TECHNICAL UNIVERSITY OF CRETE
School of Production Engineering and Management

Joint optimization of vehicle trajectories for a highway weaving area in lane-free traffic

Κοινή βελτιστοποίηση τροχιών οχημάτων για μια περιοχή σύμπλεξης κυκλοφοριακών ρευμάτων σε αυτοκινητόδρομο χωρίς λωρίδες

DIPLOMA THESIS

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A thesis submitted in fulfillment of the requirements for the diploma of Production Engineering and Management
in the
Dynamic Systems and Simulation Laboratory

October 2024

Περίληψη

Η κυκλοφοριακή συμφόρηση σε αυτοκινητόδρομους και αρτηρίες είναι ένα ολοένα και πιο σοβαρό πρόβλημα για τις περισσότερες μεγάλες πόλεις σε όλο τον κόσμο, οδηγώντας σε σημαντικές καθυστερήσεις, αυξημένη κατανάλωση καυσίμου, υπερβολική περιβαλλοντική ρύπανση και μειωμένη κυκλοφοριακή ασφάλεια. Ενώ τα μέτρα διαχείρισης της κυκλοφορίας που χρησιμοποιούν συμβατικά μέσα είναι πολύτιμα και, σε ορισμένες περιπτώσεις, ικανά να καθυστερήσουν ή ακόμα και να αποτρέψουν τη συμφόρηση, δεν είναι πάντα επαρκή για την αντιμετώπιση συνθηκών κυκλοφορίας με μεγάλη συμφόρηση. Σταδιακά αναδυόμενα και μελλοντικά πρωτοποριακά συστήματα αυτοματισμού και επικοινωνίας οχημάτων θα πρέπει να αξιοποιηθούν για την ανάπτυξη καινοτόμων λύσεων που μπορούν να εφαρμοστούν σε μια έξυπνη οδική υποδομή.

Το TrafficFluid, μια ιδέα που εισήχθη πρόσφατα, παρουσιάζει ένα νέο παράδειγμα για την κυκλοφορία οχημάτων, ιδιαίτερα εφαρμόσιμο σε υψηλά επίπεδα αυτοματισμού οχημάτων και συστημάτων επικοινωνίας. Συγκεκριμένα το TrafficFluid προτείνει: (1) κυκλοφορία χωρίς λωρίδες, σύμφωνα με την οποία τα οχήματα δεν δεσμεύονται σε σταθερές λωρίδες κυκλοφορίας, όπως στη συμβατική κυκλοφορία, και (2) ώθηση οχήματος, σύμφωνα με την οποία τα οχήματα μπορούν να ασκήσουν μια επίδραση «ωθώντας» τα οχήματα που βρίσκονται μπροστά τους. Τα οχήματα σε περιβάλλον χωρίς λωρίδες δεν ευθυγραμμίζονται απαραίτητα για να σχηματίσουν λωρίδες, αλλά αυτο-οργανώνονται σε δυναμικά μεταβαλλόμενα δισδιάστατα συμπλέγματα, ανάλογα με το μέγεθος των οχημάτων, τις επιθυμητές ταχύτητες, τις στρατηγικές κίνησης που εφαρμόζονται και την επικρατούσα κυκλοφοριακή πυκνότητα, για τη μεγιστοποίηση της χρήσης της υποδομής.

Αυτή η διπλωματική εργασία εισάγει έναν κοινό αλγόριθμο βελτιστοποίησης τροχιάς για έναν αριθμό συνδεδεμένων και αυτοματοποιημένων οχημάτων σε μια περιοχή σύμπλεξης κυκλοφοριακών ρευμάτων αυτοκινητόδρομου χωρίς λωρίδες, δηλαδή ένα κοινό στοιχείο σχεδιασμού αυτοκινητοδρόμων κοντά σε ράμπες. Μία τέτοια περιοχή μπορεί να αντιμετωπίσει προβλήματα συμφόρησης ακόμα και όταν η κυκλοφοριακή ροή είναι μικρότερη από τη χωρητικότητα λόγω του σχεδιασμού και της πολυπλοκότητας των αλληλεπιδράσεων των οχημάτων, με αποτέλεσμα χαμηλή κυκλοφοριακή ροή και πιθανά προβλήματα ασφάλειας. Ένα μοντέλο διπλής ολοκλήρωσης χρησιμοποιείται για τις διαμήκεις και πλευρικές κινήσεις κάθε οχήματος. Η αντικειμενική συνάρτηση περιλαμβάνει πολλούς επιμέρους στόχους που αντικατοπτρίζουν αντίστοιχα, εν μέρει ανταγωνιστικές, οδηγικές πτυχές και ανησυχίες, όπως η άνεση των επιβατών, η χαμηλή κατανάλωση καυσίμου, η προώθηση του οχήματος στην επιθυμητή ταχύτητα, η αποφυγή σύγκρουσης, η καταστολή ανέφικτων ελιγμών. Τα σταθερά και εξαρτώμενα από την κατάσταση όρια εισόδου ελέγχου αντιπροσωπεύουν διάφορους τεχνικούς περιορισμούς

καθώς και την τήρηση των ορίων του δρόμου. Η λύση στο διαμορφωμένο κοινό Πρόβλημα Βέλτιστου Ελέγχου (ΠΒΕ) υπολογίζεται χρησιμοποιώντας έναν εξαιρετικά αποδοτικό αλγόριθμο εφικτής κατεύθυνσης, ο οποίος εκμεταλλεύεται τη δομή των εξισώσεων κατάστασης για να αντιστοιχίσει το ΠΒΕ σε ένα μειωμένο πρόβλημα μη γραμμικού προγραμματισμού. Για να αποδειχθεί η αποτελεσματικότητα της προτεινόμενης προσέγγισης, θα εξεταστούν δύσκολα σενάρια σε έναν αυτοκινητόδρομο χωρίς λωρίδες, συμπεριλαμβανομένης μιας οδού εισόδου και μιας οδού εξόδου από τον αυτοκινητόδρομο, που βρίσκονται η μια κοντά στην άλλη και σχηματίζουν ένα τμήμα σύμπλεξης κυκλοφοριακών ρευμάτων.

Abstract

Traffic congestion on highways and arterials is an increasingly serious problem for most major cities around the world, leading to substantial delays, increased fuel consumption, excessive environmental pollution, and compromised traffic safety. While traffic management measures utilizing conventional actuators are valuable and, in some cases, capable of delaying or even preventing congestion, they are not always adequate for addressing heavily congested traffic conditions. Gradually emerging and future ground-breaking vehicle automation and communication systems should be exploited to develop innovative solutions that can be applied within a smart road infrastructure.

The recently introduced TrafficFluid concept presents a novel paradigm for vehicular traffic, particularly applicable at high levels of vehicle automation and communication systems. The TrafficFluid concept suggests: (1) lane-free traffic, whereby vehicles are not bound to fixed traffic lanes, as in conventional traffic; and (2) vehicle nudging, whereby vehicles may exert a "nudging" effect on vehicles in front of them. Vehicles in a lane-free environment do not necessarily align to form lanes but are self-organizing into dynamically changing 2-D clusters, depending on the vehicle sizes, their desired speeds, the employed vehicle movement strategies, and the prevailing density, so as to maximize the infrastructure utilization.

This thesis introduces a joint trajectory optimization algorithm for a number of connected and automated vehicles within a lane-free highway weaving area, i.e. a common highway design element near ramps. A weaving area may experience congestion problems even when traffic flow is less than capacity due to design and complexity of the vehicle interactions, resulting into low traffic flow and potential safety issues. A double-integrator model is utilized for the longitudinal and lateral movements of each vehicle. The objective function comprises several sub-objectives reflecting corresponding, partially competing, driving aspects and concerns, including passenger comfort, low fuel consumption, vehicle advancement at desired speed, collision avoidance, suppressing of infeasible maneuvers. Fixed and state-dependent control input bounds account for various technical limitations as well as road boundary adherence. The solution to the formulated joint Optimal Control Problem (OCP) is computed using a highly efficient Feasible Direction Algorithm, which exploits the structure of the state equations to map the OCP into a reduced Nonlinear Programming Problem. To demonstrate the efficiency of the proposed approach, challenging scenarios will be examined on a lane-free highway, including an on-ramp and off-ramp, located close to each other and forming a weaving area.

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 A brief introduction to CAVS

Connected and Automated Vehicles (CAVs) have the capability of operating on their own, advanced sensory, artificial intelligence, and communication technologies. In recent decades, the automobile industry and research institutes have collaborated to plan, develop, test, and deploy many technologies. These improvements are anticipated to fundamentally alter the functions and capabilities of individual vehicles [1]. Huge developments have been taking place in CAVs during the past twenty years. They are considered one of the biggest changes in transportation technology. The wide acceptance of these will bring a revolution not only in the way people travel but also in the working of urban and rural transportation systems. CAVs can identify the road surroundings and navigate autonomously, minimizing human error and avoiding collisions. Moreover, CAVs adhere to speed restrictions and other traffic regulations. CAVs are anticipated to decrease fuel consumption, improve traffic safety and passenger comforts [2].

In addressing the previously described challenges, significant efforts have been made in recent years to create various Vehicle Automation and Communication Systems (VACS) and control systems, anticipated to enhance capabilities of the individual vehicles.

VACs are systems that modify some vehicle functions across multiple levels of automation, from basic driver assistance to complete automation. Concurrently, the mobile communication sector has realized significant advancements and transformed lifestyles. The implementation of modern mobile communication technologies in cars is anticipated to implement significant advancements in transportation technology. Furthermore, the communication systems VACs facilitate varied degrees of cooperation, enabling vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication. Moreover, numerous corporations and research institutions have been engaged in technical advancements to enhance driving conditions for. Four both the operator and the occupant. Recently, numerous researchers and industries are engaged in the development and testing of fully automated vehicles utilizing approaches such as Artificial Intelligence (AI) and optimization control and reinforced learning to evaluate decisions regarding car-following lane altering [1], [3].

The connected vehicles communication systems are classified into the three following categories [4]:

Vehicle-to-vehicle (V2V) communication systems: This category includes systems that require interaction and cooperation with other similar systems to operate well. An infrastructure-based MTM system, similar to autonomous systems, is incapable of direct communication or exerting control decisions and recommendations externally. This category's VACS encompasses the cruise control system CACC, the lane change/merge assistance system CM, the combined-functionality systems CFM and IRSA, and the vehicle platooning system VPS.

Vehicle-to-Infrastructure (V2I) communication systems: This category includes systems that must interact and cooperate with the infrastructure to function effectively. Unlike the systems in the preceding categories, these systems can receive and implement control options and recommendations determined externally by an infrastructure-based MTM system, contingent upon their level of support (informative or intervening systems). The dual connection facilitates the transfer of vehicle data to the MTM system, hence enhancing the variety, quality, and volume of centrally accessible real-time information. This category's VACS comprises the speed regulation systems CVSLS and ISA, the lane change/merge assistance system CM, and the navigation system NAVS.

Vehicle-to-vehicle and infrastructure communication systems (V2X): This category includes systems that possess characteristics of both V2V and V2I system classifications. This category's VACS comprises the multifunctional systems CFM and IRSA, in addition to the vehicle platooning system VPS. [4]

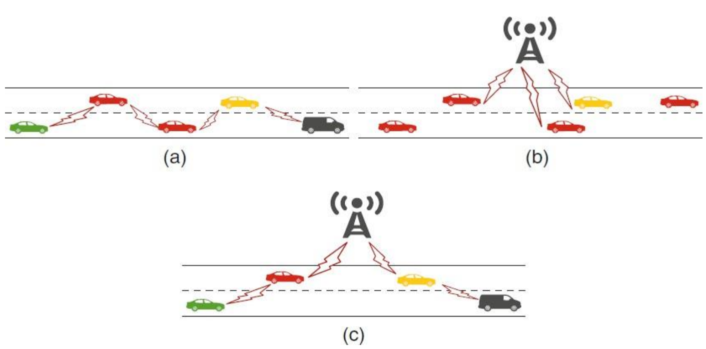


Figure 1: (a) Vehicle to Vehicle, (b) Vehicle to Infrastructure, (c) Hybrid architecture, Source:

Amongst the most critical aspects of CAV technology is that they can communicate with one another and with road infrastructure. V2V communication, a system that allows CAVs to share pertinent data regarding speed, location, and intended next moves, enables the vehicles around them to understand what is happening around them when they drive in a coordinated manner. It helps reduce the chances of accidents because the capability to perceive other drivers' actions and adjust one's behavior accordingly enhances. By contrast, V2I allows a CAV to communicate with the smart infrastructures such as traffic light systems, road signs, and sensors within the road surface. This would enable a CAV to gain updated information in real time with regard to traffic and road conditions, among other information that would further enhance its capability to make decisions on safe driving efficiently. Besides, the hybrid architecture of CAV systems, covering both V2V and V2I solutions, enables a closer-to-reality and active transportation ecosystem, in which CAVs will be more appropriately able to handle complex and dynamic traffic environments [5], [6].

So, CAV technology can be utilized to fulfill the following goals: **Safety Benefits** [7]**, Mobility and Traffic Efficiency** [8]**, Environmental Impact** [9], **Economic Development and Public Health** [10]:

Safety Benefits [7]**:**

CAVs have elevated safety expectations for future road transportation systems, the National Highway automobile Safety Administration reports that human error accounts for 94-96% of automobile collisions and crashes. Recent research suggests that CAVs may have an impact on safety. During the fully automated period, there is an estimated 90% increase in safety [11]. In recent years, several researchers have concentrated on simulation-based investigations in because of the scarcity of real-world data, it is difficult to assess the benefits of CAVs. Although there are numerous vehicle-centric applications available on the market, such as lane departure warning systems and speed upgrades, these technologies only partially aid humans. Human mistake continues to be the cause of accidents among drivers. Current research focuses on deployment of autonomous vehicles on public roads and the operation of lane-free settings in traffic without human drivers.

Enhancing road safety is essential for mitigating human error. Connected and Autonomous Vehicles (CAVs) may address the problem of human error. Despite the proliferation of vehicle-centric apps, such as lane departure warning systems and speed updates, these technologies only partially aid human drivers, and accidents due to human irresponsibility persist. The research presently concentrates on the introduction of CAVs on roadways, succeeded by the implementation of a lane-free environment in traffic devoid of human drivers. Considering the possibilities of CAV technology to enhance safety.

Mobility and Traffic Efficiency [8]**:**

Connected and Autonomous Vehicles (CAVs) possess the capability to transform future travel methods. The utilization of CAVs alleviates road congestion by identifying sudden surges in traffic volume or velocity. The lane-free idea will be implemented over a broader part of the roadway, facilitating denser and improved traffic flow. Vehicles may be directed to alternative routes to circumvent traffic. However, the most crucial aspect is that emergency responders may arrive at the area more swiftly and securely. Due to the interconnection between connected and automated vehicles (CAVs) and emergency technology [8].

Environmental Impact [9]:

In terms of environmental sustainability, the transportation sector is one of the largest emitters of greenhouse gases, particularly carbon dioxide. Climate change and urbanization have prompted the demand for a sustainable transportation infrastructure. Road transportation accounts for around 70% of all CO2 emissions in Europe, including passenger vehicles have an important function [9]. As a result, the adoption of CAVs significantly reduces fuel consumption and, hence, emissions by making their operations considerably more efficient. The steady speed without traffic jams and less stop-and-go drives will result in less fuel consumption, which will lead to a reduction in CO2 emissions. Furthermore, the environmental benefits of automated cars will be increased when electric and hybrid vehicles become more prevalent in CAV fleets. It should also be noted that CAVs' impact on the environment is not unidirectional, but rather a complicated combination. For example, comfortable CAVs are expected to increase demand for travel and greater speeds, canceling out some of the environmental benefits of lower fuel usage. As a result, the long-term environmental consequences of CAV adoption necessitate ongoing research to develop potential mitigating methods.

Economic Development and Public Health [10]:

In the framework of CAV technologies, society contributes to economic progress. Improving safety, mobility, and environmental quality benefits public health and productivity in the long run. However, the impact of this technology on the economic sector is a reliable and difficult-to-quantify transportation network is crucial in the financial well-being of society, as well as the improved quality of life for all road users.

Completely autonomous and networked vehicles (CAVs) possess the capacity to dominate the automotive sector. When CAVs are dependable and cost-effective, they have the potential to transform economies and sectors. Connected and Autonomous Vehicles (CAVs) will increasingly represent a greater proportion of vehicle value and will play a significant role in the automotive business. Vehicle pooling may lead to a reduction in annual vehicle purchases, but extended journeys could increase vehicle sales. The capacity of heavy-truck drivers to perform additional tasks or rest during extended journeys may reduce freight expenses and enhance capacity. The implementation of collaborative autonomous vehicle operations Private transportation may supplant taxis, buses, and other forms of public transit. The demand for auto repair, traffic enforcement, medical, insurance, and legal services will diminish due to a reduction in collisions and an increase in compliant vehicles on the road. Connected and Autonomous Vehicles (CAVs) will introduce novel methodologies for managing travel demand. Curbside and off-street parking will be relocated, resulting in substantial cost savings due to increased productivity from hands-free mobility and diminished expenses associated to crash injuries. The anticipated economic effect of connected and autonomous vehicles (CAVs), should they capture a substantial portion of the automotive market, is estimated at $1.2 trillion, or to $3,800 per American citizen each year.

1.2 Traffic Fluid Concept

The TrafficFluid approach represents a paradigm shift by introducing a new traffic movement concept, specially made for the era of CAVs. This is based on two basic principles of lane-free traffic and vehicle nudging [12].

The lane-free traffic concept challenges the traditional approach of organizing vehicular movement into discrete lanes and offers a revolutionary alternative suited for high-level vehicle automation and connectivity. The existence of lanes within the traffic systems is essential for manually driving because they provide to the driver structure and ensure safety. However, dealing with traffic lanes is often complicated and risky, especially in dense traffic conditions or withing merging areas, such as on-ramps and offramps.

Changing lanes becomes significantly more challenging when frequent lane alterations occur due to dips or merging on-ramps or roads. Indeed, 10% of incidents result from lane changes [13]. Moreover, the conventional lane width, intended to accommodate heavier vehicles like trucks, results in poor utilization of road space. In the United States, highways generally feature lanes that are 3.6 meters wide, yet the average automobile measures about 1.8 meters in width. This leads to lateral road usage exceeding 50%, hence diminishing the overall capacity of the road network.

Thus, lane-free traffic might substantially enhance carriageway capacity by maximizing the utilization of all available road space. Vehicle sensors in connected and automated vehicles (CAVs) enable rapid, simultaneous, continuous, and reliable monitoring of nearby cars in a 360-degree field, facilitating prompt computer-based driving decisions.

TrafficFluid represents a groundbreaking model for vehicular traffic, usable under conditions of advanced vehicle automation and communication, alongside high penetration rates anticipated in the near future. We posit that vehicles engage in communication with each other (V2V) and with infrastructure (V2I) at an adequate frequency, distance, and bandwidth, and that they operate autonomously utilizing their sensors, communication systems, and an effective movement control methodology. Vehicles can vary in kind (e.g., electric or internal combustion engine) and size (cars, vans, buses, trucks, motorcycles), possessing diverse maximum speeds and acceleration capacities.

Numerous advancements are currently occurring in research and industry to provide dependable real-time information exchange between vehicles and infrastructure (V2I) as well as between vehicles (V2V) [14] [15]. This study posits that adequate communication exists among neighboring vehicles within a restricted range, both upstream and downstream of each ego vehicle (EV), facilitating the exchange of information regarding their position, speed, and the most recently computed optimal trajectories.

1.2.1 Lane-Free Traffic

The lane-free structure enhances the utilization of lateral road space by vehicles and offers further advantages, including augmented capacity through gradual road widening over a constrained length, as well as adaptable (both spatially and temporally) incremental sharing of total road capacity between the two traffic directions in real- time [16]. By employing suitable algorithms, traffic on motorways, highways, arterials, and urban roads might be liberated from lane restrictions, thereby restoring lost capacity without jeopardizing safety. The abrupt lateral shifts between lanes become redundant, allowing for seamless lateral vehicle motion for overtaking and other driving maneuvers.

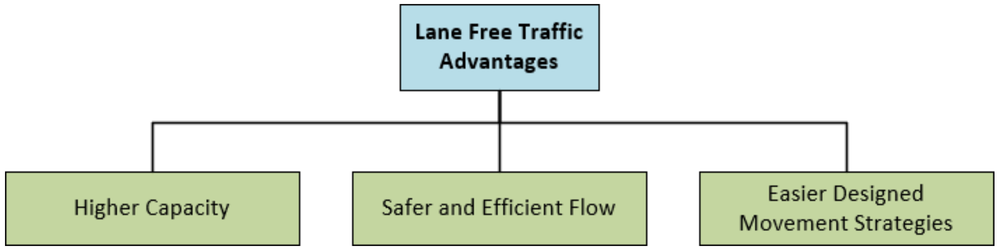


Figure 2: Lane free traffic advantages

In contrast, lane-free traffic gets rid of fixed lanes with the possibility of vehicles utilizing the full 2-D surface of the road. Equipped with advanced sensors and communication systems, CAVs will safely navigate in a lane-free environment while their position and speed are dynamically optimized with real-time traffic conditions. Static and dynamic road capacity will be better exploited since the vehicles will no longer be constrained to follow a predefined path. The entire road will be used, and there will not be congestion-triggering maneuvers like lane changes. Therefore, quite a smooth and safe ride and improvement of road efficiency by quite a lot.
[16]

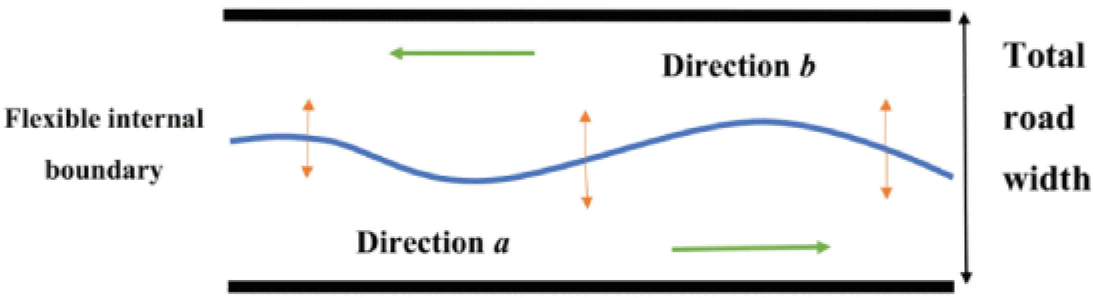


Figure 3: Lane-free traffic

In lane-free environments, CAVs are modeled as particles in an artificial fluid that can move freely by continuously making changes in trajectory according to traffic density and interaction between vehicles. These behaviors flow like a fluid allow vehicles to maintain safe distances while maximizing road capacity, with very high densities, as occurs in motorways and urban road infrastructures. The lane-free model removes the limitation brought by human-driven traffic.

1.2.2 Nudging

The second principle of TrafficFluid is nudging—a concept wherein faster-moving vehicles are allowed to "nudges" the slower vehicles ahead. Nudging removes this limitation and lets the vehicle employ either sensors or a communication system in order to "nudge" slower-moving vehicles out of the way for a smooth flow of traffic, thereby preventing bottlenecks.

In real-world terms, nudging would take place when a vehicle is able to detect a slower-moving vehicle in its path and send a signal to the slower vehicle to change its course or reduce speed so the faster vehicle can overtake. This process improves the traffic flow and increase the roadway capacity, as the vehicles are no longer constrained by human-driven reactions limited to downstream inputs.

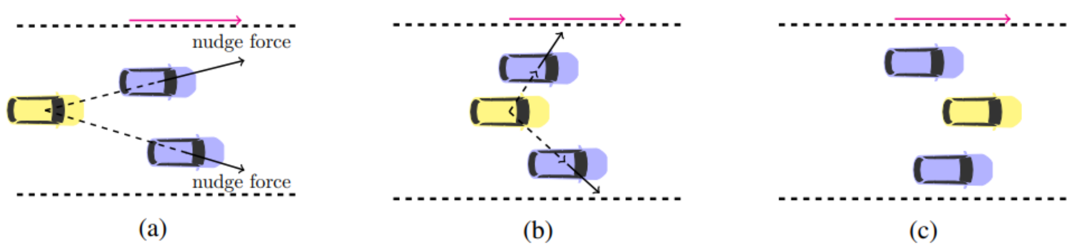


Figure 4: A minuscule impact of vehicular nudging: The yellow vehicle pushes the slower blue vehicles aside to overtake [14].

In Fig. 4a, a fast car is caught behind two slower vehicles traveling in parallel symmetry. Without vehicle nudging, the car would follow slow cars, resulting in a locally optimal trajectory. In our OCP, if nudging is present, the car may nudge both vehicles aside (Fig. 4b) before passing (Fig. 4c).

TrafficFluid

The objective of the project TrafficFluid is to devise a completely novel traffic system; it certainly goes beyond the mere description and modeling of traffic flow. Lane-free traffic combined with nudging offers for the first time the opportunity to engineer traffic flow as an artificial fluid that could be optimized for efficiency, safety, and convenience. It allows traffic managers to devise strategies applicable to various road infrastructures, ranging from urban environments to motorways, with the objective of maximizing the flow while simultaneously minimizing congestion [1].

Although there is considerable risk in building a completely new system of traffic, the possible benefits are quite revolutionary. Removing the traditional lane-based constraints and redefining the movement of the vehicles within the road, TrafficFluid ushers in a more efficient and safe traffic environment in the CAV era. Also, the project forecasts that this approach will act as a catalyst for further development and research on the transportation area.

1.3 Vehicular Traffic, Traffic Congestion and Weaving Areas

Vehicular Traffic

When roads were initially created, there was no necessity for lanes due to minimal road usage. With the increasing affordability and popularity of automobiles, the necessity for lane markings emerged to delineate opposing traffic directions and prevent crashes. Subsequently, in the 1950s, dashed lines were introduced to separate parallel lanes in the same direction, accompanied by regulations governing lane changes. Parallel lanes enhance traffic safety by facilitating human drivers' ability to maintain their course and avert collisions with other cars. A driver in a lane concentrates exclusively on the velocity of the car in front, ensuring a safe following distance is preserved. Changing lanes introduces additional complexity, as the driver must identify an available gap in the target lane, anticipate its future position based on the velocities of multiple vehicles (including their own), and continuously monitor the distance to the vehicle in front. Frequent lane changes, often due to lane reductions, freeway configurations, or merging on-ramps, further increase the risk associated with lane changes.

The mass production of affordable cars led to increased motorization in the first half of the 20th century, along with high levels of fatalities on roads. The need for highway safety led to demands to further develop road markings. Dashed lines began to be used in the 1950s to separate parallel traffic lanes going in the same direction, and guidelines were adopted that required the vehicle's motion should be made upon change of lane. These improvements provided enhanced traffic safety by allowing human drivers to steer easily and avoid collisions with surrounding vehicles.

The traffic lanes also seem to have imposed more structure on most drivers, who had only one clear concern: the car in front of them. That made going close or fast-risky behaviors. But driving gets complex as we account for changing lanes — especially with lane drops, merging on-ramps and off-ramps. Change detection is one of the most known high accident precursors being 10% and it is lane change [17]. They get drivers to consider a number of features — whether there is room in the adjacent lane, how fast other traffic is moving and what distance that vehicle following behind has become.

In summary, unidirectional traffic lanes emerged in the mid-twentieth century as an essential strategy for enhancing traffic safety during manual driving, even at the cost of reducing highway capacity.

Vehicular traffic is a fundamental element of modern transportation systems as it is connected with the movement of people and goods. The problem is more important, though, especially in unfavorably congested and populated parts of cities that have grown up anew as to so many folks owning cars.

Traffic Congestion

Whatever the reason, lane changes pick up as traffic volume increases, resulting in more braking for everyone else and less overall road carrying capacity.

Lane Usage Affects Traffic Flow: The more considerable the number of lanes that a road has, the higher the throughput can be. On American interstates, the lanes are about 3.6 meters around, while in Germany, they measure closer to 3.5 — 3.75 meters across. Excluding vehicles that are larger than the lane itself (300 mm for trucks and 100 mm for medium-sized cars), there is room to spare! In other words, an average of just 50% of each motorway's width is actually put to use. Therefore, any fractional use of the lateral space such as this is projected to have a substantial impact of flow-capacity on no lanes traffic. Lastly, dynamic effects such as lane change maneuvers introduce additional fleet capacity losses on top of what was already a less efficient system to begin with from manual driving.

Unidirectional lanes, in conclusion, marked the beginning of improving traffic safety in a time where people were primarily manual drivers. On the other hand, such a system also decreased highway capacity due to cases of frequent lane changes like interchange areas.

Urbanization has impacted the problem of traffic congestion even more dramatically in heavily populated cities. Ttraffic congestion occurs when the volume of vehicles on a road results in an excessive flow of traffic. This can lead to slower speeds, longer trip times and an increased risk of accidents. The negative effects of congestion are found in large sections of society:

- Health: Traffic congestion leads to a long list of stress-related health problems and late response times for emergency vehicles.

• Pollution: A large increase in air pollution occurs from traffic jams.

• Economy: Time spent in traffic equals economic loss through lost productivity as well as the fuel consumption increase.

Although a considerable effort has been put into the research of traffic congestion over the last few decades, road congestion and traffic accidents are still widely present across global urban areas. In response to this, creative and innovative solutions are needed.

The causes of traffic congestion are as varied as the many day-to-day challenges we face as drivers. These include bottlenecks, merging areas, accidents or breakdowns and poorly designed or improperly managed intersections. Capacity, additions and traffic signaling are no longer efficient to handle the rapid expansion in vehicle population been witnessed. Modern solutions have thus, in turn, evolved towards Intelligent Transportation Systems (ITS), Connected and Automated Vehicles (CAVs) as well as the implementation of advanced traffic flow optimization techniques [1].

On the other hand, other experts have indicated that CAVs could diminish capacity if they function conservatively at reduced speeds and extended headways [18].

Weaving Areas

Weaving is defined as the intersection of two or more traffic flows moving in the same direction across an extended stretch of road, typically a freeway, without the assistance of traffic control systems (Transportation Research Board 2000). Weaving zones arise when an on-ramp (merging region) is located near an off-ramp (diverging area). One or more auxiliary lanes connect the on- and off-ramp.

The length of a weaving stretch constrains the spatial and temporal opportunities for drivers to merge or diverge from the primary freeway traffic, resulting in turbulence in the traffic flow owing to lane shifts. The merging and diverging maneuvers, contingent upon entry volume, exit volume, and configuration type, may result in intricate scenarios during crowded traffic conditions. The weaving section's operational performance and safety are determined by its length, total entry volume, total departure volume, non-weaving traffic volume, and width, specifically the number of lanes inside the actual weaving area.

The vehicles participate in complicated lateral movements while merging, diverging, and changing lanes in weaving areas. Quite often, such lane-change maneuvers result in abrupt slows-downs and delays because the vehicles are struggling not to clash with one another in traffic streams. Pretty regularly, lane changes in weaving sections also highly contribute to increased crash risks and act as common bottlenecks of traffic.

Traditionally, traffic management in weaving areas has been done by implementing a few traffic control strategies like traffic signals or ramp metering at on/off ramps that are designed to control the flow of vehicles entering or leaving the highway. However, all these are not always enough to solve the dynamic problems produced by continuous lane changes and the merging/crossing of vehicles. This is attributed to the constant merging and diverging of traffic, which interferes with the smooth flow of the other vehicles by reducing their speeds and further increasing congestion.

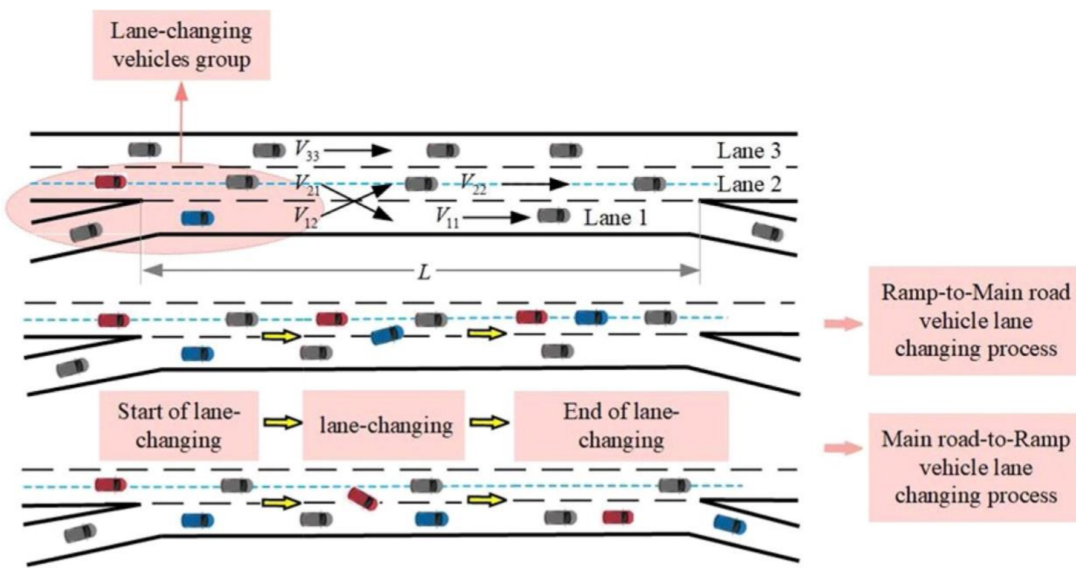


Figure 5: Weaving area [19]

1.4 The impact of On-ramps and Off-ramps

Weaving sections often function as bottlenecks within freeway systems and have been identified as areas with an increased crash probability [20]. At weaving sections, both mandatory lane changes (for vehicles entering or exiting the freeway) and discretionary lane changes (for through vehicles) occur [2].

Forced lane changes are more aggressive and highly influence the general performance in flow. Lane changing introduces traffic flow turbulence, which propagates into throughput and vehicle speeds. This creates an unstable flow in the use of available road space at the section where weaving is occurring longer headways when lane-changing and shorter headways during merging. From the driver's comfort perspective, lane changes in weaving sections can increase the driver's workload and stress; this can lead to a potential human error and associated safety risks [21].

These areas have been a long-term focus in studies of traffic operations for decades and many attempts have been built to overcome the difficulties related to them. This includes areas of research into driver behavior at merging and diverging, acceleration and deceleration profiles, lane changing behavior and gap acceptance etc. For instance, to understand the complexities of lane-changing maneuvers, vehicle interactions during freeway ramp merging in weaving area under congestion were studied by formulating a class of acceleration-deceleration models, see [22]. Furthermore, unconventional methods have been utilized to analyze crash-prone sites that include driver behavior studies in work zone merging areas [23] and freeway interchange merging zones [24]. These research studies suggest that other factors, such as geometric characteristics of the road and traffic conditions, can improve safety and efficiency in weaving sections [25].

Weaving areas are at an even higher risk for congestion because of the constant lateral movements vehicles must make to enter or leave the mainline traffic flow. Most of these movements require sudden deceleration or acceleration, causing loose flow of traffic which leads to congestion. Sections to be weaved also mean higher density and therefore more accidents, more complex overtaking maneuvers, in which cars pass at significantly reduced distances from one another.

The introduction of lane-free traffic supported by nudging techniques may thus become an effective relief for congestion and safety problems in weaving areas. Lane-free traffic-free traffic enables transitions to be made much more smoothly, with much fewer abrupt lane changes and leads to significantly fewer congestion levels than common nowadays. In lane-free traffic, vehicles can maintain higher speeds while achieving smoother traffic flow since sections with multiple entries and/or exits no longer need to impede the traffic stream. This will also prevent the possibility of an accident occurring due to a sudden stop or acceleration of vehicles when merging or diverging from traffic streams.

1.5 Thesis Structure

This thesis aims to provide a joint optimization approach and the requisite algorithm to enhance the trajectories of connected and automated vehicles (CAVs) in a fully automated setting during highway weaving stretches, in addition to modeling and assessing the suggested methodology. The infrastructure presumes that all vehicles inside the network are completely automated and capable of communicating with the infrastructure. A lane-free simulated network is created to assess the efficiency of the weaving algorithm under various demand scenarios.

The rest of the thesis is organized as follows: chapter 2 provides a concise overview of the necessary background to comprehend this diploma thesis by examining the essential principles and concepts of contemporary transportation systems. Furthermore, it delineates the problem together with its restrictions, including the objective function, the OCP solution, and the boundary formulation, while providing an expanded overview of the context in which the simulations occur. Chapter 3 establishes experimental configurations and benchmarks for our methodology across various difficulty levels to assess its performance. Chapter 4 concludes with a summary of our strategy and the enhancement of the CAVs performance, while also outlining potential future directions for the project.

CHAPTER 2

OPTIMAL CONTROL PROBLEM (OCP) FORMULATION

This section introduces a discrete-time Optimal Control Problem (OCP) that handles the movement of multiple vehicles in a coordinated way while considering the aforementioned features of the Traffic fluid context. State-dependent constraints on control inputs are enforced to ensure the vehicle stays within road boundaries and to prevent collisions. A multi - objective function with several sub-objectives is formulated, the minimization of which enables vehicle advancement at the desired velocity. An optimal nonlinear control problem (OCP) is formulated for the objective of minimization. The goal function is bound by the limits applicable to each vehicle. A Feasible Direction Algorithm (FDA) is used to compute the numerical solution. The methodology is applied to a group of vehicles while executing simulations in a lane-free motorway stretch, accommodating diverse traffic densities.

2.1 Vehicle Dynamics, Constraints and Boundary Controller

Vehicle Dynamics

The issue pertains to a dynamic model with an objective function that must be minimized under constraints to achieve specific characteristics, such as navigation in a lane-free setting while avoiding collisions with adjacent vehicles. Four state equations, comprising two in the longitudinal direction and two in the lateral direction, are employed to characterize each vehicle [12]. The state equations are derived from the rules of kinematics.

A discrete-time vehicle model is examined, regulated by discrete-time control inputs. Moreover, the lateral movement employs real-valued states due to the lane-free road configuration, enabling cars to be continually positioned within the road borders in the lateral direction. The motion of each vehicle, assuming several vehicles, is characterized by four state equations. By employing a two-dimensional plane and discrete time, the subsequent equations can be derived:

$$x_i(k+1) = x_i(k) + T \cdot V_{x,i}(k) + \frac{1}{2} \cdot T^2 \cdot a_{x,i}(k) \quad (2.1)$$

$$y_i(k+1) = y_i(k) + T \cdot V_{y,i}(k) + \frac{1}{2} \cdot T^2 \cdot a_{y,i}(k) \quad (2.2)$$

$$V_{x,i}(k+1) = V_{x,i}(k) + T \cdot a_{x,i}(k) \quad (2.3)$$

$$V_{y,i}(k+1) = V_{y,i}(k) + T \cdot a_{y,i}(k) \quad (2.4)$$

where x_i and y_i denote the longitudinal and lateral positions, respectively; $V_{x,i}$ and $V_{y,i}$ describe the longitudinal and lateral velocities, while the control inputs $a_{x,i}$ and $a_{y,i}$ indicate the longitudinal and lateral accelerations, respectively k is the discrete time integer index and T is the step size, related to time via $kT = t$. Figure 6 depicts an automobile's parameters in a two-dimensional plane.

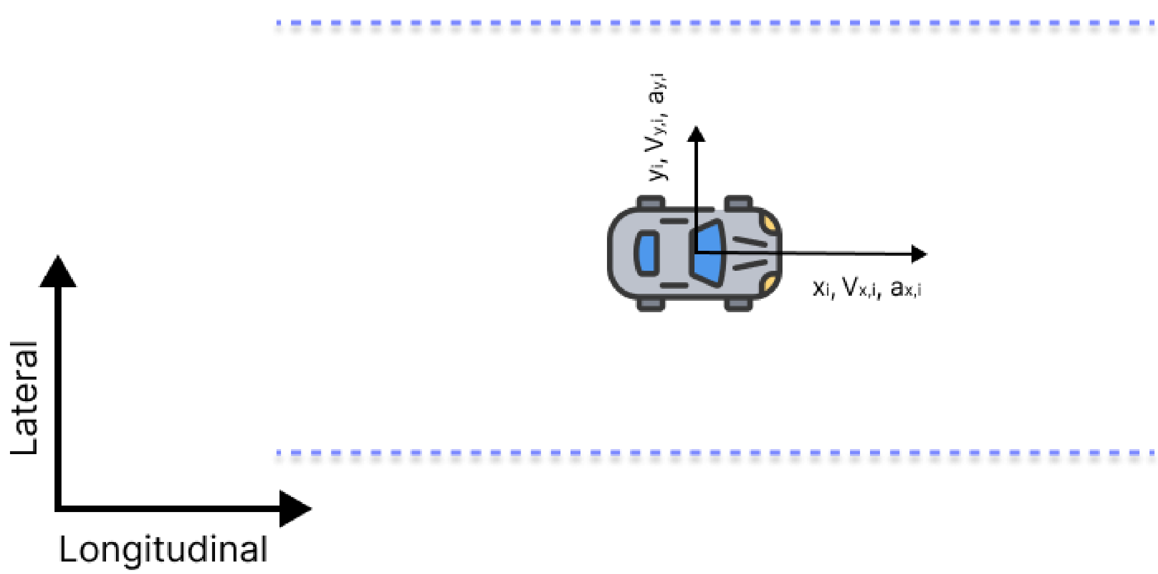


Figure 6: Vehicle Dynamics

Constraints

Limitations of this issue include avoiding negative longitudinal speeds for the next time step. So, the longitudinal and lateral accelerations are bounded, and the following equations provide the general form.

$$a_{x,i}^{min}(x(k),k) \leq a_{x,i}(k) \leq a_{x,i}^{max}(k)$$

$$a_{y,i}^{min}(x(k),k) \leq a_{y,i}(k) \leq a_{y,i}^{max}(x(k),k) \tag{2.5}$$

Let x be the vector comprising all the states referenced in (2.1 – 2.4). The maximum limit for longitudinal acceleration is $a_{x,i}^{max}(k) = A^{max}$, where A^{max} a constant determined by the vehicle's acceleration capabilities. The minimum threshold of longitudinal acceleration is established based on two criteria: the non-negativity of longitudinal velocity and a constant A^{min}, which indicates the vehicle's deceleration capacity as follows:

$$a_{x,i}^{min}(x(k),k) = max\left\{-\frac{1}{2}V_{x,i}(k), A^{min}\right\} \tag{2.6}$$

Boundary Controller

The lateral acceleration limits guarantee that cars operate inside or precisely on the road boundaries. According to (2.7), every vehicle situated within the road boundaries at time steps k and k+1 must also remain within the road boundaries at time step k+2:

$$\tilde{y}_i \leq y_i(k+2) \leq \hat{y}_i \tag{2.7}$$

where $\hat{y}_i$ and $\tilde{y}_i$ are the lateral positions of the left and right road boundary, respectively. Whenever the vehicle reaches the left or right road boundary from an admissible lateral position, the corresponding constraint in (2.7) is activated, then we must have for the lateral vehicle speed $u_{y,i} = 0$, as otherwise the vehicle would eventually exit the road. In other words, any approach of a road boundary must be asymptomatical. Substituting lateral position y_i and lateral speed $u_{y,i}$ from (2.2) and (2.4), yields after some rearrangements, state-dependent bounds on lateral acceleration, which can be interpreted as dead-beat controllers that drive the vehicle towards the corresponding road boundary and its lateral speed to zero in two time-steps. By generalizing the dead-beat controller to allow for smoother and asymptomatic boundary approach, we have the state-dependent bounds on lateral acceleration given by:

$$a_y \geq -K(y - y_b(x)) - (2\sqrt{K} - \frac{T}{2}K)(v_y - v_{y,b}(x)) + a_{y,b}(x) \quad (2.8)$$

Where $0 \leq K_{lat} \leq \frac{1}{T^2}$ is a feedback controller gain that may be tuned appropriately for smooth boundary approach, while asymptomatic behavior is inherently guaranteed.

2.2 Boundary Formulation for On-ramp and Off-ramp Traffic

In merging scenarios where the vehicle needs to exit an on-ramp and enter off-ramp it is essential to use a smooth boundary control mechanism. The provided function offers a sigmoid-based approach to compute the boundary values for a vehicle's longitudinal position. This function adjusts the lateral boundaries depending on the vehicle's position relative to a predefined offset, allowing for smooth transitions.

This boundary formulation provided here adjusts the lateral boundaries of the vehicle by incorporating the vehicle's longitudinal position x, velocity, and lateral positioning y with respect to a predefined offset. The smooth transition is modeled by using a hyperbolic tangent function tan h, which is particularly well suited to model gradual changes because of its asymptomatic behavior. This smooth behavior ensures that as vehicles approach merging or diverging point, they gradually adapt their lateral movements without sudden shifts, thereby preventing instability or aggressive maneuvers.

The primary role of this function is to ensure that the vehicle's lateral movement is bounded by a continuously differentiable curve, preventing abrupt changes and providing a more controlled maneuver as the vehicle either merges onto the highway or diverges from it. By incorporating a hyperbolic tangent function, the model allows for a soft transition between the boundaries, making the lane-changing process smooth and predictable. The role of each variable in the equation is explained below:

$$y_{on}^{b} = \begin{cases} y_1^l \cdot (1 - H_1^l) + y_2^l \cdot H_1^l + H_1^l \cdot (y_2^l - y_1^l) \cdot \tanh\left(s_1^l \cdot (\mathrm{x} - o_1^l)\right), & x < o_1^l \\ y_1^l \cdot (1 - H_1^l) + y_2^l \cdot H_1^l + H_1^l \cdot (y_2^l - y_1^l) \cdot \tanh\left(\frac{H_1^l}{1 - H_1^l} \cdot s_1^l \cdot (\mathrm{x} - o_1^l)\right), & x \geq o_1^l \end{cases} \tag{2.9}$$

$$y_{off}^{b} = \begin{cases} y_2^l \cdot (1 - H_2^l) + y_1^l \cdot H_2^l + H_2^l \cdot (y_1^l - y_2^l) \cdot \tanh\left(\frac{H_2^l}{1 - H_2^l} \cdot s_1^l \cdot (\mathrm{x} - o_2^l)\right), & x < o_2^l \\ y_2^l \cdot (1 - H_2^l) + y_1^l \cdot H_2^l + H_2^l \cdot (y_1^l - y_2^l) \cdot \tanh\left(s_1^l \cdot (\mathrm{x} - o_2^l)\right), & x \geq o_2^l \end{cases} \tag{2.10}$$

The notation y_{on}^{b} refers to the boundary conditions associated with the on-ramp, while y_{off}^{b} denotes the boundary conditions for the on-ramp. The sub-script l in the values signifies that the function represents the lower boundary for both the on-ramp and the off-ramp, respectively.

In contrast, if we aim to simulate the upper boundary, we need to utilize the corresponding values, which are presented in the following table. These upper values are used to represent the upper bound for the on-ramp and off-ramp scenarios, ensuring a comprehensive definition of both lower and upper bounds for vehicle movement in these areas.

Table 1: Sigmoid boundary parameters

Boundary Parameters	
y_1^l	0
y_1^u	4.0
y_2^l	4.0
y_2^u	14.0
H_1^l	0.5
H_1^u	0.5
o_1^l	1100
o_1^u	800

o_2^l	800
o_2^u	1110
s_1^l	0.1
s_1^u	0.1

Parameters of the function

$\boldsymbol{H_1}$: Controls where the sigmoid function reaches its midpoint. It determines the influence of the slope on lateral positioning, effectively adjusting how rapidly the transition occurs.

$\boldsymbol{y_1}$: The 1st distinct lateral level. Three levels need to be connected via a smooth line to form the complete left boundary of the itinerary to be considered by the corresponding vehicles, as they move longitudinally through the road network. T. The $\mathrm{y_1}$ level is set according to the boundary of the on-ramp and off-ramp acceleration lane.

$\boldsymbol{y_2}$: The 2nd distinct lateral level. The $\mathrm{y_2}$ level is set according to the boundary of main road.

$\boldsymbol{s_1}$: the slope value s_1 at the offset point calibrates how steeply the boundary moves from y_1 to y_2 and via versa.

$\boldsymbol{o_1}$: To connect the three levels, longitudinal offset locations are specified, imposing where (longitudinally) the smooth change between two consecutive lateral levels takes place. For instance, the first offset o_1 in the figure determines the merging area into the main highway.

$\boldsymbol{o_2}$: The second offset o_2 in the figure determines the diverging area into the off-ramp section.

$\boldsymbol{X}$: The x represents the vehicle's current longitudinal position, which serves as the input to the function. This value is used as reference to determine whether is before or after the offset.

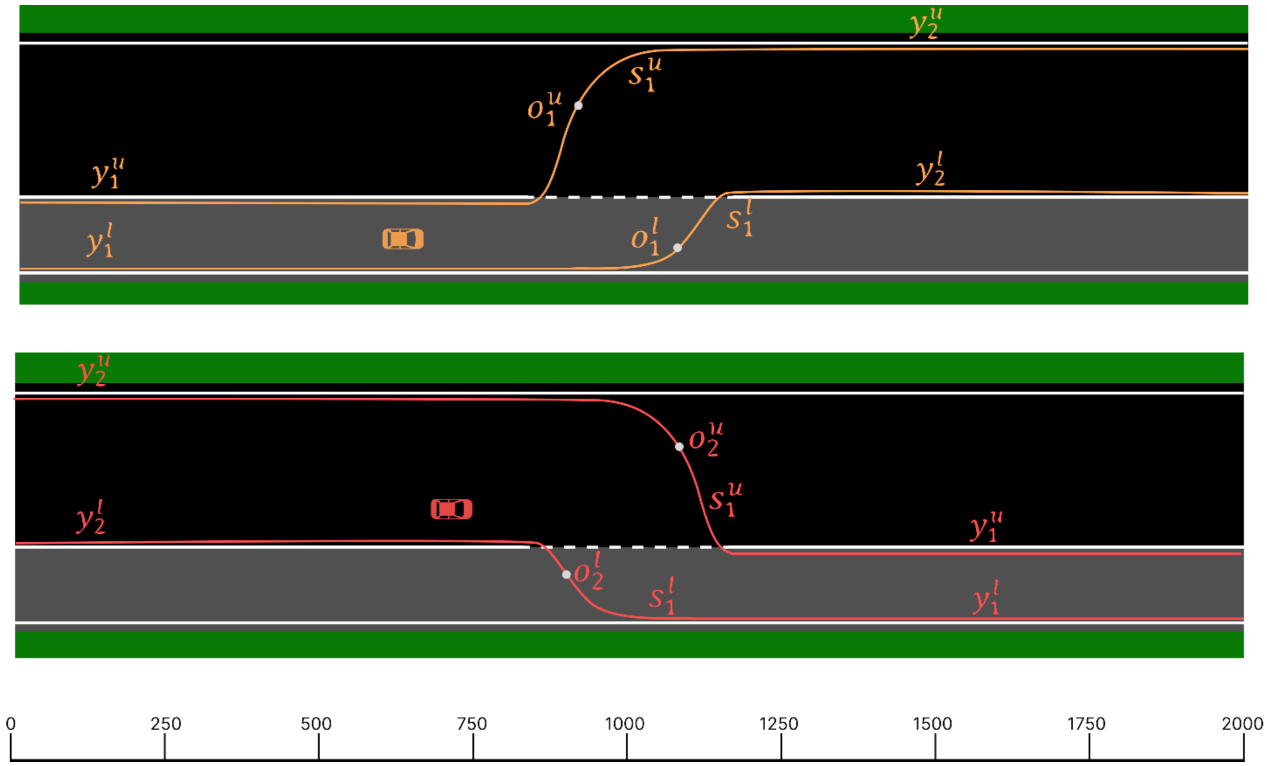


Figure 7: Road boundaries

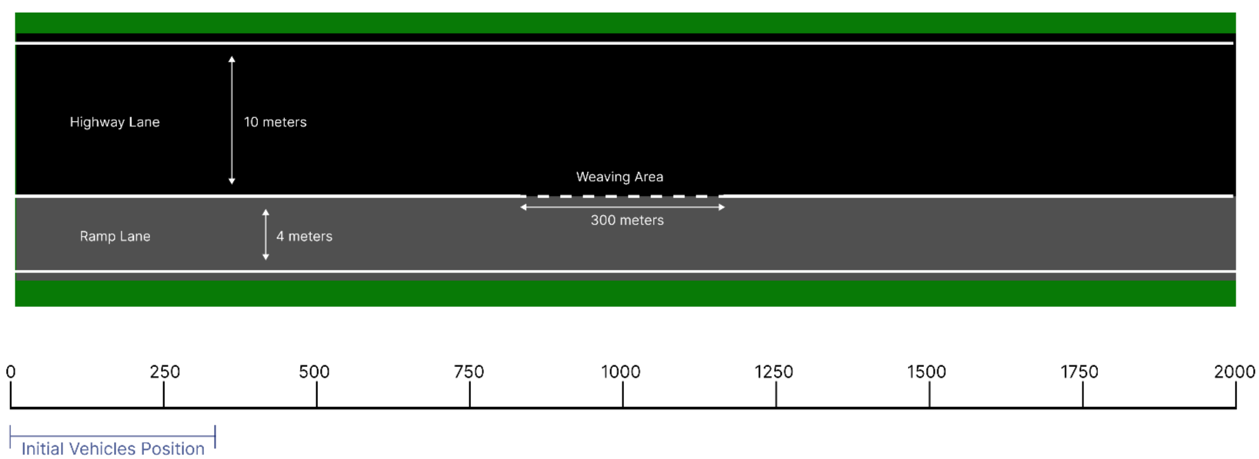


Figure 8: Highway design with on-ramp, off-ramp and weaving section

Functions Behavior:

The function calculates the boundary values based on whether the vehicle's longitudinal position is before or after the offset.

- Before the offset: If the vehicle's position is before the offset, the function computes the boundary value using smooth sigmoid curve, transitioning from y_1 and y_2 on the case of the on-ramp and via versa on the case of the off-ramp.
- After the offset: Once the vehicle passes the offset, the function continues the transition using a modified version of the sigmoid, ensuring the boundary progression.

The function returns the calculated lateral boundary, providing a smooth interpolation between the lower and upper bound. This is achieved through the use of the tanh function, which ensures smooth transitions without abrupt changes.

Function Adjustment:

The function is totally adaptable and can be used in various road configurations. By adjusting parameters such as y_1, y_2 and o_1, o_2 we are able to control the lateral and longitudinal positioning of the weaving section respectively. Alternatively, by adjustments on s_1 and H_1 occur on geometrical changes of the bounds.

In summary, this function offers a flexible and robust mechanism for calculating smooth transitions for Connected and Automated Vehicles, particularly in challenging road sections like on-ramps and off-ramps. It's ability to interpolate lateral positions smoothly helps improve traffic safety and efficiency.

The primary condition for lateral movement is for the vehicle to remain within the road boundaries or to traverse precisely along a border without exiting upon it. Furthermore, certain hypothetical borders may be established under exceptional emergency situations.

2.3 Objective Function for Joint Trajectory Optimization

The objective function seeks to minimize five sub-objectives: passenger comfort, fuel consumption, attainment of target speeds for all cars, collision avoidance, and mitigation of infeasible maneuvers. Every sub-objective must be continuous and differentiable [12].

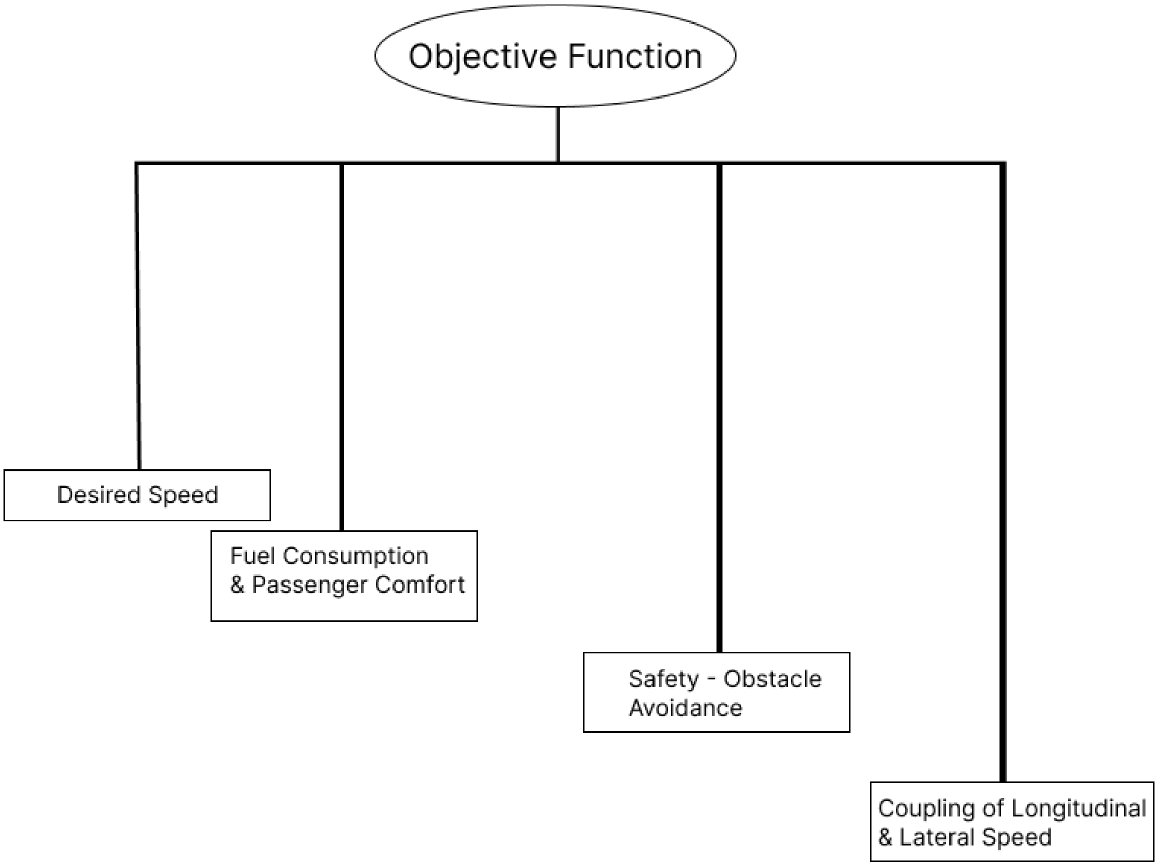


Figure 9: Objective function

Sub-objectives: fuel consumption, reaching the desired speed, passenger comfort, collision avoidance and mitigation of infeasible maneuvers. Several vehicles are considered, each with predetermined longitudinal and lateral speeds.

Sub-objectives: fuel consumption and passenger comfort, desired speed, collision avoidance, coupling of longitudinal and lateral speeds

1) **Fuel Consumption and Passenger Comfort**

Two quadratic cost terms of longitudinal and lateral accelerations, $a_{x,i}(k)^2$ and $a_{y,i}(k)^2$, respectively, are considered in the objective function for each vehicle i, which is known to reduce the fuel consumption [26] and also results in moderate and smooth accelerations and improved passenger convenience.

2) **Desired Speed**

All vehicles aim to achieve their desired longitudinal and lateral speeds, ($\dot{V}_{x,i}$, $\dot{V}_{y,i}$), in a coordinated way. For each vehicle i, two quadratic cost terms, $(V_{x,i}(k) - \dot{V_{x,\iota}})^2$ and $(V_{y,i}(k) - \dot{V_{y,\iota}})^2$ are used to penalize the deviations between longitudinal and lateral velocity from their corresponding desired values. $\dot{V_{x,\iota}}$ is assigned to a positive value, while $\dot{V_{y,\iota}}$ is assigned to zero in order to prevent needless lateral movements.

3) Collision Avoidance

A sub-objective term is included in the cost function to prevent each group vehicle i from colliding with other vehicles of the group or out of the group (obstacles). Whenever vehicles i and j approach each other, the group vehicle i faces a cost around vehicle j, and this incurs a corresponding positive cost value that increases with decreasing inter-vehicle distance. The same applies to vehicle j, if it is a group vehicle, vis-à-vis vehicle. This cost aims at discouraging vehicles i and j from approaching too close to each other or collide.

More specifically, the collision-avoidance cost (or potential) function reads:

$$c_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j) = exp^{-d_{ij}} \tag{2.11}$$

where d_{ij} is a notion of inter-vehicle elliptical distance that depends on the physical dimensions, the positions and the speeds of both vehicles, the latter being included to account for increased collision risk at higher speeds. We opt for an elliptical relative distance $v(\sqrt{(\frac{\Delta x_{ij}}{L_{ij}})^2 + (\frac{\Delta y_{ij}}{W_{ij}})^2}) = v\frac{(\sqrt{(\Delta x_{ij})^2 + p_{ij}(\Delta y_{ij})^2}}{L_{ij}})$ (where $\Delta x_{ij} = x_i - x_j$ and $\Delta y_{ij} = y_i - y_j$) with ellipse eccentricity factor $p_{ij} = \frac{(L_{ij}}{W_{ij}})^2$, that determines the shape (length versus width) of the iso-distance ellipses of d_{ij}, while v is a constant scaling parameter.

Since vehicle speeds on highways may differ significantly, one may augment the physical vehicle dimensions and constant space-gaps to also consider speed-dependent longitudinal and lateral time gaps. An effective length is created L_{ij}^{eff} by use of a longitudinal time-gap function.

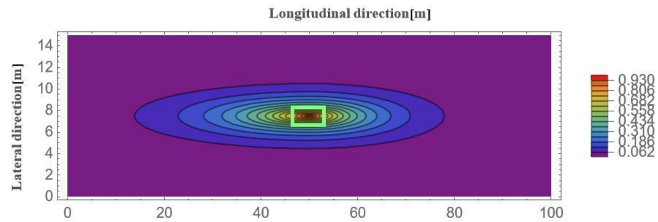


Figure 10: Illustration of the collision avoidance

ω_x, which is multiplied with the longitudinal speed of whichever vehicle is the follower, i.e. if vehicle i is following the vehicle j, the effective length equals $L_{ij} + \omega_x v_{x,i}$; and in case vehicle j is the follower, the effective length becomes $L_{ij} + \omega_x v_{x,j}$. Therefore, the effective length reads:

$$L_{ij}^{\text{eff}} = L_{ij} + \omega_x \left(\frac{(v_{x,i}+v_{x,j})}{2} + \frac{\tanh(x_i - x_j)(v_{x,j} - v_{x,i})}{2} \right) \tag{2.12}$$

In the lateral direction, a lateral time-gap ω_y is defined and activated, only when vehicles are approaching each other laterally. We reach to the effective width W_{ij}^{eff} :

$$W_{ij}^{\text{eff}} = W_{ij} + 0.5\Big(\omega_y\big((v_{y,i} - v_{y,j})\, tanh(\, y_j - y_i) + \sqrt{(v_{y,i} - v_{y,j})^2\, tanh(\, y_j - y_i)^2 + \varepsilon_\omega}\big)\Big) \tag{2.13}$$

where ε_ω is a small positive value.

Based on (2.12) and (2.13), the time-gap dependent, hence speed-dependent ellipse eccentricity factor p_{ij} is given by:

$$p_{ij} = \left(\frac{L_{ij}^{\text{eff}}}{W_{ij}^{\text{eff}}}\right)^2. \tag{2.14}$$

Thus, by utilizing this p_{ij} factor, the distance d_{ij} (and the collision avoidance potential function) is longitudinally elongated to account for rectangular vehicle shapes, as well as for higher longitudinal vehicle speeds, which imply higher longitudinal collision risk.

Based on the above, the elliptical relative distance d_{ij} with time-gap consideration is defined as:

$$d_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j) = v \frac{\sqrt{(x_i - x_j)^2 + p_{ij}(y_i - y_j)^2}}{L_{ij}^{\text{eff}}}. \tag{2.15}$$

The elliptical distance in (2.15) is used in the potential function (2.11) to penalize vehicles approaching too close to each other. Note that the potential function is symmetric, meaning that $c_{ij} = c_{ji}$. Thus, when considering a group of n vehicles, the collision avoidance sub-objective is

$$\sum_{j=i+1}^{n} c_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j). \tag{2.16}$$

Fig. 1 illustrates the collision avoidance function seen by vehicle i around vehicle j, for a case where vehicles i and j are moving at 35 m/s and 25 m/s longitudinal speeds, respectively. The green rectangle represents the vehicle's j physical dimensions.

4) Coupling of Longitudinal and Lateral Speeds

The longitudinal and lateral movements are presumed to be decoupled, as indicated by the vehicle dynamics in (2.1 – 2.4), a mainly reasonable assumption at medium to high longitudinal speeds without significant turning angles. At extremely low longitudinal speeds, infeasible lateral movements may occur. Such conditions correspond to a high ratio of lateral velocity to longitudinal velocity. To mitigate such infeasible maneuvers, a sub-objective that penalizes deviations of that rate beyond a specific parameter β, is provided as follows:

$$f_i^c = \begin{cases} \left(\beta v_{x,i}(k) - \left|v_{y,i}(k)\right|\right)^2 & if \left|v_{y,i}(k)\right| > \beta v_{x,i}(k) \\ 0 & otherwise \end{cases} \tag{2.17}$$

Where β is a small, positive value.

The objective is to minimize the overall function (2.18), encompassing all sub-objectives, which account for all vehicle-related expenditures as follows:

$$J = \sum_{k=0}^{K-1} \sum_{i=1}^{n} \left(\frac{1}{2} b_1 \left(a_{x,i}(k)\right)^2 + \frac{1}{2} b_2 \left(a_{y,i}(k)\right)^2 + \frac{1}{2} b_3 \left(v_{x,i}(k) - v_{x,i}^*\right)^2 + \frac{1}{2} b_4 \left(v_{y,i}(k) - v_{y,i}^*\right)^2 + \frac{1}{2} b_5 f_i^c + b_6 \sum_{j=i+1}^{n} c_{ij}\left(x_i, x_j\right) \right) \quad (2.18)$$

b_1 to b_6 represent the weighting factors for each subobjective, which must be selected suitably, and K is the time horizon. The acceleration $a_{x,i}$ and $a_{y,i}$ are dependent on the new control variables u_x and u_y.

The vehicle's performance is monitored and assessed in initial simulations comprising various driving scenarios; weights are adjusted if specific sub-objectives are found to exceed expectations (to the detriment of other sub-objectives) or to fall short; this process continues until a satisfactory overall vehicle performance is achieved. Once suitable weights have been designated in this manner, the sensitivity of optimal control decisions concerning varying weight values of the same magnitude is typically minimal.

The Optimal Control Problem (OCP) is characterized by the minimization of the objective function, which integrates all the aforementioned sub-objectives, while adhering to the equality requirements (state equations) specified in (2.1 – 2.4) and the constraints.

2.4 Formulation of the Optimal Control Problem (OCP)

A centralized nonlinear constrained Optimal Control Problem (OCP) featuring fixed and state-dependent limits on control inputs, is solved for a group of Connected and Automated Vehicles (CAVs) numerically using an efficient Feasible Direction Algorithm (FDA). The Optimal Control Problem (OCP) is solved.

The general form of the consider Optimal Control Problem is:

$$J = \sum_{k=0}^{K-1} \Phi[x(k), u(k)] \quad (2.19)$$

$$x(k+1) = f\,[x(k), u(k)] \quad (2.20)$$

$$u^{\min} \leq u(k) \leq u^{\max} \quad (2.21)$$

Where u^{max} and u^{min} are constant lower and upper bounds on controls, respectively. The Hamilton function of this OCP reads:

$$H[x(k), u(k), \lambda(k+1)] = \lambda(k+1)^T f[x(k), u(k)] + \Phi[x(k), u(k)] \tag{2.22}$$

Where $\lambda(k)$ are the co-states, associated with the state equations. The necessary conditions for a optimality is given below. The state equation is:

Where $\lambda(k)$ represents the co-states linked with the state equations. The necessary conditions for a optimality are outlined below. The state equation is as follows:

$$x(k+1) = \frac{\partial H}{\partial \lambda(k+1)} = f[x(k), u(k)] \tag{2.23}$$

The control condition is as follows:

$$\frac{\partial H}{\partial u_i(k)} = \begin{cases} < 0 \ \ if & u_i(k) = u_i^{max} \\ = 0 \ \ if & u_i^{min} \leq u_i(k) \leq u_i^{max} \\ > 0 \ \ if & u_i(k) = u_i^{max} \end{cases} \tag{2.24}$$

Taken for all control variables $u_i(k)$. The co-state equation is:

$$\lambda(k) = \frac{dH}{dx(k)} \tag{2.25}$$

And the boundary conditions are given by:

$$\begin{aligned} x(0) &= x_0 \\ \lambda(K) &= 0 \end{aligned} \tag{2.26}$$

2.5 Numerical Solution

An efficient Feasible Direction Algorithm (FDA) is utilized to numerically address this Optimal Control Problem (OCP) is employed to tackle the Optimal Control Problem. The application of FDA is aided by the smoothness of all relevant functions, the established OCP exhibits box-type characteristics. Regulate restrictions with fixed limits. The approach utilizes the explicit structure of the state equations and employs the essential criteria of optimality to address the OCP in the significantly diminished scope of the control variables, namely in an mk-dimensioanl space, where m denotes the quantity of controls variables. . The aim is to achieve a control vector u(k), where k=0,…,K-1, that corresponds to a local minimum of the cost function, while adhering to the state equations and all limitations. Consequently, the FDA utilizes the equation $dH/d_u(k) = [df/d_u(k)]^T \lambda(k+1) + d\Phi/d_u(k)$ is equivalent to the decreased gradient in the mk-dimensional control space, if the states and co-states implicated in the partial derivative fulfill the state and co-state equations. The regulations variables must adhere to the constant bound requirements. Consequently, the saturation vector function is delineated by the elements:

$$sat_i(\boldsymbol{u}(k)) = \begin{cases} u_i^{max} & if \quad u_i(k) > u_i^{max} \\ u_i(k) & if \quad u_i^{min} \le u_i(k) \le u_i^{max} \\ u_i^{min} & if \quad u_i(k) < u_i^{min} \end{cases} \quad (2.27)$$

Also, the projected gradient γ, to account for active constraints is defined as:

$$\gamma_i(k) = \begin{cases} 0 & if \quad u_i(k) = u_i^{min} and \quad g_i(k) > 0 \\ 0 & if \quad u_i(k) = u_i^{max} and \quad g_i(k) < 0 \\ & g_i(k) else \end{cases} \quad (2.28)$$

The FDA is an iterative process that begins with a feasible initial guess for the control trajectories. In each iteration, employing reduced gradient information, an appropriate step is chosen in the mK-dimensional control space to optimize control trajectories. This study develops a possibly improved control trajectory using Resilient Backpropagation (RPROP) [26]. The RPROP method calculates the necessary modifications for the control variables, $\Delta u^{(l)}$(k), exclusively based on the signs of the gradient components as detailed below:

$$\Delta u_i^{(l)}(k)=\begin{cases}-\text{sign}(g_i^{(l)}(k)) & \text{if } g_i^{(l-1)}(k)g_i^{(l)}(k)>0\\ \eta^{+}\Delta u_i^{(l-1)}(k) & \\ -\text{sign}(g_i^{(l)}(k)) & \text{if } g_i^{(l-1)}(k)g_i^{(l)}(k)<0\\ \eta^{-}\Delta u_i^{(l-1)}(k) & \\ \Delta u_i^{(l-1)}(k) & \text{otherwise}\end{cases} \tag{2.29}$$

Where $0 < \eta^{-} < 1 < \eta^{+}$. The method targets each control $u_i(k)$ and advances in the negative direction of the corresponding gradient $g_i(k)$.If the gradient has reversed in sign since the last iteration, a local minimum has likely been overpassed; hence, the step size is reduced by η^{-}. If the gradient maintains the same sign as in the prior iteration, the step size is increased by $\eta^{+}$to accelerate convergence to the component minimum. The updated control trajectory is fed into the next iteration until a sufficiently low projected-gradient norm is attained. The FDA's algorithmic techniques are outlined in the following table.

[1]	Receive initial states
[2]	Guess a feasible initial control trajectory $u^0(k)$ for k=0,…,K-1
[3]	Compute states x^0 (k) for k=0,…,K-1 from [2.18] with initial state value [2.25]
[4]	Compute co-states $\lambda^0(k)$ for k=0,…,K-1 from [2.25] starting from [2.26]
[5]	Compute $g^0(k)$
[6]	Set iteration index l = 0
[7]	While l < max-iterations
[8]	Compute $u^{l+1}(k) = sat[u^l(k) + \Delta u^l(k)] and\ \ x^{(l+1)}(k)\ for\ k = 0, \ldots, K-1\ from$ [17] $with\ initial\ value$ [2.29]
[9]	Compute co-states $\lambda^{(l+1)}(k)$ for k=0,…,K-1 from [2.25], starting with [2.26]
[10]	Compute $g^{(l+1)}(k)$
[11]	Compute projected gradient $\gamma^{(l+1)}(k)$
[12]	If not converge
[13]	Index increment $l\ =\ l + 1$

[14]	Continue
[15]	else
[16]	Break
[17]	End if
[18]	End while
[19]	Generate optimal control input $u(k)$ for k=0,…,K-1

The FDA iterations may be terminated at any point, even prior to convergence, resulting in a control trajectory that is feasible—fulfilling all state equations and constraints—but potentially suboptimal. The current OCP is non-convex; hence, the obtained minimum meets the problem's criteria, as elaborated in the subsequent section.

2.6 Simulation Tools

We conducted simulation tests to evaluate the effectiveness of the proposed approach. The simulations took place in a lane-free environment, utilizing a custom extension developed for the Dynamic Systems and Simulation Laboratory's visualization needs, which is coded in a Javascript environment. We created this extension based on two concepts from the innovative paradigm known as TrafficFluid. Firstly, the environment in which a vehicle operates lacks designated lanes, allowing cars to occupy any position on the roadway. Secondly, the cars exert a nudging effect on other vehicles inside their contact zone. This simulator aims to facilitate the implementation of diverse bespoke movement tactics in a lane-free environment, using the nudging effect. Programming allows the user to create and evaluate scenarios. Ultimately, the simulator delivers a variety of information to the user, including vehicle attributes such as width and length, current status indicators like position and speed, as well as details regarding the operational road conditions.

2.7 Scenario Description: Weaving Areas with On-ramps and Off-ramps

To demonstrate the efficiency of the proposed approach, three challenging scenarios are tested, considering a group of 12 and 20 vehicles driving lane-free on a straight road while some of them are trying to merge via an offramp and some other to escape via an off-ramp.

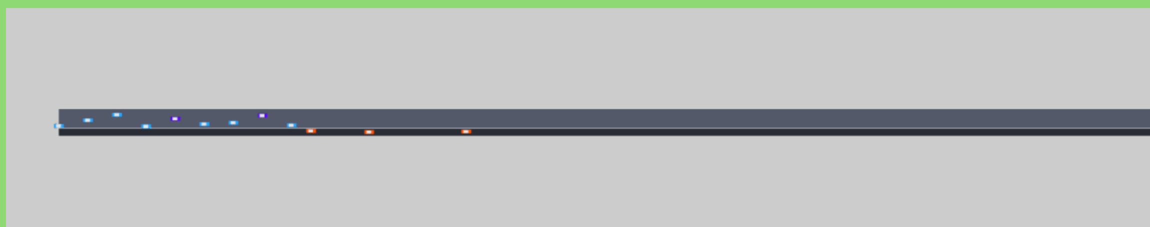
Every vehicle has its own longitudinal desired speed and zero lateral desired speed. At the initial state, the vehicles are positioned behind each other at random lateral positions, with inter-vehicle distances ranging from 15-30 meters, therefore, vehicles need via appropriate maneuvers to manage to merge in the main road and reach their desired speeds simultaneously.

A lane-free straight road with a 10 m width parallel under which is a 4 m width road that drives into a weaving section with a common acceleration deceleration area is considered for the tests. All the vehicles have the same dimensions of 5 m length and 2 m width. A 10.0-meter road width accommodates three lanes of normal traffic.

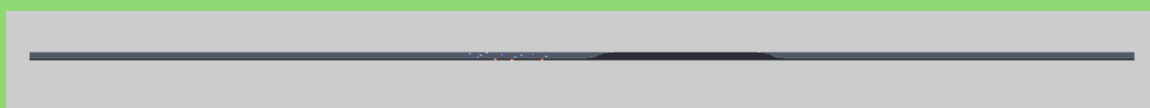
The parameters used for RPROP are $\eta^{+} = 1{,}2$ and $\eta^{-} = 0{,}5$. In (2.18), the penalty weights are $\{b_1,\ b_2, b_3, b_4, b_5, b_6\} = \{0.1,\ 0.01,\ 4.0,\ 1.0,\ 3.0,\ 0.1, 0.0,\ 6.0\}$ and the discrete time step is T=0.25 s. All lateral initial speeds are zero. The longitudinal desired speeds are given in the table that refers to each of the scenarios, while the lateral desired speeds are zero.

Freeway weaving portions refer to the intersection of two or more traffic flows moving in the same general direction across an extended stretch of highway, lacking traffic control devices.

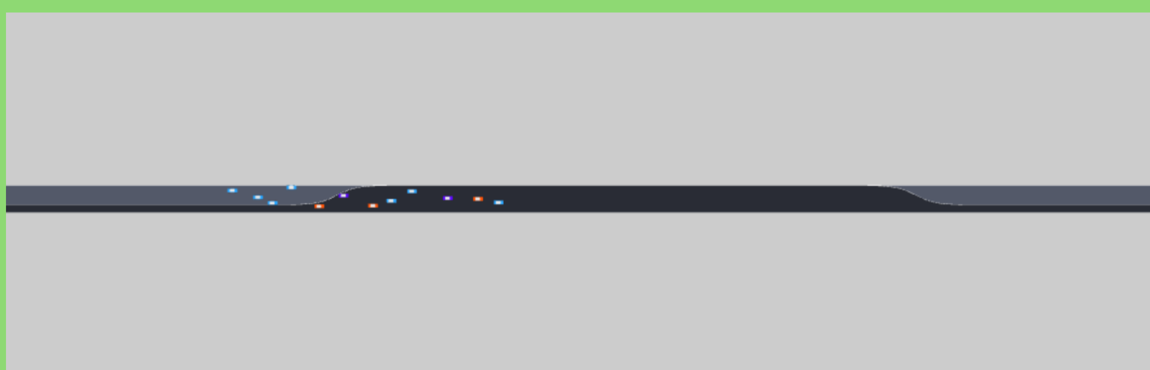
An image of the visualizer layout used can be seen in the figures above:



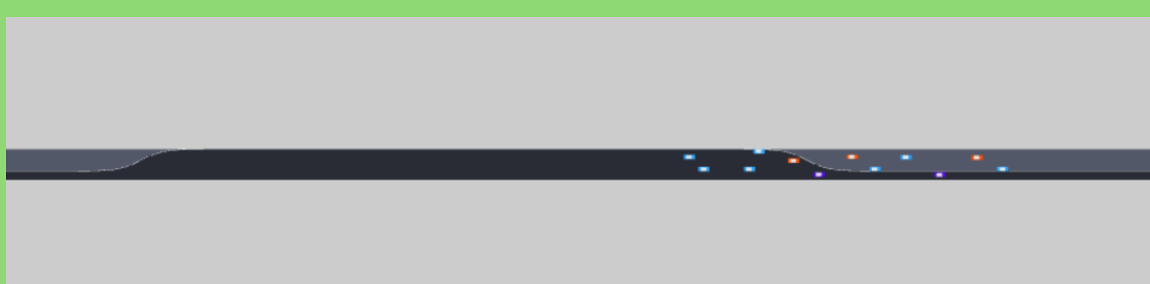
(a) Simulation layout vehicles initial positions – zoomed in view



(b) Simulation layout - full view setup



(c) Start of weaving zone - zoomed in view



(d) End of weaving zone – zoomed in view

Figure 11: Simulation environment

2.7.1 Scenario 1

In the first scenario, a total of 12 vehicles are considered. Nine of these vehicles are initially positioned at the beginning of a 2000-meter highway. The remaining three vehicles are positioned on a parallel road adjacent to the main highway, leading into a weaving section. The objective in this scenario is for the three on-ramp vehicles to successfully merge onto the main highway, while simultaneously, two other vehicles diverge from the main road, creating a complex crossing scenario.

To better reflect real-world conditions, the vehicles on the on-ramp have an initial longitudinal velocity lower than that of the vehicles on the main highway, mimicking the speed limits commonly enforced in merging sections. These on-ramp vehicles begin accelerating a few steps before merging onto the highway, further increasing the complexity of the scenario by simulating realistic acceleration behavior as they prepare to enter the main traffic flow. In Scenario 1, the cars successfully attain their designated longitudinal speeds and navigate through the other vehicles without any collisions.

All the initial and desired parameters used in this scenario are presented in the following table:

Table 2: Scenario 1 parameters

Position x	Position y	Initial x velocity	Desired x velocity
Vehicle 1			
0.0	5.4	12.0	12.0
Vehicle 2			
15.0	8.3	12.0	12.0
Vehicle 3			
30.0	11.1	12.0	12.0
Vehicle 4			
45.0	5.2	12.0	12.0
Vehicle 5 (Off-ramp vehicle)			
60.0	9.1	12.0	12.0
Vehicle 6			
75.0	6.3	12.0	12.0
Vehicle 7			
90.0	7.0	12.0	12.0
Vehicle 8 (Off-ramp vehicle)			
105.0	10.7	12.0	12.0
Vehicle 9			

120.0	5.7	12.0	12.0
Vehicle 10 (On-ramp Vehicle)			
130.0	2.8	12.0	12.0
Vehicle 11 (On-ramp Vehicle)			
160.0	2.2	12.0	12.0
Vehicle 12 (On-ramp Vehicle)			
210.0	2.5	12.0	12.0

2.7.2 Scenario 2

This scenario introduces a higher level of complexity compared to the previous one, as each vehicle must not only interact with other vehicles within the group but also navigate around obstacles in the area. The obstacles in this scenario are vehicles that are not included in the optimization process. These vehicles are initialized with a predetermined position and an initial longitudinal velocity lower than that of the optimized vehicles. Additionally, the obstacles are strategically placed on the lower part of the highway to further complicate the merging process. Obstacles in this context can be considered as independent, non-automated vehicles or any other entities present within the infrastructure, adding an additional layer of complexity to the process.

As a result, the merging vehicles are required to maneuver around the obstacles to successfully merge onto the highway. These additional maneuvers increase the demands on both longitudinal and lateral vehicle movements. Despite this, the control and trajectory responses of the vehicles remain smooth and controlled throughout the process.

All the initial and desired parameters used in this scenario for the vehicles and obstacles are presented in the following tables:

Table 3: Scenario 2 parameters

Position x	Position y	Initial x velocity	Desired x velocity
Vehicle 1			
0.0	9.4	14.0	14.0
Vehicle 2			
15.0	8.3	14.0	14.0
Vehicle 3			
30.0	6.1	14.0	14.0
Vehicle 4			
45.0	8.2	14.0	14.0
Vehicle 5			
60.0	10.1	14.0	14.0
Vehicle 6			
75.0	9.3	14.0	14.0
Vehicle 7			
90.0	7.0	14.0	14.0
Vehicle 8			
105.0	10.7	14.0	14.0
Vehicle 9			
120.0	5.7	14.0	14.0
Vehicle 10			
80.0	2.8	12.0	14.0
Vehicle 11			
130.0	2.2	12.0	14.0
Vehicle 12			
180.0	2.5	12.0	14.0

Table 4: Scenario 2 obstacle parameters

Position x	Position y	Initial x velocity	Desired x velocity
Obstacle 1			
280.0	5.2	10.0	10.0
Obstacle 2			
340.0	5.5	10.0	10.0
Obstacle 3			
410.0	5.2	10.0	10.0

2.7.3 Scenario 3

In our final scenario, to evaluate vehicle performance and trajectories under higher traffic density, we positioned 20 vehicles on the road. Of these 20 vehicles, 5 attempt to merge onto the highway via the on-ramp, while another 3 diverge via the off-ramp. Despite the increased complexity, no collisions were observed during this scenario.

Table 5: Scenario 3 parameters

Position x	Position y	Initial x velocity	Desired x velocity
Vehicle 1			
0.0	5.4	12.0	12.0
Vehicle 2			
30.0	8.3	12.0	12.0
Vehicle 3			
60.0	11.1	12.0	12.0
Vehicle 4			
90.0	5.2	12.0	12.0
Vehicle 5			
120.0	9.1	12.0	12.0
Vehicle 6			
150.0	11.3	12.0	12.0
Vehicle 7			
180.0	6.4	12.0	12.0
Vehicle 8 (Off-ramp vehicle)			
210.0	10.7	10.0	12.0
Vehicle 9			
240.0	7.7	12.0	12.0
Vehicle 10 (Off-ramp vehicle)			
270.0	6.8	10.0	12.0
Vehicle 11			
300.0	8.2	12.0	12.0
Vehicle 12			
330.0	7.5	12.0	12.0
Vehicle 12 (On-ramp Vehicle)			
360.0	5.3	10.0	12.0
Vehicle 13			
390.0	8.6	12.0	12.0
Vehicle 14			
420.0	11.2	12.0	12.0
Vehicle 15			
450.0	5.6	12.0	12.0
Vehicle 16 (On-ramp Vehicle)			
195.0	2.2	10.0	12.0
Vehicle 17 (On-ramp Vehicle)			

280.0	1.4	10.0	12.0
Vehicle 18 (On-ramp Vehicle)			
330.0	2.7	10.0	12.0
Vehicle 19 (On-ramp Vehicle)			
380.0	1.7	10.0	12.0
Vehicle 20 (On-ramp Vehicle)			
440.0	1.2	10.0	12.0

In the following figure, the arrangements for the three scenarios are depicted. The blue color represents the vehicles on the main road, the red indicates the vehicles diverging onto the off-ramp, and the orange represents the vehicles merging from the on-ramp. Lastly, on the scenario 2 the obstacles are the purple vehicles.

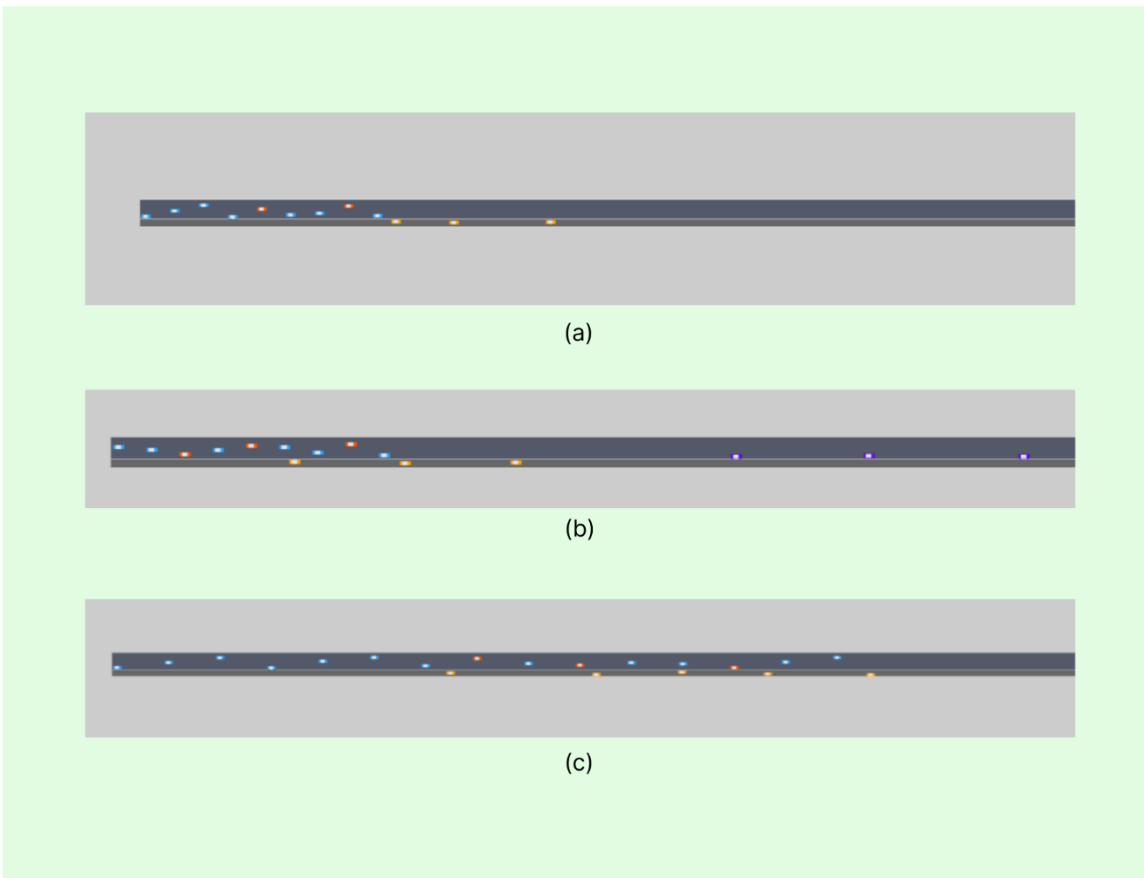


Figure 12: Illustration of scenario arrangements: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3.

CHAPTER 3

Simulation Results and analysis

3.1 Results from On-ramp and Off-ramp scenarios

Focusing on the movements of individual cars, it is important to note that there are no collisions in any of the provided findings, although there are infrequent emergencies, specifically instances where the initially estimated vehicle trajectory indicates a collision, necessitating the design of a new course. In the subsequent chapter, typical trajectories of all vehicles will be analyzed to assess their microscopic behavior. In all simulations, the vehicles on the roadway commence at an initial velocity. Vehicles entering the highway ramp commence with a reduced initial velocity that corresponds to the speed restriction in such zones. Vehicles on the entrance ramp, just before to the highway entrance elevation, can accelerate to match the desired longitudinal speed of other vehicles. Furthermore, executing suitable moves in relation to the other vehicles encountered.

It is evident that, for all necessary maneuvers across all scenarios, the acceleration values in the longitudinal direction remain within a reasonable range, which is crucial for passenger comfort and enhances fuel efficiency. The histograms of data at the points of highway entry and exit need specific observation. Histograms of lateral speed and acceleration values for all vehicles affirm a comfortable driving experience across the board.

The established lateral vehicle motion is adaptable and fluctuates among various vehicle densities and situations. At subcritical concentrations, the car seems to merge into the freeway along the lower lateral boundary established by the lateral boundary function. Lateral motions are notably more extensive in the weaving zone. In such instances, the pronounced lateral displacements indicate the vehicle's endeavor to identify favorable opportunities for acceleration to achieve the intended speed. The vehicle occasionally maneuvers just to the left or right of the road boundaries, approaching them asymptotically without crossing them. Efficient vehicle maneuvers enhance vehicle progression and overall traffic efficiency, resulting in increased flow rates. At critical or supercritical densities, vehicles on the ramp interact, often leading to earlier entry onto the freeway if regarded more advantageous. The last presentation of this thesis features videos from the simulator that illustrate the vehicle movements and the weaving process. In the videos, the camera focuses on a single car, indicated by the mouse pointer, causing all other vehicles to seem to move in relation to the tracked vehicle's movements. Illustrations from the aforementioned arrangement are presented in the figure.

3.1.1 Vehicle Trajectories in Lane-free Weaving Areas

Scenario 1

The figure displays the longitudinal and lateral movements of all 12 vehicles in the group. It can be observed that all vehicles successfully reached their desired speeds and completed the highway segment. Additionally, in the graphs depicting lateral movements—such as lateral velocity and lateral acceleration—there are noticeable peaks for the vehicles that are merging and diverging. This is expected, as these vehicles are constrained by the hyperbolic tangent function boundary we implemented. It is also important to note that no collisions occurred, as the optimization effectively minimized the high cost associated with the collision avoidance term, whose value in the results remains close to zero. For a detailed visualization, you can watch the https://bit.ly/3Ycbi8y to see the movements of the vehicles in real-time.

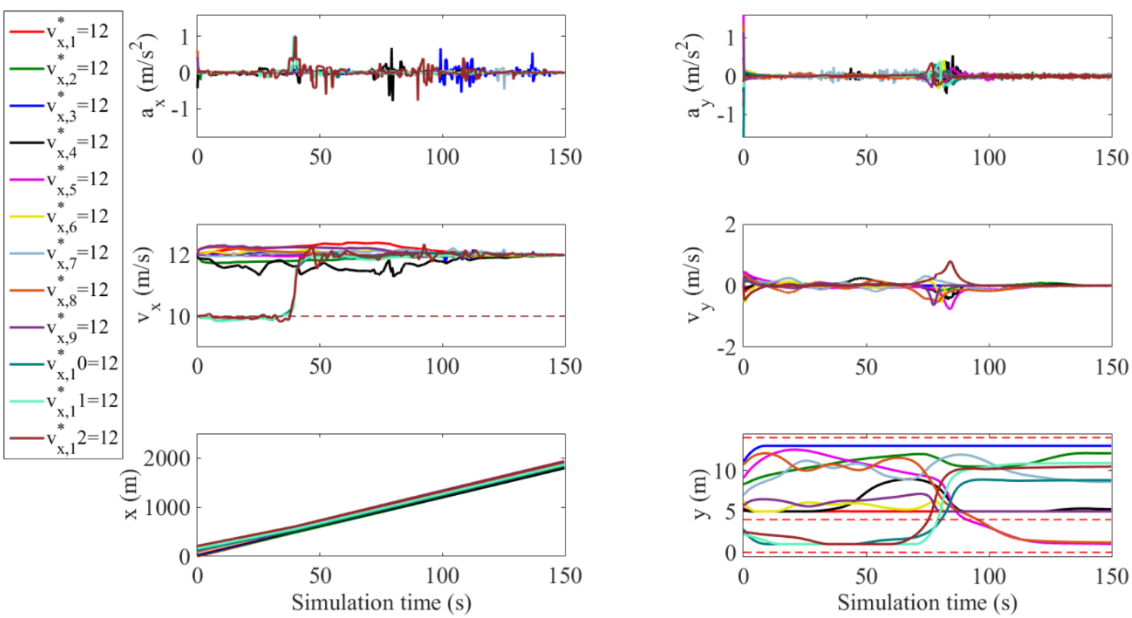


Figure 13: Vehicle trajectories scenario 1

Scenario 2

In Scenario 2, additional obstacles were strategically placed at key positions where merging and diverging vehicles would encounter them while attempting to enter or exit the highway, respectively. In the trajectory diagram, it is evident that the graphs for lateral acceleration and lateral velocity display more peaks. This behavior is expected, as the vehicles perform more complex maneuvers to avoid the obstacles. Additionally, the longitudinal velocity graph shows some fluctuations, particularly in the merging zone, which likely results from the intensive maneuvers required to navigate around the obstacles. Overall, these observations reflect the increased complexity of this scenario compared to Scenario 1, with the vehicles successfully managing both merging and obstacle avoidance. For a detailed visualization, you can watch the https://bit.ly/4eUTNiz to see the movements of the vehicles in real-time.

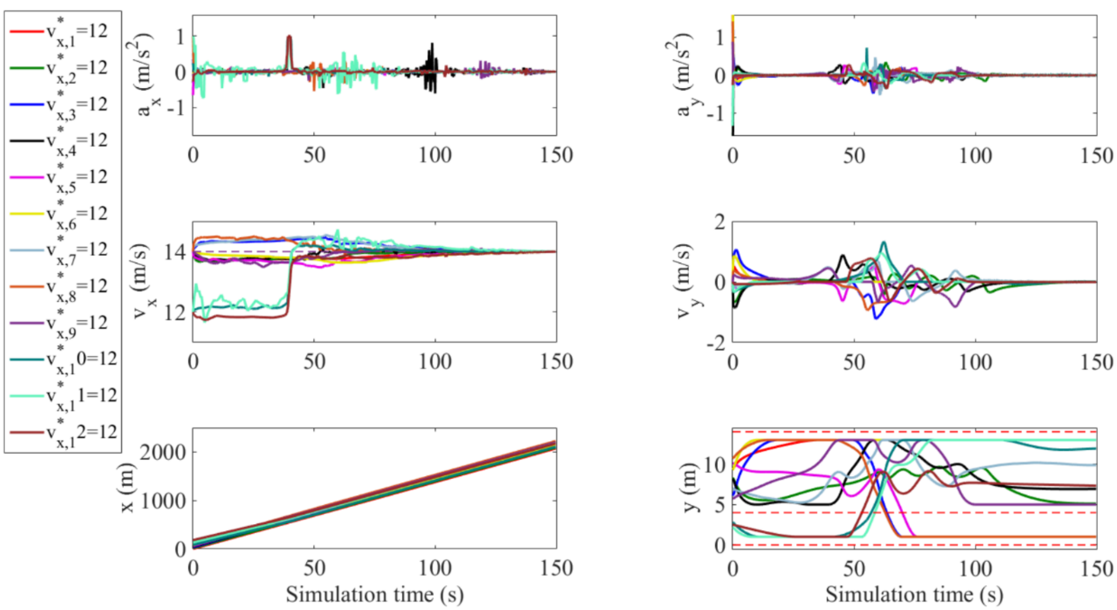


Figure 14: Vehicle trajectories scenario 2

Scenario 3

In the third scenario, which involves a total of 20 vehicles, we observe several notable peaks in the graphs depicting lateral velocity and lateral acceleration, particularly during the periods when the on-ramp and off-ramp vehicles are merging and diverging from the highway. Although the maximum values of lateral velocity do not differ significantly from the previous scenarios, the graph representing lateral acceleration over time displays considerably more activity. This increase in lateral acceleration is primarily attributed to the higher traffic density on the road in this scenario, as well as the increased number of vehicles attempting to diverge into the off-ramp. These diverging vehicles interacted more frequently with others, leading to the greater fluctuations seen in the lateral acceleration. The scenario, therefore, underscores the impact of higher traffic density and more complex vehicle interactions on the system's performance, particularly in terms of lateral movements and maneuvers. For a detailed visualization, you can watch the https://bit.ly/4eDtfD6 to see the movements of the vehicles in real-time.

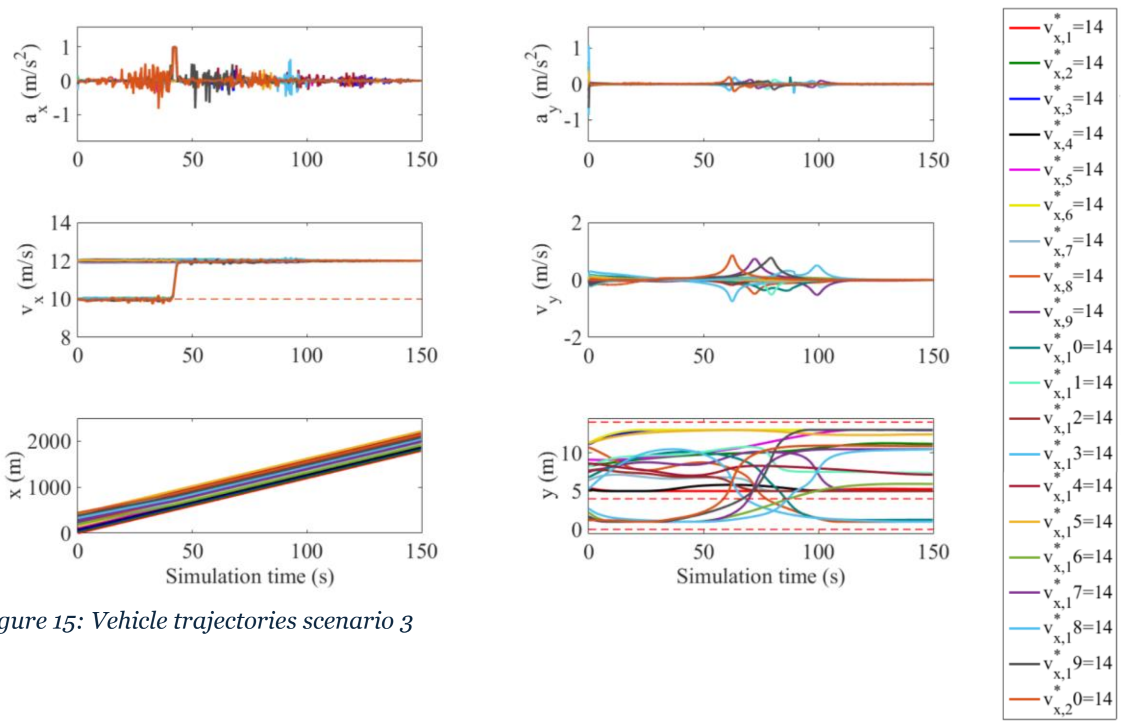


Figure 15: Vehicle trajectories scenario 3

3.1.2 Performance Under Various Traffic Densities

Additionally, to examine the highway merging approach developed, we conducted several tests under varying traffic densities. Each time the number of vehicles on the highway was increased, the number of merging and diverging vehicles was proportionally adjusted. As expected, the results indicate that increasing the number of vehicles also increases the CPU time required for the optimization horizon. The exact CPU time in seconds, corresponding to the number of vehicles, is presented in the following figure.

Table 6: CPU time under various traffic densities (CPU unit: Intel Core i5-6300HQ Base Clock: 2.3GHz)

Scenario	Total Vehicles	CPU Time (seconds)
Scenario 1	12 Vehicles	17.990
Scenario 2	20 Vehicles	58.678
Scenario 3	30 Vehicles	188.658
Scenario 4	50 Vehicles	559.358
Scenario 5	100 Vehicles	2468.863

The method used to collect CPU time values involved running the most complex scenario multiple times, progressively reducing the number of iterations each time. This process continued until the simulation produced collisions, excessive accelerations or decelerations, and abnormal vehicle behaviors. Once these issues were observed, we increased the number of iterations and halted further adjustments when the system stabilized. The CPU time required to achieve this stable state was then matched with the longitudinal gradient reached during the scenario. Subsequently, we recorded the CPU time for the other scenarios required to reach the same longitudinal gradient value.

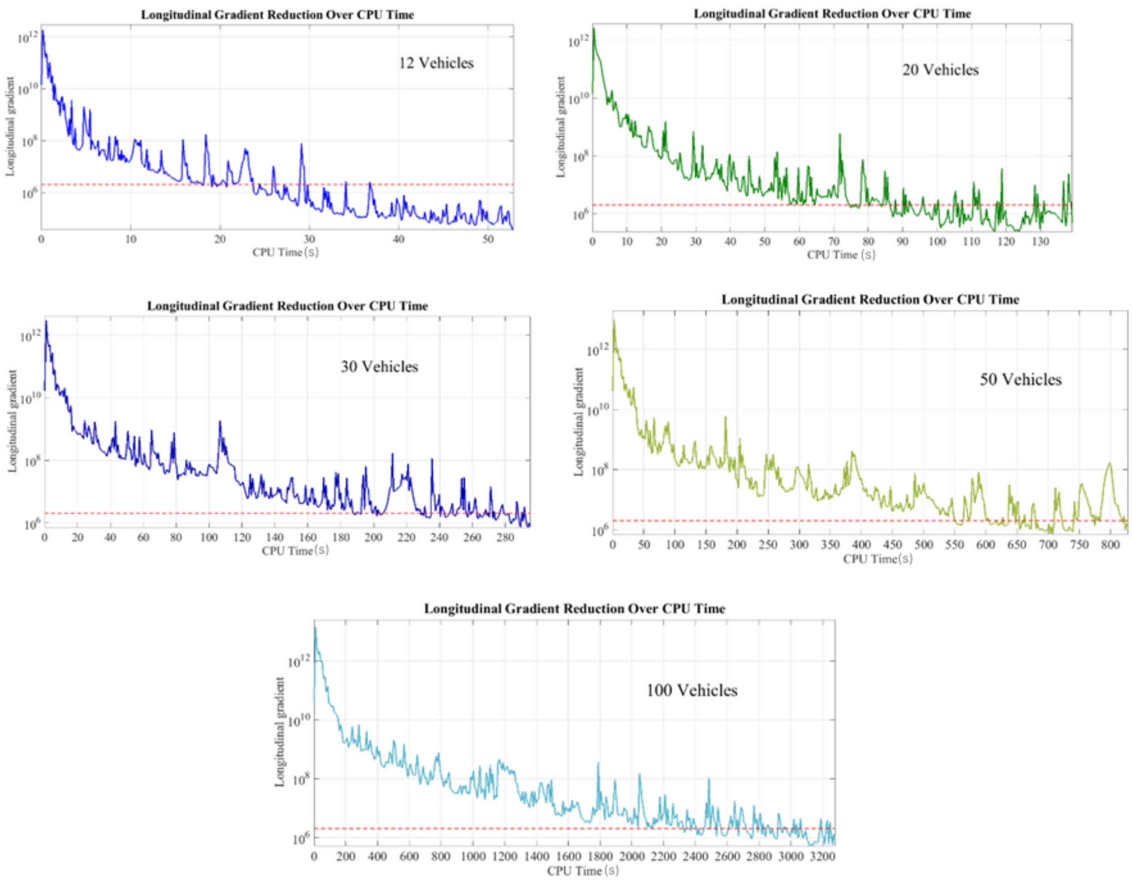


Figure 16: CPU time comparison under various traffic densities

In the CPU time analysis for different traffic densities, we observed that both the CPU time per iteration and the number of iterations were higher in scenarios where the goal was to achieve the same longitudinal gradient value.

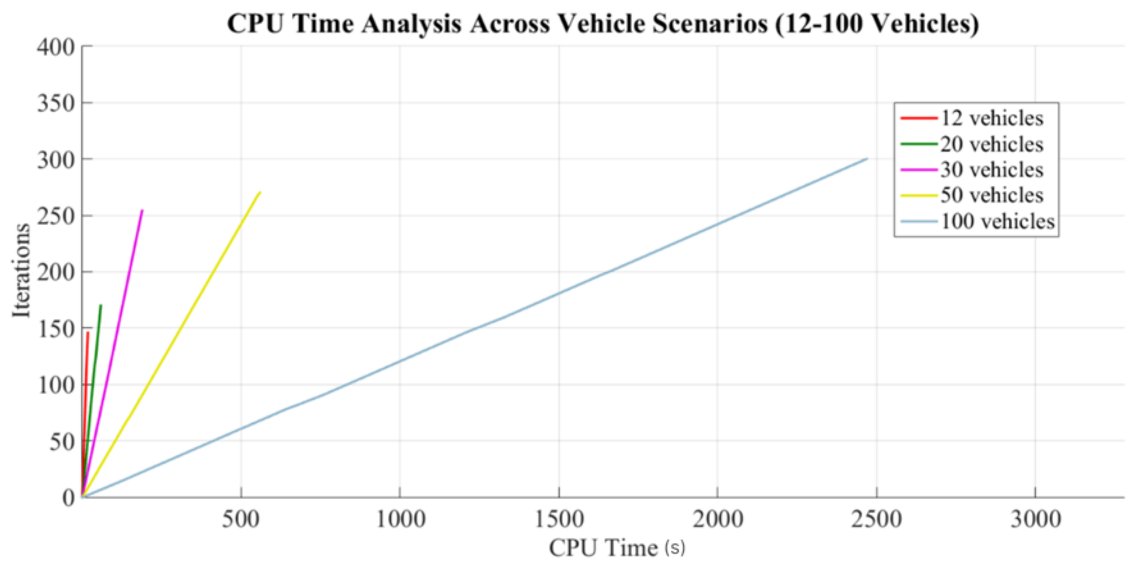


Figure 17: CPU time comparison under various traffic densities

After collecting the CPU time (in seconds) required for each scenario, we plotted the data to better observe the scalability of CPU time relative to traffic density. From the two figures below, it is evident that the relationship follows a quadratic trend, demonstrating how CPU time scales as traffic density increases.

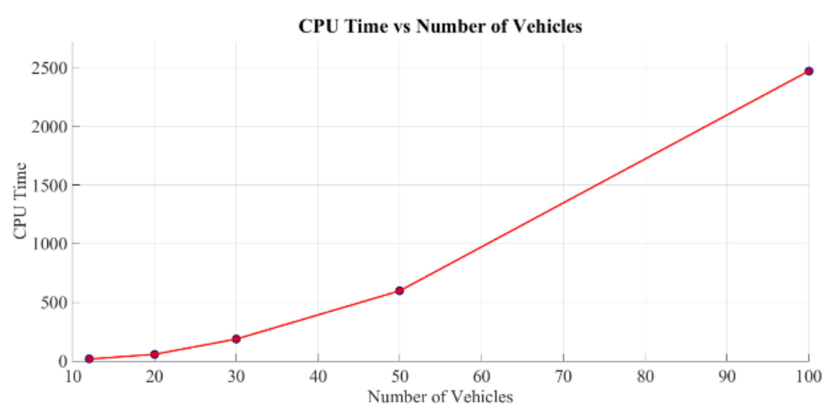


Figure 18: CPU time scaling

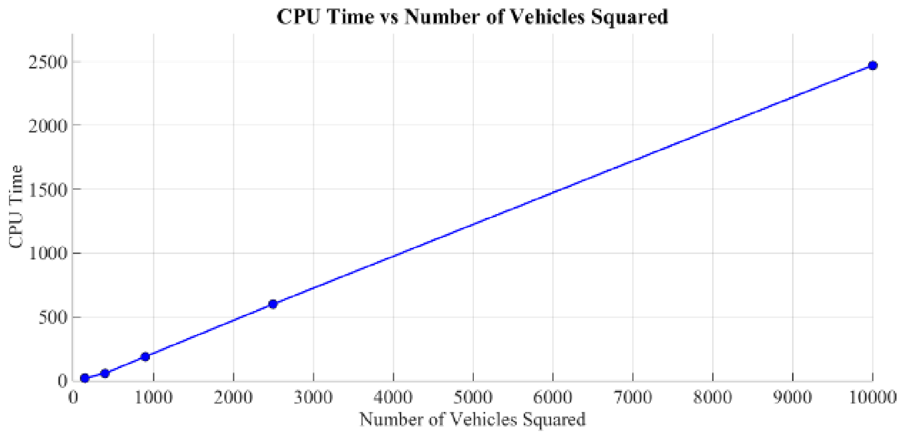


Figure 19: CPU time scaling

CONCLUSION AND FUTURE WORK

4.1 Summary of Contributions

This thesis presents a centralized trajectory optimization method for several connected and automated vehicles in a lane-free highway weaving zone, a typical highway design feature to ramps. A weaving area may encounter congestion challenges even when traffic flow is below capacity, attributable to the design and complexity of vehicle interactions, leading to reduced traffic flow and associated safety concerns. A double double-integrator model is employed for the longitudinal and lateral dynamics of each vehicle. The objective function consists of multiple sub-objectives that represent partially conflicting driving factors and considerations, including passenger comfort, minimal fuel consumption, vehicle progression at the desired speed, collision avoidance, and the suppression of infeasible moves. Fixed and state-dependent control input constraints consider multiple technical limits and compliance with road boundaries. The resolution of the established joint Optimal Control Problem (OCP) is achieved by a very efficient Feasible Direction Algorithm, which leverages the structure of the state equations to transform the OCP into a reduced Nonlinear Programming Problem. To illustrate the efficiency of the proposed methodology, complex scenarios were tested on a lane-free highway, encompassing an on-ramp and off-ramp that create a weaving zone. The vehicles operate effectively and safely, while ensuring passenger comfort. The flow values and critical density values are markedly superior to those in lane-based traffic.

4.2 Potential future directions

The proposed technique will be evaluated under increased traffic densities. Future considerations for our team's work on enhancing the technique outlined in this thesis include evaluating its efficacy during bottlenecks that may arise from heavy flow and on-ramps, both at the vehicle and traffic levels. Finally, we may compare centralized and decentralized path-planning to assess the efficiency advantages provided by the proposed coordinated approach and implement this approach within a Model Predictive Control framework.

Model predictive control, or moving horizon control, is a widely utilized advanced control technique. MPC relies on continuous real-time optimization of a mathematical system model. The fundamental aspect of Model Predictive Control (MPC) is to forecast the system's future behavior over a specified time horizon and to determine an optimal control input, u. The ideal control input can be determined by an optimization process, taking into account specified system restrictions and a cost function. The ideal control trajectory can be computed within a finite rolling horizon. Subsequently, the optimal input trajectory is implemented in the system until the subsequent sampling moment, at which point the horizon is adjusted and the entire process is reiterated [4]. The event-triggered MPC technique is implemented independently for all vehicles, while the FDA generates real-time numerical solutions for the optimization component. The time frame for this issue is 8 seconds, however the application duration of the generated trajectories is 4 seconds, as finer accelerations are necessary. Consequently, only half of a trajectory is utilized, while the remainder serves as the beginning value for the subsequent iteration of FDA. These incomplete paths

Specifically, while OCP has numerous potential applications that regard independent MPC-based driving for individual vehicles, the current tool, utilized in an MPC mode, facilitates the consideration of various entities in lane-free traffic, each controlled independently. Such entities may include individual vehicles, clusters of vehicles, and vehicle platoons in various formations (e.g., vehicle flocks, two-dimensional serpentine platoons), among others.

The MPC is implemented autonomously for each individual CAV operating on a lane-free circular roadway, where cars may be in close proximity to one another. An intriguing subject within this lane-free environment is the combined trajectory specification for a group of vehicles, regardless of size. This is especially attractive in lane-free driving, where all variables are real-valued, and FDA is recognized for its polynomial complexity.

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