\frac{|r||b_2|}{\lambda_2}e^{\left(1-\frac{\kappa}{\lambda_2}\right)}+\frac{2|b_1b_2|}{\lambda_1\lambda_2}e^{\left(\frac{|\kappa a|+|b_1|}{|b_1|}\right)}e^{\left(1-\frac{\kappa}{\lambda_2}\right)}\right)$$
$$+\beta_1\left(\frac{2|b_1|}{\lambda_1}e^{\left(\frac{|\kappa a|+|b_1|}{|b_1|}\right)}\frac{|b_2|}{\kappa}+\frac{2|b_1|}{\lambda_1}e^{\left(\frac{|\kappa a|+|b_1|}{|b_1|}\right)}\right)$$
$$+\beta_2\left(\frac{|b_2|}{\lambda_2}e^{\left(1-\frac{\kappa}{\lambda_2}\right)}+\frac{|b_2|}{\kappa}\right)<1.$$

By arguments of continuity, there exists a sufficiently small $\varsigma>0$, with $\varsigma<\kappa$, such that

$$\begin{aligned}&\beta_1\Lambda^2(\varsigma)\left(\frac{|b_2||r|}{\kappa-\varsigma}+\frac{|r||b_2|}{\lambda_2}e^{\left(1-\frac{\kappa}{\lambda_2}\right)}\right)\\&+\beta_1\Lambda^2(\varsigma)\frac{2|b_1|}{\lambda_1}e^{\left(\frac{|\kappa a|+|b_1|}{|b_1|}\right)}\left(\frac{|b_2|}{\lambda_2}e^{\left(1-\frac{\kappa}{\lambda_2}\right)}+\frac{|b_2|}{\kappa-\varsigma}\right)\\&+\beta_1\Lambda(\varsigma)\frac{2|b_1|}{\lambda_1}e^{\left(\frac{|\kappa a|+|b_1|}{|b_1|}\right)}\\&+\beta_2\Lambda(\varsigma)\left(\frac{|b_2|}{\lambda_2}e^{\left(1-\frac{\kappa}{\lambda_2}\right)}+\frac{|b_2|}{\kappa-\varsigma}\right)<1,\end{aligned}\tag{31}$$

where

$$\Lambda(\varsigma)=e^{\varsigma\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)}.\tag{32}$$

By organizing terms in (31), we define the following quantities,

$$\phi_1:=\frac{2|b_1|}{\lambda_1}e^{\left(\frac{|\kappa a|+|b_1|}{|b_1|}\right)},\phi_2:=\frac{|b_2|}{\lambda_2}e^{\left(1-\frac{\kappa}{\lambda_2}\right)},\phi_3(\varsigma):=\frac{|b_2|}{\kappa-\varsigma}.\tag{33}$$

Let us also define

$$\begin{aligned}\Phi:=&\beta_1\Lambda(\varsigma)^2\left(1-\beta_1\Lambda(\varsigma)\phi_1\right)^{-1}\left(1-\beta_2\Lambda(\varsigma)(\phi_2+\phi_3(\varsigma))\right)^{-1}\\&(\phi_2+\phi_3(\varsigma))(|r|+\beta_2\phi_1+\phi_1).\end{aligned}\tag{34}$$

If the condition in (31) is satisfied, we can guarantee that $\Phi<1$. In the following, we are ready to state our main result.

Theorem 1. *Assume $\beta_1,\beta_2>0$ are chosen such that the condition (31) is satisfied. Then, for any initial conditions $(z_0,v_0)\in(L^\infty(0,1))^2$, $\eta(0)\in\mathbb{R}$, there exist constants $\varsigma>0$ and $\Theta>0$ such that the following estimates hold for the solutions to the closed-loop system (17)-(20) with the event-triggered control law (16), (27):*

$$\begin{aligned}&\|z(t,\cdot)\|_\infty+\|v(t,\cdot)\|_\infty+|\eta(t)|\\&\le\Theta\exp(-\varsigma t)\Big(\|z_0\|_\infty+\|v_0\|_\infty+|\eta(0)|\Big),\forall t\in[0,\lim_{j\to\infty}(t_j)).\end{aligned}\tag{35}$$

Proof. From Lemma 1, the ISS estimates (28)-(30) imply the following more conservative estimates, for all $t\in[0,\lim_{j\to\infty}(t_j))$,

$$\begin{aligned}&\|z(t,\cdot)\|_\infty\le e^{-\varsigma t}e^{\frac{\varsigma}{\lambda_1}}\|z(0,\cdot)\|_\infty\\&+\phi_1\Lambda(\varsigma)\max_{0\le s\le t}\Big(\left(\|d^*(s,\cdot)\|_\infty+\|v(s,\cdot)\|_\infty\right)e^{-\varsigma(t-s)}\Big)\\&+\Lambda(\varsigma)|r|\max_{0\le s\le t}\Big(\|v(s,\cdot)\|_\infty e^{-\varsigma(t-s)}\Big),\end{aligned}\tag{36}$$

$$\begin{aligned}&\|v(t,\cdot)\|_\infty\le e^{-\varsigma t}e^{\frac{\varsigma}{\lambda_2}}\|v(0,\cdot)\|_\infty\\&+\phi_2\Lambda(\varsigma)\max_{0\le s\le t}\Big(\|d^*(s,\cdot)\|_\infty e^{-\varsigma(t-s)}\Big)\\&+\Lambda(\varsigma)\max_{0\le s\le t}\Big(|\eta(s)|e^{-\varsigma(t-s)}\Big),\end{aligned}\tag{37}$$

$$|\eta(t)|\le e^{-\varsigma t}|\eta(0)|+\phi_3(\varsigma)\max_{0\le s\le t}\Big(|d^*(s,1)|e^{-\varsigma(t-s)}\Big),\tag{38}$$

where $\Lambda(\varsigma)$ is defined in (32), and ϕ_1, ϕ_2 and $\phi_3(\varsigma)$, are defined in (33). We define the following quantities for all $t\in[0,\lim_{j\to\infty}(t_j))$, $x\in[0,1]$:

$$\begin{aligned}\|z\|_{[0,t]}&:=\max_{0\le s\le t}\left(\|z(s,\cdot)\|_\infty e^{\varsigma s}\right),\\\|v\|_{[0,t]}&:=\max_{0\le s\le t}\left(\|v(s,\cdot)\|_\infty e^{\varsigma s}\right),\\\|\eta\|_{[0,t]}&:=\max_{0\le s\le t}\left(|\eta(s)|e^{\varsigma s}\right),\\\|d^*\|_{[0,t]}&:=\max_{0\le s\le t}\left(\|d^*(s,\cdot)\|_\infty e^{\varsigma s}\right).\end{aligned}\tag{39}$$

Using (36)-(38) and (39), we obtain

$$\begin{aligned}\|z\|_{[0,t]}\le&e^{\frac{\varsigma}{\lambda_1}}\|z_0\|_\infty+\Lambda(\varsigma)|r|\,\|v\|_{[0,t]}\\&+\phi_1\Lambda(\varsigma)\left(\|d^*\|_{[0,t]}+\|v\|_{[0,t]}\right),\end{aligned}\tag{40}$$

$$\|v\|_{[0,t]}\le e^{\frac{\varsigma}{\lambda_2}}\|v_0\|_\infty+\Lambda(\varsigma)\|\eta\|_{[0,t]}+\phi_2\Lambda(\varsigma)\|d^*\|_{[0,t]},\tag{41}$$

$$\|\eta\|_{[0,t]}\le|\eta(0)|+\phi_3(\varsigma)\|d^*\|_{[0,t]}.\tag{42}$$

From Definition 1, the triggering condition ensures that $\forall t\in[0,\lim_{j\to\infty}(t_j))$,

$$\|d^*(t,\cdot)\|_\infty\le\beta_1\|z(t,\cdot)\|_\infty+\beta_2\|v(t,\cdot)\|_\infty.\tag{43}$$

Thus, from (40)-(42) along with definitions (39), it follows

$$\begin{aligned}\|z\|_{[0,t]}\le&e^{\frac{\varsigma}{\lambda_1}}\|z_0\|_\infty+(\Lambda(\varsigma)|r|+\beta_2\Lambda(\varsigma)\phi_1\\&+\Lambda(\varsigma)\phi_1)\|v\|_{[0,t]}+\beta_1\Lambda(\varsigma)\phi_1\|z\|_{[0,t]},\end{aligned}\tag{44}$$

$$\begin{aligned}\|v\|_{[0,t]}\le&e^{\frac{\varsigma}{\lambda_2}}\|v_0\|_\infty+\Lambda(\varsigma)\|\eta\|_{[0,t]}\\&+\beta_1\Lambda(\varsigma)\phi_2\|z\|_{[0,t]}+\beta_2\Lambda(\varsigma)\phi_2\|v\|_{[0,t]},\end{aligned}\tag{45}$$

$$\|\eta\|_{[0,t]}\le|\eta(0)|+\beta_1\phi_3(\varsigma)\|z\|_{[0,t]}+\beta_2\phi_3(\varsigma)\|v\|_{[0,t]}.\tag{46}$$

Moreover, the following estimates hold

$$\begin{aligned}\|z\|_{[0,t]}\le&(1-\Phi)^{-1}\left(1-\beta_1\Lambda(\varsigma)\phi_1\right)^{-1}\Lambda(\varsigma)\|z_0\|_\infty\\&+(1-\Phi)^{-1}\left(1-\beta_1\Lambda(\varsigma)\phi_1\right)^{-1}\big(\Lambda(\varsigma)|r|\\&+\beta_2\Lambda(\varsigma)\phi_1+\Lambda(\varsigma)\phi_1\big)\\&\times(1-\beta_2\Lambda(\varsigma)\psi_{2,3}(\varsigma))^{-1}\Lambda(\varsigma)\|v_0\|_\infty\\&+(1-\Phi)^{-1}\left(1-\beta_1\Lambda(\varsigma)\phi_1\right)^{-1}\big(\Lambda(\varsigma)|r|\\&+\beta_2\Lambda(\varsigma)\phi_1+\Lambda(\varsigma)\phi_1\big)\\&\times(1-\beta_2\Lambda(\varsigma)\psi_{2,3}(\varsigma))^{-1}\Lambda(\varsigma)|\eta_0|,\end{aligned}\tag{47}$$

and

$$\begin{aligned}\|v\|_{[0,t]}\le&(1-\Phi)^{-1}(1-\beta_2\Lambda(\varsigma)\psi_{2,3}(\varsigma))^{-1}\Lambda(\varsigma)\|v_0\|_\infty\\&+(1-\Phi)^{-1}(1-\beta_2\Lambda(\varsigma)\psi_{2,3}(\varsigma))^{-1}\Lambda(\varsigma)|\eta_0|\\&+(1-\Phi)^{-1}(1-\beta_2\Lambda(\varsigma)\psi_{2,3}(\varsigma))^{-1}\beta_1\Lambda(\varsigma)\psi_{2,3}(\varsigma)\\&\times\left(1-\beta_1\Lambda(\varsigma)\phi_1\right)^{-1}\Lambda(\varsigma)\|z_0\|_\infty,\end{aligned}\tag{48}$$

where

$$\psi_{2,3}(\varsigma):=\phi_2+\phi_3(\varsigma),\tag{49}$$

and Φ is expressed as

$$\begin{aligned}\Phi=&\beta_1\Lambda(\varsigma)^2\left(1-\beta_1\Lambda(\varsigma)\phi_1\right)^{-1}(1-\beta_2\Lambda(\varsigma)\psi_{2,3}(\varsigma))^{-1}\\&\times\psi_{2,3}(\varsigma)(|r|+\beta_2\phi_1+\phi_1),\end{aligned}\tag{50}$$

exactly as defined in (34) and verifies $\Phi<1$ since (31) holds. Combining (47), (48) and (46), we conclude that (35) holds for appropriate constant Θ, for all $t\in[0,\lim_{j\to\infty}(t_j))$. This completes the proof. •

4.2 Avoidance of the Zeno phenomenon and exponential stability result

Theorem 2. *Under the event-triggering condition* (27)*, any inter-sampling interval is lower bounded by a positive constant (depending on the current state at t_j).*

The proof of Theorem 2 can be found in the extended journal version (Wang et al. (2025)), and is omitted here due to space limitations.

Theorem 2 allows to conclude that $\lim_{j\to\infty}(t_j) = \infty$. Hence we have the following corollary.

Corollary 1. *Let $\beta_1, \beta_2 > 0$ (design parameters involved in the triggering condition* (27)*) that are selected such that* (31) *holds. Let ς be such that* (31) *holds. Then, for any initial conditions $(z_0, v_0) \in (L^\infty(0,1))^2$, $\eta(0) \in \mathbb{R}$, the closed-loop system* (17)-(20) *with event-triggered control* (16), (27) *is exponentially stable, that is, there exists a constant $\Theta > 0$ such that the following estimate holds:*

$$\begin{aligned}&\|z(t,\cdot)\|_\infty + \|v(t,\cdot)\|_\infty + |\eta(t)| \\ \leq &\Theta \exp(-\varsigma t)\Big(\|z_0\|_\infty + \|v_0\|_\infty + |\eta(0)|\Big), \forall t \geq 0. \quad (51)\end{aligned}$$

5. NUMERICAL SIMULATIONS

In this part, we present a numerical example to demonstrate the validity of our results. The event-triggered control strategy described in (16) and (27) is implemented on the linearized system (9)-(14). The numerical values of the parameters are listed in Table 2. The initial values are chosen as $\rho(0,x) = \bar{\rho} + 10\cos\left(\frac{8\pi x}{L}\right)$, and $v(0,x) = \frac{q_{\text{in}}}{\rho(0,x)}$. Fig. 1 shows the numerical solutions of (1)-(4), with event-triggered control (16), (27). By using line search method, the parameters of the triggering condition are chosen to be $\beta_1 = 1.2 \times 10^{-3}$ and $\beta_2 = 0.2$ that verifies the small-gain condition (31). Furthermore, Fig. 2 depicts the in-domain controller based on event-triggered feedback with a control gain $\kappa = 0.1$ (left subfigure), and also illustrates the error dynamics of systems (5)-(8), displayed using the sup-norm to highlight their convergence to zero (right subfigure).

Table 2. System parameters

$\bar{\rho} = 105.8(\frac{\text{vel}}{\text{km}})$	$\bar{v} = 11.35(\frac{\text{km}}{\text{h}})$	$\bar{h}_{acc} = 1.5(\text{s})$
$\bar{h}_{mix} = 1.39(\text{s})$	$q_{in} = 1200(\frac{\text{vel}}{\text{h}})$	$\tau_{acc} = 2(\text{s})$
$\tau_m = 60(\text{s})$	$h_m = 1(\text{s})$	$L = 1000(m)$
$l = 5(m)$	$\alpha = 0.15$	

6. CONCLUSION

This paper proposed an ETC scheme to regulate mixed traffic flow, incorporating ACC-equipped vehicles, under the ARZ modeling framework. The event-triggered scheme makes use of a small-gain based triggering condition. We performed the stability anaylsys though ISS estimates and small-gain arguments. The ETC design ensures exponential stability, and we state the avoidance of the Zeno phenomenon (though lacking uniformity as there is dependence on the information of the current inter-sampling interval). Future work will focus on reducing the frequency of updating by addressing the conservativeness of the event-triggering conditions, thereby improving system efficiency and further optimizing the control strategy, and compare the performance evaluation in terms e.g., of fuel consumption. In addition, spatial discretization can also be studied based on recent advancements in sampled-data control by Zhao et al. (2024).

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REFERENCES

Aw, A. and Rascle, M. (2000). Resurrection of “second order” models of traffic flow. *SIAM journal on applied mathematics*, 60(3), 916–938.

Bayen, A., Delle Monache, M.L., Garavello, M., Goatin, P., and Piccoli, B. (2022). *Control problems for conservation laws with traffic applications: modeling, analysis, and numerical methods.* Springer Nature.

Bekiaris-Liberis, N. and Delis, A.I. (2021). PDE-based feedback control of freeway traffic flow via time-gap manipulation of ACC-equipped vehicles. *IEEE Transactions on Control Systems Technology*, 29(1), 461–469.

Espitia, N., Karafyllis, I., and Krstic, M. (2021). Event-Triggered boundary control of constant-parameter reaction-diffusion PDEs: a small gain approach. *Automatica*, 128(109562).

Espitia, N., Yu, H., and Krstic, M. (2020). Event-triggered varying speed limit control for stop-and-go traffic. *Proc of the IFAC World Congress.*

Goatin, P., Göttlich, S., and Kolb, O. (2016). Speed limit and ramp meter control for traffic flow networks. *Engineering Optimization*, 48(7), 1121–1144.

Karafyllis, I. and Krstic, M. (2018). *Input-to-State Stability for PDEs.* Springer.

Koudohode, F., Espitia, N., and Krstic, M. (2024). Event-triggered boundary control of an unstable reaction diffusion PDE with input delay. *Systems & Control Letters*, 186, 105775.

Li, J., Yu, C., Shen, Z., Su, Z., and Ma, W. (2023). A survey on urban traffic control under mixed traffic environment with connected automated vehicles. *Transportation research part C: emerging technologies*, 154, 104258.

Molnar, T.G. and Orosz, G. (2024). Destroying phantom jams with connectivity and automation: Nonlinear dynamics and control of mixed traffic. *Transportation Science*, 58(6), 1319–1334.

Papageorgiou, M. (1980). A new approach to time-of-day control based on a dynamic freeway traffic model. *Transportation Research Part B: Methodological*, 14(4), 349–360.

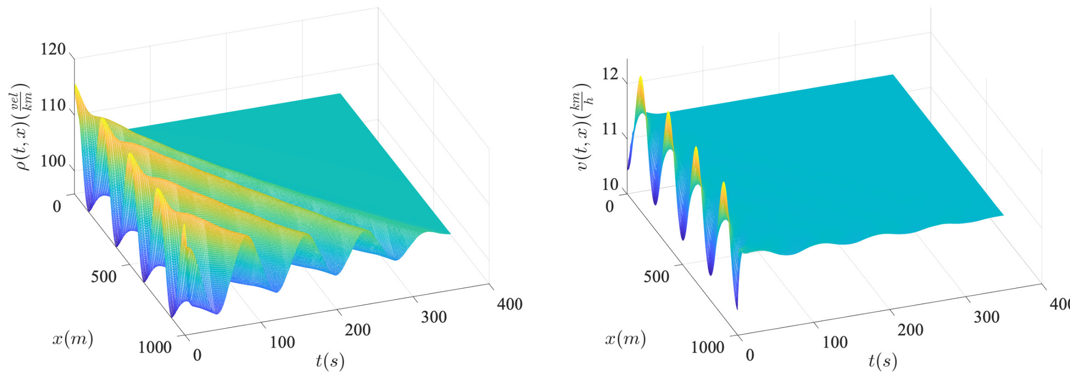

Fig. 1. Solution of ρ and v in system (1)–(4) with small-gain event-triggered in-domain controller (16) satisfying (27).

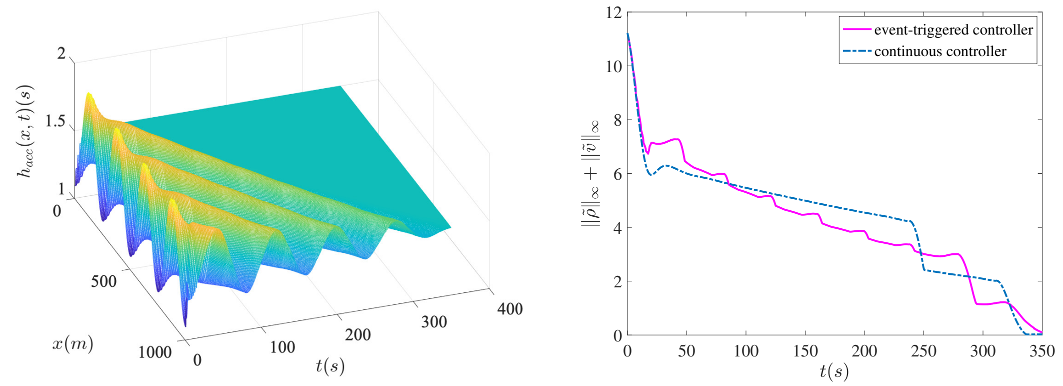

Fig. 2. Profile of the event-triggered in-domain controller (16) (on the left) and time evolution of $\|\tilde{\rho}\|_\infty + \|\tilde{v}\|_\infty$ by both a continuous controller (15) and a small-gain event-triggered in-domain controller (16) (on the right) .

Papageorgiou, M. (1995). An integrated control approach for traffic corridors. *Transportation Research Part C: Emerging Technologies*, 3(1), 19–30.

Payne, H. (1971). Model of freeway traffic and control. *Mathematical Model of Public System*, 51–61.

Prieur, C., Girard, A., and Witrant, E. (2014). Stability of switched linear hyperbolic systems by Lyapunov techniques. *IEEE Transactions on Automatic Control*, 59(8), 2196–2202.

Qi, J., Mo, S., and Krstic, M. (2023). Delay-compensated distributed PDE control oftraffic with connected/automated vehicles. *IEEE Transactions on Automatic Control*, 68(4), 2229–2244.

Rathnayake, B. and Diagne, M. (2024). Observer-based periodic event-triggered and self-triggered boundary control of a class of parabolic PDEs. *IEEE Transactions on Automatic Control*.

Rathnayake, B., Diagne, M., Cortes, J., and Krstic, M. (2024). Performance-barrier-based event-triggered boundary control of a class of reaction-diffusion PDEs. In *2024 American Control Conference (ACC)*, 5313–5319. IEEE.

Selivanov, A. and Fridman, E. (2016). Distributed event-triggered control of transport-reaction systems. *Automatica*, 68, 344–351.

Somathilake, E. and Rathnayake, B. Diagne, M. (2024). Output feedback periodic-event and self-triggered control of coupled 2 × 2 linear hyperbolic PDEs. *arXiv:2404.02298v2*.

Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, 52(9), 1680–1685.

Wang, J. and Krstic, M. (2021). Event-triggered output-feedback backstepping control of 2x2 hyperbolic pde-ode systems. *IEEE Transaction on Automatic Control*.

Wang, X., Tang, Y., Espitia, N., and Bekiaris-Liberis, N. (2025). Event-triggered PDE-based control of freeway traffic flow with connected/automated vehicles. *IMA Journal of Mathematical Control and Information.*, Submitted.

Whitham, G. (1974). *Linear and nonlinear waves*. John Wiley & Sons.

Yu, H. and Krstic, M. (2019). Traffic congestion control for Aw-Rascle-Zhang model. *Automatica*, 100, 38–51.

Yu, L. and Wang, R. (2022). Researches on adaptive cruise control system: A state of the art review. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 236(2-3), 211–240.

Zhang, H.M. (2002). A non-equilibrium traffic model devoid of gas-like behavior. *Transportation Research Part B: Methodological*, 36(3), 275–290.

Zhang, P., Rathnayake, B., Diagne, M., and Krstic, M. (2024). Performance-barrier event-triggered PDE control of traffic flow. *arXiv:2501.00722*.

Zhao, H., Zhan, J., and Zhang, L. (2024). Sampled-data distributed control of mixed traffic flow with ACC-equipped vehicles (I). In *2024 IEEE 63th Conference on Decision and Control (CDC)*. IEEE.