Technical University of Crete

Controlling Automated Vehicles on Lane-free Roundabouts Using Nonlinear Feedback Control

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Έλεγχος αυτόματων οχημάτων σε κυκλικούς κόμβους χωρίς λωρίδες με χρήση μη γραμμικού ελεγκτή ανάδρασης

Diploma Thesis

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Abstract

Road traffic congestion has emerged as a progressively severe issue, leading to extensive delays, substantial environmental pollution, and degraded traffic safety. Connected and automated vehicles (CAVs) present a potential solution to this issue, as they can utilize information from infrastructure, other vehicles, and their own sensors to make more efficient decisions. Given the significance of roundabout infrastructure in urban traffic, it is crucial to develop effective approaches for managing CAVs driving on complex roundabouts. This is a challenging task because of the roundabout geometrical features and frequent conflicts among entering, rotating, and exiting vehicles. The main control goals include avoiding collisions and violation of the roundabout boundaries, smooth and reasonable vehicle movements, and good exploitation of infrastructure. In particular, large roundabouts operate with no lanes; hence, they can be considered as lane-free elements. In this diploma thesis, we aim to employ a decentralized nonlinear feedback controller to navigate CAVs towards their respective destinations while reaching the abovementioned goals. To this end, we will modify a nonlinear controller which was previously designed for lane-free highways with varying lateral boundaries to be used for controlling vehicles on the roundabout. Place Charles de Gaulle, a famous roundabout in Paris, is considered as an overly complex case study featuring an inner (outer) radius of 46 m (84 m) and comprising a dozen radial two-way streets.

Περίληψη

Η κυκλοφοριακή συμφόρηση έχει αναδειχθεί ως ένα σοβαρό πρόβλημα, προκαλώντας μεγάλες καθυστερήσεις, περιβαλλοντική ρύπανση και μείωση της οδικής ασφάλειας. Τα συνδεδεμένα και αυτόματα οχήματα (ΣΑΟ) προσφέρουν μια πιθανή λύση σε αυτό το πρόβλημα, καθώς μπορούν να χρησιμοποιήσουν πληροφορίες από υποδομές, άλλα οχήματα και τους αισθητήρες τους για να λάβουν πιο αποτελεσματικές αποφάσεις. Δεδομένης της σημασίας της υποδομής κυκλικών κόμβων στην αστική κυκλοφορία, είναι σημαντικό να αναπτυχθούν αποτελεσματικές μέθοδοι για τον έλεγχο ΣΑΟ που κινούνται σε πολύπλοκους κυκλικούς κόμβους. Αυτό αποτελεί μία πρόκληση λόγω των γεωμετρικών χαρακτηριστικών των κυκλικών κόμβων και των συχνών συγκρουόμενων συμφερόντων μεταξύ οχημάτων που εισέρχονται, περιστρέφονται και εξέρχονται των κόμβων. Οι κύριοι στόχοι ελέγχου περιλαμβάνουν την αποφυγή συγκρούσεων και παραβιάσεων των ορίων του κυκλικού κόμβου, την ομαλή και λογική κίνηση των οχημάτων και τη καλή εκμετάλλευση της υποδομής. Πιο συγκεκριμένα, μεγάλοι κυκλικοί κόμβοι λειτουργούν χωρίς λωρίδες κυκλοφορίας. Επομένως, μπορούν να θεωρηθούν ως στοιχεία χωρίς λωρίδα. Σε αυτή τη διπλωματική εργασία, σκοπός είναι να χρησιμοποιήσουμε έναν αποκεντρωμένο μη γραμμικό ελεγκτή με ανάδραση για να καθοδηγήσουμε τα ΣΑΟ προς τους αντίστοιχους προορισμούς τους, ενώ ταυτόχρονα πληρούνται οι παραπάνω στόχοι. Για τον σκοπό αυτό, θα τροποποιήσουμε έναν μη γραμμικό ελεγκτή που σχεδιάστηκε πρόσφατα για δρόμους χωρίς λωρίδες με ποικίλα πλευρικά όρια, ώστε να χρησιμοποιηθεί για τον έλεγχο των οχημάτων στον κυκλικό κόμβο. Ο περίφημος κυκλικός κόμβος Place Charles de Gaulle στο Παρίσι θεωρείται ως μια αρκετά περίπλοκη περίπτωση μελέτης με εσωτερική (εξωτερική) ακτίνα 46 μέτρα (84 μέτρα) και περιλαμβάνει δώδεκα διπλές εισόδους-εξόδους

Acknowledgement

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1. Introduction

The increase in traffic congestion in urban centers has emerged as a serious issue, causing delays, increased environmental pollution, and degradation of road safety. Connected and Automated Vehicles (CAVs) offer promising solutions to address these challenges, as they can leverage data from infrastructure and other vehicles to make more effective decisions. The management of CAVs at roundabouts, which are critical elements of urban infrastructure, is particularly significant. Given the complex geometric characteristics and frequent conflicts among vehicles entering, circulating, or exiting the roundabout, the development of reliable control methods is essential to ensure traffic safety and efficiency.

In recent decades, substantial advancements have been made in vehicle automation, indicating the possibility of achieving full automation in the near future. In order to address the issue of traffic congestion and its associated negative effects, such as prolonged delays, environmental pollution, and compromised traffic safety, various forms of traffic control [1, 2] have been developed for decades. More recently, there has been a focus on the advancement of Vehicle Automation and Communication Systems (VACS) that greatly enhance the individual capabilities of vehicles. These systems are now being considered in the development of a new generation of traffic management tools [3, 4]. This trend carries on as high-automation or virtually driverless vehicles are subjected to testing in real traffic conditions, as exemplified in [5]. Recent studies have highlighted the benefits of utilizing connected vehicles, which may soon become the prevailing trend. In the near future, it is possible that vehicles will establish communication both among themselves and with the infrastructure.

The TrafficFluid concept was recently proposed [6], which is a novel paradigm for vehicular traffic, applicable at high levels of vehicle automation and communication. Two core principles serve as the foundation for the TrafficFluid concept: (a) Lane-free traffic, whereby vehicles are not bound to fixed traffic lanes, as in conventional traffic, but may drive anywhere on the 2-D surface of the road; and (b) Vehicle nudging, whereby vehicles communicate their presence to other vehicles in front of them (or are sensed by them), and this may influence the movement of vehicles in front. Over the past few years, several movement strategies have been developed for automated vehicles on lane-free highways, following the TrafficFluid paradigm. These strategies include ad-hoc approaches [6, 7], optimal model predictive control [8], reinforcement learning [9], and nonlinear feedback control [10,11]. A microscopic simulator for lane-free traffic is designed in [12].

The roundabout is a crucial component of urban traffic, enhancing traffic efficiency particularly in situations of light traffic [13]. Nevertheless, as demands increase, it has the potential to become a bottleneck. Hence, efficient operation of roundabouts, which is indeed considered challenging because of the geometric complexities, can enhance traffic in its surrounding area. Numerous studies in the literature examine the control of automated vehicles on roundabouts, with a majority of them emphasizing uncomplicated infrastructures. Specifically, many studies focus on single-lane roundabouts and explore different methods of control. Some research [14-16] suggests priority management approaches, whereby a suitable policy, like “First-Come-First-Served”, is utilized to assign the priorities to vehicles. In the event of a conflict between two vehicles in the roundabout, the vehicle with lower priority is expected to yield or decrease speed to allow the higher-priority vehicle to proceed. Moreover, several methodologies [17-22] have been suggested to conceptualize the motion of vehicles on roundabouts as an optimal model predictive control problem. The objective of these methodologies is to reduce various factors, including travel time, fuel consumption, and distance from the destination. Additionally, [23] suggests a hierarchical arrangement designed to secure vehicle safety and establish the ideal inflow for roundabouts.

In other works, various control approaches have been suggested for two-lane roundabouts. In reference [24], two fuzzy controllers are utilized to regulate the steering angle and angular speed in a two-lane roundabout, employing real data, drivers' knowledge, and common reasoning for their design. Moreover, the utilization of optimal control and game theory allows for strategic decision-making at the merging points or when changing lanes on the roundabout [25-27]. Moreover, [28] introduces a reinforcement learning technique that is embedded with optimization for making lane changing decisions at a four-lane roundabout.

The concept of lane-free roundabout was first addressed in [29], where a comprehensive control strategy was presented for vehicles on the basis of the bicycle model for vehicle dynamics. The nonlinear controller that was developed in [10] and ensures various features for straight lane-free roads, such as collision and boundary-violation avoidance, desired speed tracking, and convergence of acceleration and orientation to zero, was adapted to effectively control vehicles in the roundabout. The method was tested for a limited number of vehicles driving on Place Charles de Gaulle roundabout. Then, in [30,31], an optimal approach is proposed to determine the state-dependent desired orientation for vehicles driving on roundabouts. Finally, a nonlinear controller which was designed for ring roads [11] got employed in [32] to control vehicles on large lane-free roundabouts while employing the optimal desired orientation proposed in [30,31]. Applying necessary modifications, they could manage a large number of vehicles on Place Charles de Gaulle roundabout.

While the proposed approach in [32] proves effective, it is prudent to explore alternative methods that may offer advantages in terms of simplicity or efficiency. In this study, we employ the nonlinear controller developed in [33] which is able to control vehicles in roads with curvy boundaries. In addition, it integrates several units, which were used in [32], like OD corridors, adaptive desired speed according to the surrounding density, lateral boundary controllers, and position desired orientation. As a result, employing the new nonlinear controller can significantly simplify the control system for vehicles driving on large roundabouts. However, the controller cannot be directly used for the case of roundabouts because of some theoretical limitations. In this diploma thesis, we propose appropriate modifications for the nonlinear controller to use it to control entering, rotating, and exiting automated vehicles on roundabout.

This thesis is composed of 5 chapters. The vehicle dynamics and the employed nonlinear controller are presented in chapter 2. The proposed transformation, enabling us to use the controller for vehicles on roundabout is given in chapter 3. Chapter 4 contains the entrance and exit strategies for two distinct cases, visible and invisible destinations. Finally, concluding remarks and future works are given in chapter 5.

2. Vehicle Dynamics and Feedback Law

In this chapter, the vehicle dynamics and nonlinear feedback controller for roads with curvy roads are introduced.

2.1. Kinematic bicycle model

There are several models to represent vehicle dynamics, among which kinematic bicycle model is appropriate for the roundabouts where vehicles have frequent and strong turnings. The schematic of bicycle model is shown in Fig. 1 and its state-space equations are as below:

$$\begin{aligned} \dot{x} &= v\cos(\theta) \\ \dot{y} &= v\sin(\theta) \\ \dot{\theta} &= \sigma^{-1} v\tan(\delta) \\ \dot{v} &= F \end{aligned} \tag{1}$$

where x and y are the longitudinal and lateral position coordinates of the rear axle midpoint of the vehicle, v is the vehicle speed, and θ is its orientation. The model has two control inputs: acceleration F and steering angle δ. Finally, σ is the length of the vehicle.

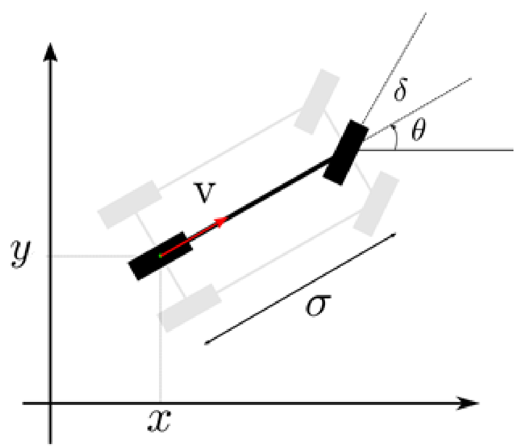

Figure 1- Illustration of the bicycle model variables

Not only for easier simulation, but also for testing the application of discrete-time control to the real system, it is necessary to employ the discrete-time version of the bicycle model. For this purpose, the exact sampled-data model can be obtained by integrating the ODEs presented in (1), while the control inputs are constant for a sample period. The discrete-time model is calculated for the following two conditions:

If $\delta \neq 0, \delta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$:

$$\begin{aligned}
x_i(k+1) &= x_i(k) + \frac{\sigma}{\tan(\delta_i(k))}\left(\sin\left(\theta_i(k) + v_i(k)\frac{\tan(\delta_i(k))}{\sigma}T + F_i(k)\frac{\tan(\delta_i(k))}{2\sigma}T^2\right) - \sin\left(\theta_i(k)\right)\right) \\
y_i(k+1) &= y_i(k) + \frac{\sigma}{\tan(\delta_i(k))}\left(\cos\left(\theta_i\right) - \cos\left(\theta_i(k) + v_i(k)\frac{\tan(\delta_i(k))}{\sigma}T + F_i(k)\frac{\tan(\delta_i(k))}{2\sigma}T^2\right)\right) \\
\theta_i(k+1) &= \theta_i(k) + v_i(k)\frac{\tan(\delta_i(k))}{\sigma}T + F_i(k)\frac{\tan(\delta_i(k))}{2\sigma}T^2 \\
v_i(k+1) &= v_i(k) + F_i(k)T
\end{aligned} \tag{2}$$

and if $\delta_i = 0$:

$$\begin{aligned}
x(k+1) &= x(k) + v(k)\cos(\theta(k))T + F\cos(\theta(k))\frac{T^2}{2} \\
y(k+1) &= y(k) + v(k)\sin(\theta(k))T + F\sin(\theta(k))\frac{T^2}{2} \\
\theta(k+1) &= \theta(k) \\
v(k+1) &= v(k) + F(k)T
\end{aligned} \tag{3}$$

where T is the sampling period.

2.2. Nonlinear Feedback Controller (NLFC)

In this thesis, we employ a decentralized two-dimensional nonlinear cruise controller for vehicles driving on roads with curvy boundaries that rigorously guarantees some important features such as collision-free movement, remaining within boundaries, and tracking of (different) desired speeds for all vehicles. Importantly, every vehicle may have its own (curvy) boundaries of its movement corridor, which may or may not coincide with the road. The feedback law reads

$$\delta_i = \tan^{-1}(\sigma_i u_i / v_i), \quad i = 1, \ldots, n \tag{4}$$

$$F_i = \frac{1}{Q\left(v_i, \theta_i, f_i(w)\right)}\left(-\mu_1(v_i\cos(\theta_i) - f_i(w)) + \frac{Z_i(w) + v_i\sin(\theta_i)u_i}{v_i(v_{\max} - v_i)} - \Phi_i(w) - g_i(x_i, y_i)\Xi_i(w)\right)$$

where:

$$u_i = \frac{2\left(\cos(\theta_i)-\cos(\Theta)\right)^2}{Rh_i(x_i,y_i,\theta_i)}\left(\begin{array}{l} -\mu_2 v_i^2\left(\sin(\theta_i)-g_i(x_i,y_i)\cos(\theta_i)\right)+R\dfrac{v_i\cos(\theta_i)a_i(x_i,y_i,\theta_i)}{\cos(\theta_i)-\cos(\Theta)} \\ -U_i'\left(\dfrac{2y_i-\left(\beta_i(x_i)+\alpha_i(x_i)\right)}{\beta_i(x_i)-\alpha_i(x_i)}\right)\dfrac{2v_i}{\beta_i(x_i)-\alpha_i(x_i)}-v_i\Xi_i(w) \end{array}\right)$$

$$g_i(x,y) := \frac{\left(\beta_i(x)-y\right)\alpha_i'(x)+\left(y-\alpha_i(x)\right)\beta_i'(x)}{\beta_i(x)-\alpha_i(x)}.$$

$$f_i(w) = v_i^* b\left(\Phi_i(w)+g_i(x,y)\Xi_i(w)\right)$$

$$\Phi_i(w) = \sum_{j\neq i} V_{i,j}'(d_{i,j})\left(x_i-x_j\right)/d_{i,j}$$

$$\Xi_i(w) = \sum_{j\neq i} p_{i,j} V_{i,j}'(d_{i,j})\left(y_i-y_j\right)/d_{i,j}$$

$$h_i(x,y,\theta) = 2\cos(\theta)\left(\cos(\theta)-\cos(\Theta)\right)+\sin^2(\theta)+\sin(\theta)g_i(x,y)\left(\left(\cos(\theta)-2\cos(\Theta)\right)\right)$$

$$a_i(x_i,y_i,\theta_i) = \cos(\theta_i)\left(\frac{y-\alpha_i(x_i)}{\beta_i(x_i)-\alpha_i(x_i)}\beta_i''(x_i)+\left(1-\frac{y_i-\alpha_i(x_i)}{\beta_i(x_i)-\alpha_i(x_i)}\right)\alpha_i''(x_i)\right)$$
$$+\left((\beta_i(x_i)-\alpha_i(x_i))\sin(\theta_i)-g_i(x_i,y_i)\cos(\theta_i)\right)\frac{\beta_i'(x_i)-\alpha_i'(x_i)}{\left(\beta_i(x_i)-\alpha_i(x_i)\right)^2}$$

$$Q(v,\theta,l) = \frac{v_{\max} v\cos(\theta)+lv_{\max}-2lv}{2v^2(v_{\max}-v_i)^2}$$

$$Z_i(w) = v_i^* b'\left(\Phi_i(w)+g_i(x,y)\Xi_i(w)\right)\frac{d}{dt}\left(\Phi_i(w)+g_i(x,y)\Xi_i(w)\right)$$

$$b(x) = \begin{cases} 1 & x \le \varepsilon \\ \exp(-(x-\varepsilon)^3) & x > \varepsilon \end{cases}$$

where v_{max} is the maximum allowable speed, μ_1, μ_2, and ε are positive design parameters. In particular, $f_i(w)$ is the adaptive desired speed indirectly dependent on the density in the surrounding area; $\Phi_i(w)$ and $\Xi_i(w)$ includes the longitudinal and lateral repulsions from the adjacent vehicles, respectively. Moreover, the vehicle-repulsive and boundary-repulsive potential functions are:

$$V_i(d) = \begin{cases} q\dfrac{(\lambda-d)^3}{d-L} & L < d \le \lambda \\ 0 & d > \lambda \end{cases}$$

$$U_i(y) = \begin{cases} \left(\frac{1}{1-y^2}-c\right)^2 & y \in \left(-1,-\sqrt{\frac{c-1}{c}}\right)\cup\left(\sqrt{\frac{c-1}{c}},1\right) \\ 0 & y \in \left[-\sqrt{\frac{c-1}{c}},\sqrt{\frac{c-1}{c}}\right] \end{cases} \tag{5}$$

where $c \ge 1$ and $q > 0$ are design parameters. The inter-vehicle distance is calculated as below:

$$d_{i,j} := \sqrt{\left(x_i - x_j\right)^2 + p_{i,j}\left(y_i - y_j\right)^2} \tag{6}$$

where $p_{i,j} \geq 1$ are weight parameters that satisfy $p_{i,j} = p_{j,i}$ for all $i, j = 1, \ldots, n$. In this work, we consider the same value for all pair of vehicles.

Finally, the functions $\alpha(x)$ and $\beta(x)$ are used to respectively determine the right and left road boundaries that could be curvy.

3. Using NLFC for vehicles rotating on roundabouts

In this chapter, the proposed modifications that enables us to use the nonlinear feedback control, introduced in Chapter 2, for vehicles driving on roundabouts are presented. Then, simulation results, which confirm the effectiveness of the proposed approach, are given.

3.1. Moving coordinates

The feedback law, presented in Section 2, cannot be directly employed for controlling vehicles on roundabouts due to the following reasons:

1- The vehicle orientation must be in the range $(-\Theta,\Theta)$ where $0<\Theta<\pi/2$. Clearly, vehicles exceed the orientation limit while driving on the entire area of a roundabout i.e., their orientation will be between $[-\pi,\pi]$.

2- The speed-tracking term appears as $(v_i\cos(\theta_i)-f_i(w))$ where $f_i(w)$ is the adaptive speed. Hence, a vehicle follows the desired speed correctly only if $\theta_i=0$.

Actually, the introduced NLFC is a generalized controller for straight roads that can handle temporary curvature as well as on-ramp and off-ramps as shown in Fig. 2.

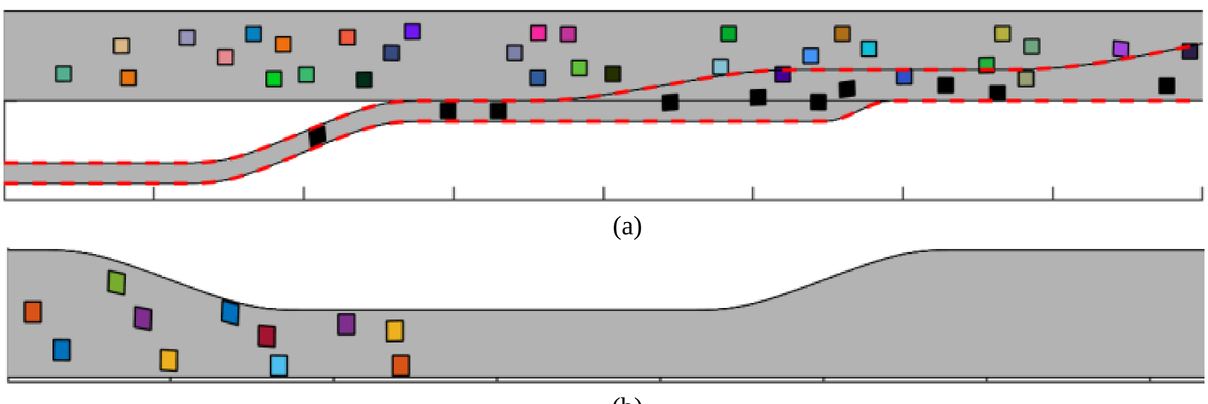

Figure 2- Applications of NLFC: a) on-ramp, b) varying road width

To address these difficulties, we introduce new moving coordinates for each vehicle, whose x'-axis is aligned with the roundabout's circular angle at the current position; and the perpendicular y'-axis points towards the roundabout center. Also, the middle of the circular road is considered as the origin of coordinates, see. Fig. 3. Thus, in the new coordinates, the longitudinal position of ego vehicle is always zero, while the lateral position is its radial distance from the middle of the circular road. The latter has a radius $R_a=(R_{in}+R_{out})/2$. Finally, θ' is the orientation with respect to the x'-axis or, in other words, the deviation from the circular angle at the current position of the vehicle.

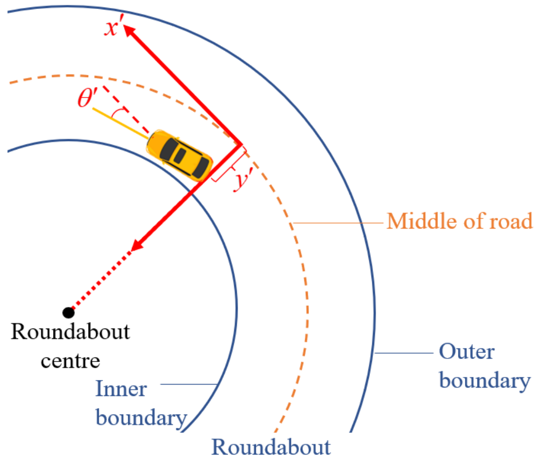

Figure 3 – Moving coordinates of each vehicle

In practical applications, the state variables in the moving coordinates can be obtained from CAV sensors or infrastructure. But, in order to conduct simulations and display the trajectories we may need relation between original Cartesian coordinates and the moving coordinates which is as below:

$$\begin{cases} x_i'(k) = x_i(k)\,cos(\,\varphi_{e,i}(k)) + y_i(k)\,sin(\,\varphi_{e,i}(k)) \\ y_i'(k) = y_i(k)\,cos(\,\varphi_{e,i}(k)) - x_i(k)\,sin(\,\varphi_{e,i}(k)) \\ \theta_i'(k) = \theta_i(k) - (\varphi_{e,i}(k)) \\ v_i'(k) = v_i(k) \end{cases} \tag{7}$$

where $\varphi_{e,i}(k)$ is the circular angle at the current position of vehicle *i* and can be found by:

$$\varphi_{e,i}(k) = tan^{-1}(\,y_i(k)/x_i(k)) + \pi/2 \tag{8}$$

Consequently, the relative longitudinal and lateral distance between ego and adjacent vehicles, which appear in the feedback law, must be calculated in the new coordinates. Hence, in addition to the state variables of ego vehicles, position of adjacent vehicles also should be transformed into the moving coordinates of vehicle *i*. Then, with these changes, the control inputs of ego vehicle can be calculated by the feedback law, i.e. (2), while replacing $x_i, y_i, \theta_i, x_j, y_j$ with the transformed variables $x_i', y_i', \theta_i', x_j', y_j'$, respectively.

3.2. Roundabout boundaries in moving coordinates

The roundabout's inner and outer circular boundaries must also be expressed as the left and right boundaries, $\alpha(x')$ and $\beta(x')$, in new coordinates. Since the first and second derivatives of $\alpha(x)$ and $\beta(x)$ appear in feedback law, these functions should be continuous and smooth around the mentioned area. Hence, we introduce a part of inner and outer circles around $x' = 0$ as left and right curvy boundaries, although we always calculate them at $x' = 0$. As illustrated in Fig. 4, they can be determined by projecting circles into the coordinates of each vehicle:

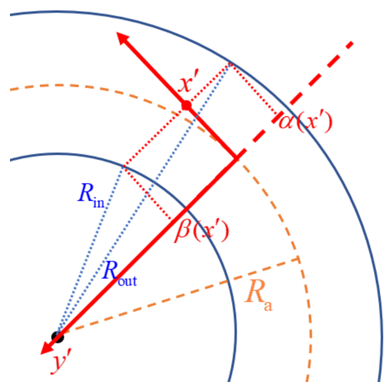

Figure 4– Left and right boundaries in the transformed coordinates

$$\alpha(x') = R_a - \sqrt{R_{out}^2 - x'^2}$$
$$\beta(x') = R_a - \sqrt{R_{in}^2 - x'^2} \qquad (9)$$

3.3. Simulation results

To investigate the effectiveness of the controller as well as the proposed transformation, several simulations for vehicles circulating on a roundabout are conducted. The NLFC has several parameters which should be properly tuned. Some of parameters are selected based on their functionality and the others are obtained by trial and errors. In particular, in collision avoidance repulsion function *V(d)*, there are three parameters: *q* determines the intensity of the repulsive function, *L* is the critical safety distance and λ determine the maximum interaction distance. In other words, vehicles affect each other if their distance is less than λ and the controller avoids inter-vehicle distances become less than *L*. The interaction and safety zones for $p = 5$, $\lambda = 30$, and $L = 5$ are shown in Fig. 5. Note that if $p = 1$, the areas would be circular rather than elliptical.

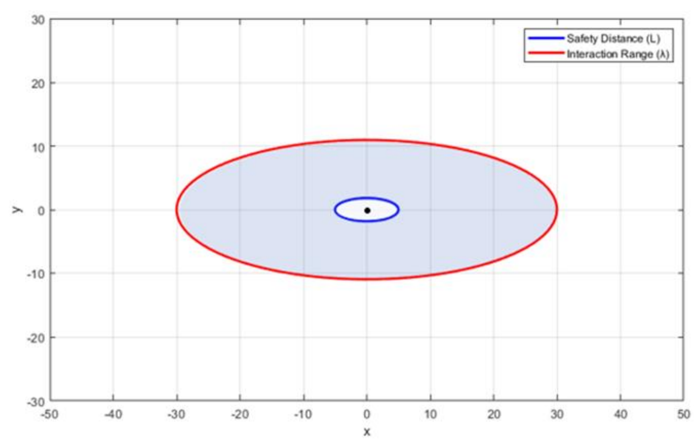

Figure 5- Vehicle safety and interaction zones

Furthermore, parameter *c* in boundary repulsion function determines the area in which vehicles are not pushed from the boundary. The bigger *c*, the narrower repulsion space in which vehicles are repulsed from the road boundary. Fig. 6 depicts the relaxed area for two different values for *c*.

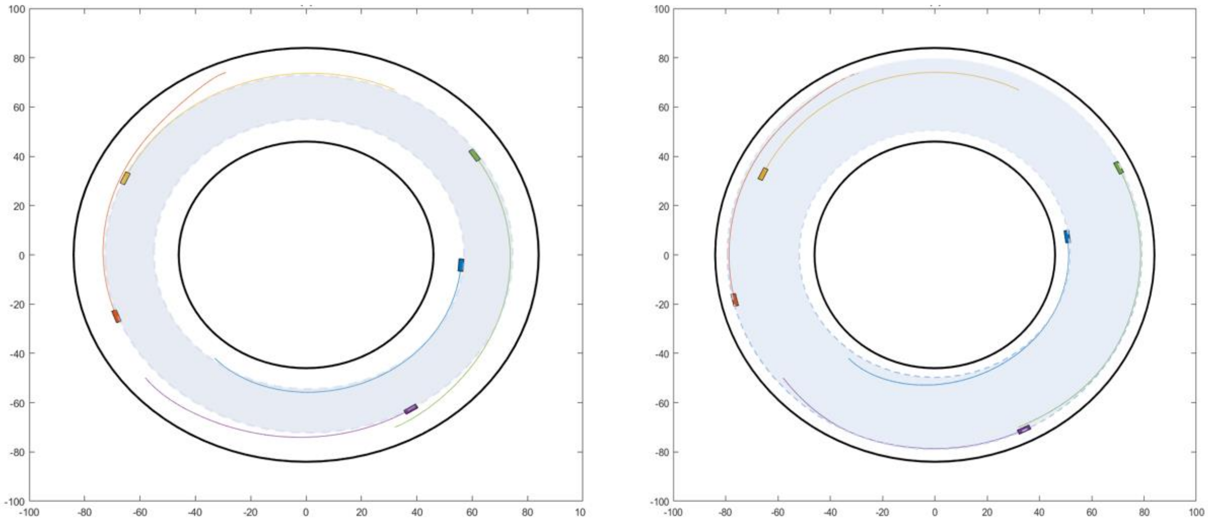

Figure 6- The impact of parameter c: Left: *c*=1.1, Right: *c*=2.1

Also, μ_1 and μ_2 respectively determine the convergence rate to the desired speed and desired orientation. As the speed tracking is not the most important goal in our work, we usually choose a relatively small value for μ_1.

Among the conducted simulations, results of one simulation including 100 vehicles will be presented in this part. The simulation is done by a sampling period of 0.01 s. To test the proposed method in a complicated situation, we assign a random desired speed for each vehicle in the range of [8,12] m/s. Having different desired speed while rotating on roundabout causes frequent conflicts with other vehicles. Vehicles also have different initial speeds and orientations; they may have deviation from their desired speed and orientation at the beginning. The initial position of vehicles is illustrated in Fig. 7. We consider the actual dimension of Charles de Gaulle roundabout, and the branches connected to it. Specifically, inner and outer radii are respectively 46 m and 84 m, meaning than it is 38 m wide. The values used for vehicle and controller parameters are:

$\varepsilon = 0.1 \quad \mu_1 = 0.05$

$c = 2 \quad \mu_1 = 0.5$

$\Theta = 0.5 \quad q = 1e-3$

$p = 8 \quad R = 1.9$

$L = 5 \quad \sigma = 4.5$

$\lambda = 30 \quad v_{\max} = 20$

The results of the simulation are illustrated in Figures 8 to 11. Additionally, you can watch a video showcasing the simulation at https://bit.ly/4gK0ANY. As seen in the video, vehicles move smoothly and efficiently throughout the roundabout, overtaking one another without any disruption. Fig. 8 shows the vehicle speed trajectories where some vehicles deviate from their desired speed to address other important goals like avoiding collisions. The acceleration trajectories can be seen in Fig. 9. In initial samples some big accelerations and decelerations are observed which are due to either short inter-vehicle distances or big deviation from desired speed. Furthermore, the vehicles may accelerate or decelerate to avoid potential collisions, even if this leads to deviations from their desired speeds. The acceleration of each vehicle consistently remains within an acceptable range, contributing to a comfortable driving experience. Note that, if desired speed of all vehicles were the same, their accelerations and speeds would converge to zero and desired speed, respectively. In this simulation, conflicts among vehicles frequently arise as they navigate with different desired speeds, resulting in the need for frequent acceleration or deceleration.

Additionally, their steering angles, after some initial efforts to get aligned with the circular angles, converge to a non-zero constant value, which is needed to maintain circular motion, see Fig. 10. The smaller radius they drive on, the bigger constant steering angle they should have. Finally, Fig. 11 illustrates the minimum inter-vehicle distance between all pairs of vehicles which is always above the critical threshold L, ensuring safe movement. It is worth noting that the exact collision avoidance algorithm, presented in [32], is employed to ensure that no collisions occur during the simulation.

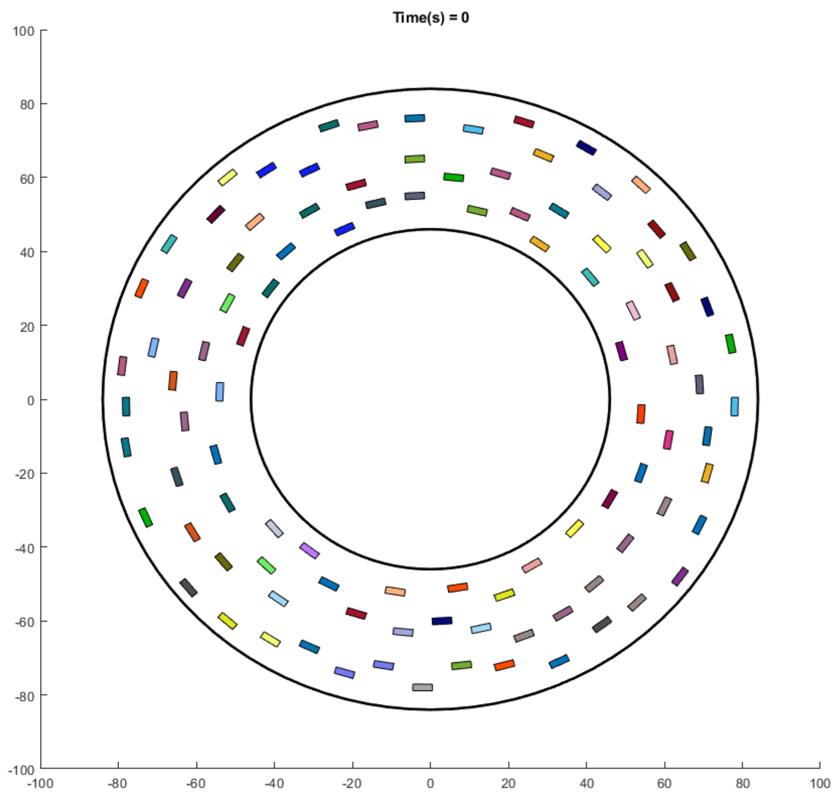

Figure 7 - The initial positions of 100 vehicles

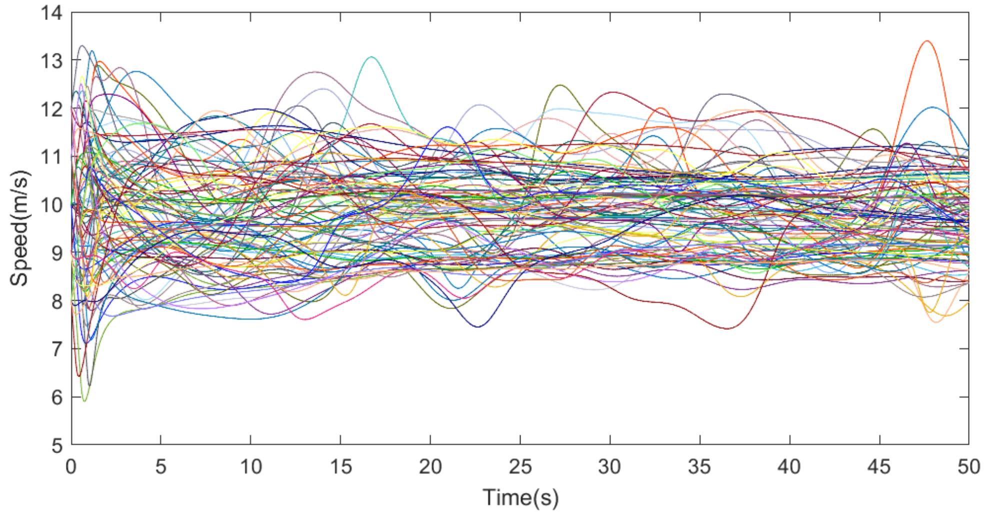

Figure 8 – Vehicle speeds

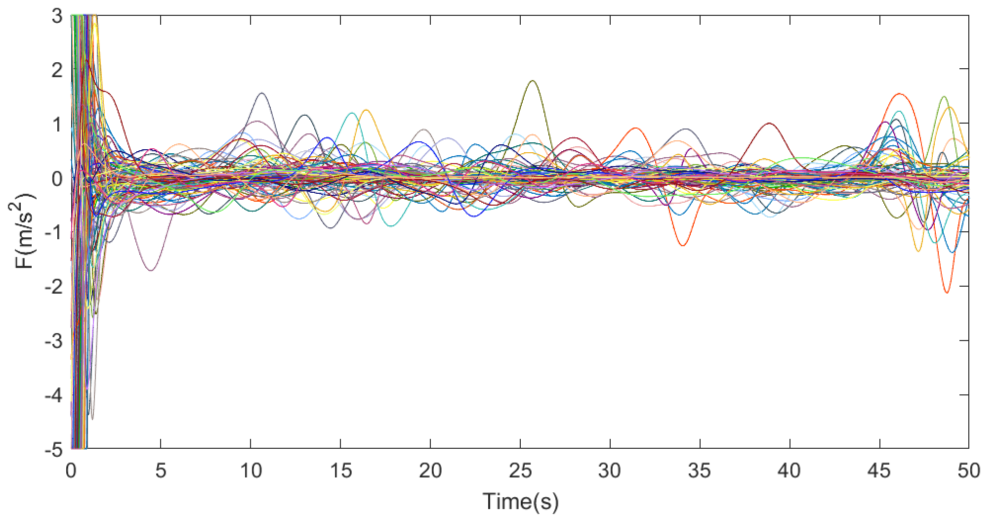

Figure 9- Vehicle accelerations

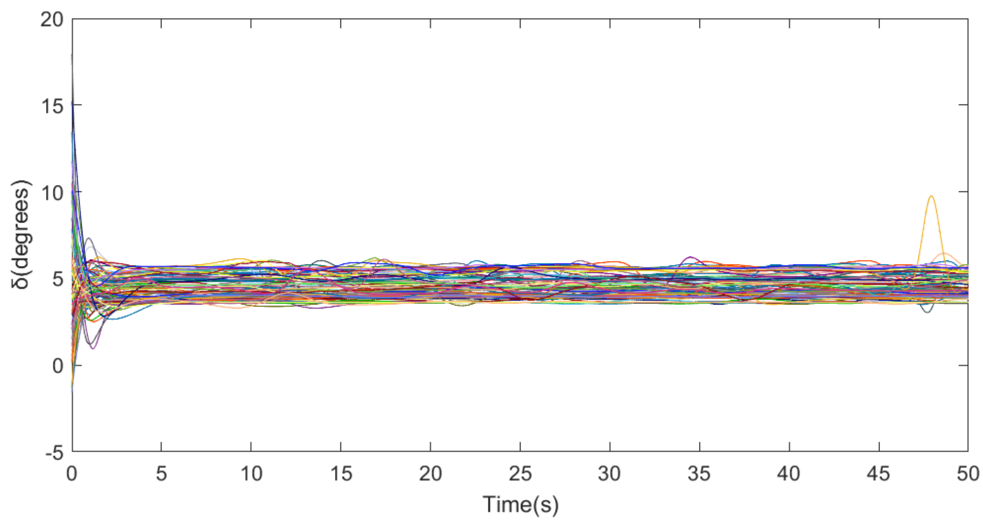

Figure 10- Vehicle steering angles

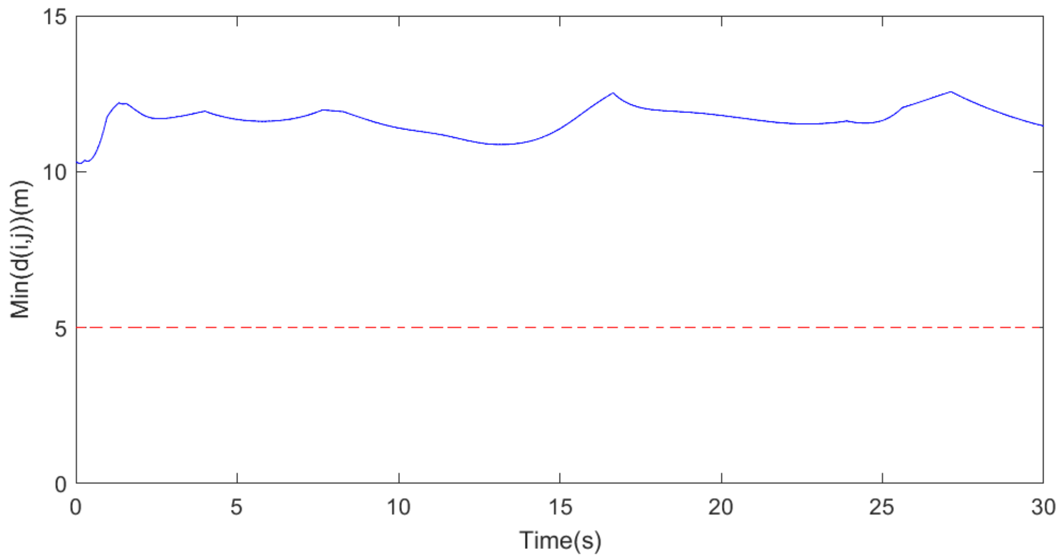

Figure 11- Minimum inter-vehicle distance

4. Developing Entrance, Navigation, and Exit Scenarios

In this chapter, appropriate strategies for different phases, namely entrance, navigation, and exit, are developed. A vehicle firstly drives on the entrance branch, then should join the roundabout traffic. Depending on the distance from its corresponding destinations, suitable corridor within the roundabout should defined for it. In the next step, it should smoothly get close to the outer boundary and exit from the corresponding branch without missing it. We do all these steps through defining proper right and left boundaries, i.e. $\alpha(x')$ and $\beta(x')$. Our case study, Place Charles de Gaulle roundabout has 12 bidirectional branches with various width, as can be seen in Fig. 12.

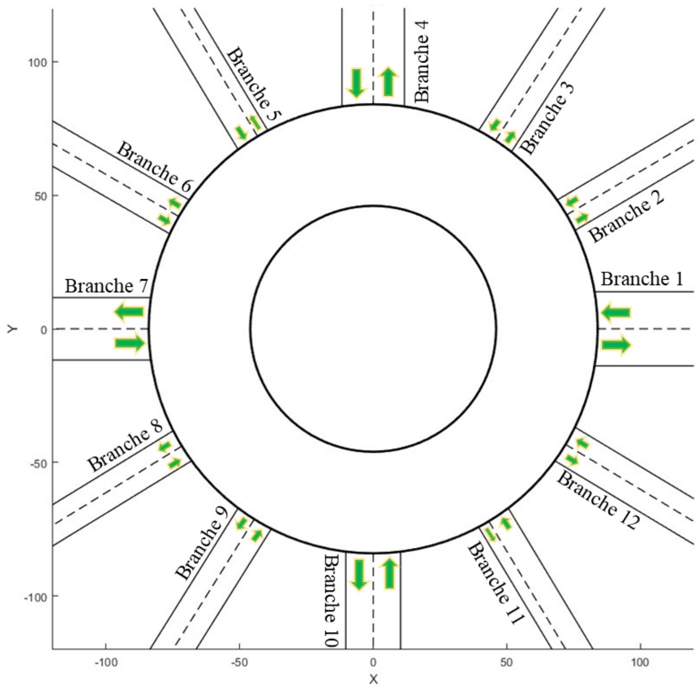

Figure 12- Roundabout's Branches and Structure

4.1. Driving on Entrance and Exit branches

Given the fact that each entrance and exit branch has a different orientation, we have to use rotated coordinates to properly control them. The transformation is illustrated in Fig. 13, where (x', y') is the position of vehicle in the rotated coordinates which is derived as below:

$$x' = x\cos\xi + y\sin\xi$$
$$y' = y\cos\xi - x\sin\xi \quad (10)$$

where ξ is the orientation of entrance or exit branch. The relative orientation in the rotated coordinates is deviation from the branch orientation, i.e. $\theta' = \theta - \xi$. Using the transformed variables in the rotated coordinates, the control signal for each vehicle can be calculated. The entrance and branch boundaries can be introduced by $\alpha(x') = -W_{\text{en/ex}}$ and $\beta(x') = 0$ with $W_{\text{en/ex}}$ being width of the branch.

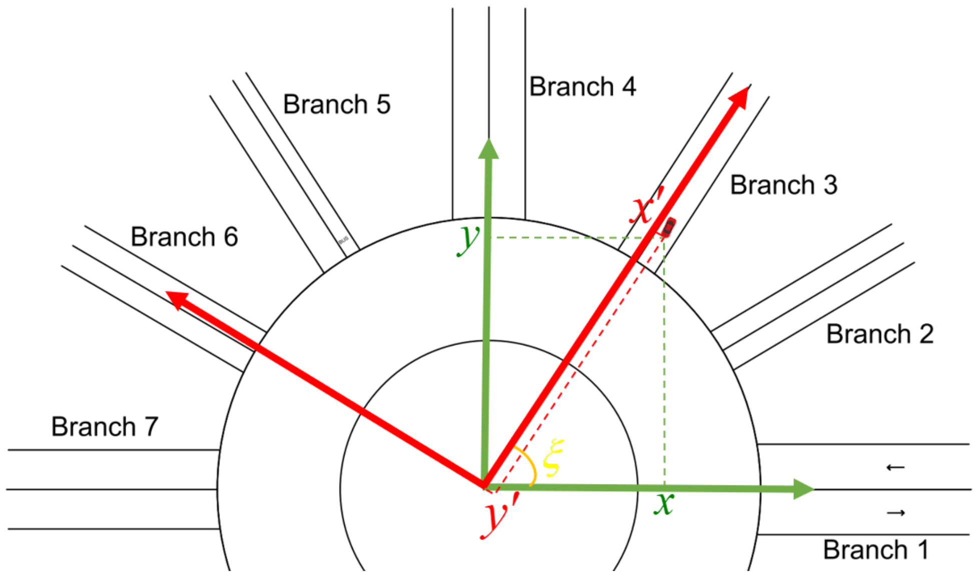

Figure 13- The rotated coordinates for entrance and exit branches

If vehicles keep the direction of the entrance branch, they will join roundabout traffic with a perpendicular orientation which may cause difficulties and collision risk. Hence, before reaching the roundabout, it could be beneficial to decrease the orientation difference between entering vehicles and the traffic orientation on roundabout. To this end, we change the boundaries in the last few meters of the entrance branch to the following ones:

$$\beta_{in}(x') = \sqrt{W_{en}^2 - (x' + 80 + W_{en})^2} - W_{en} \quad (11)$$

Figure 14 illustrates the boundaries of the corridor around an entrance where the left boundary at the end of entrance branch is curved to the right to facilitate a smoother entry of the vehicles into the roundabout. Additionally, a proper skewed line, depending on the distance from the destination, is considered as the left boundary, which will be explained in detail in the next sections. Also, a typical joining of a vehicle is depicted in Fig. 15 where the conflicts between the entering and rotating vehicles is much less than perpendicular entrance.

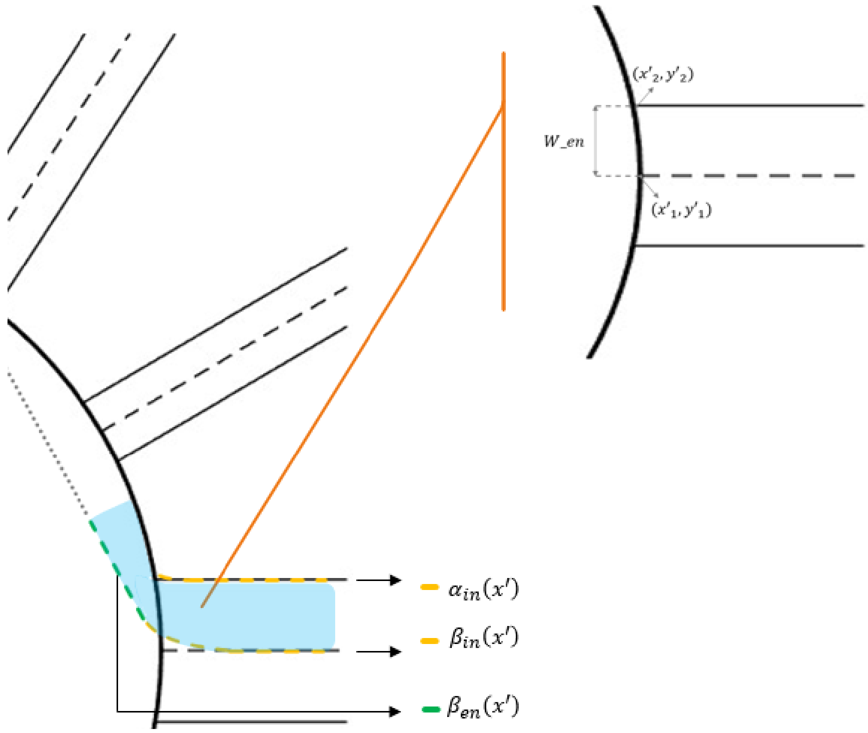

Figure 14 – Smoothening the entrance phase

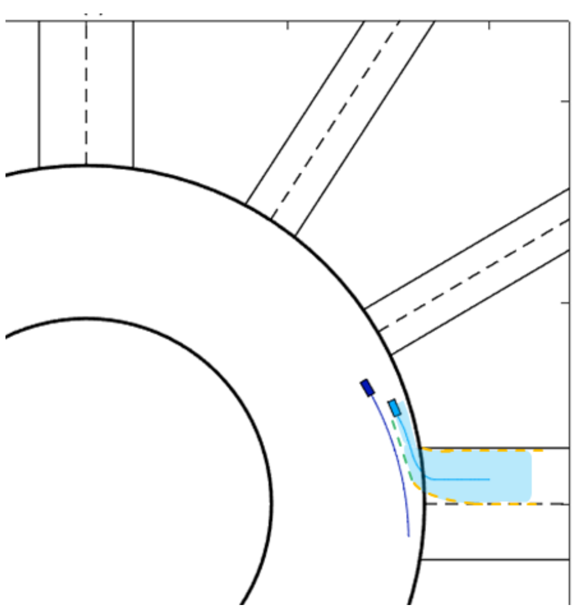

Figure 15– Safe and smooth joining to the roundabout traffic

4.2. Guiding Vehicles towards the inner boundary

While driving on roundabouts, if the destination is far, it would be useful if vehicle drives in the inner parts of the roundabout as it passes a shorter distance, and the space of roundabout would be better exploited. Using the circular boundaries in Chapter 3, vehicles do not have any preference in the sense of radius. Therefore, to guide them towards the inner boundary, we have to properly modify the outer boundary function, $\alpha(x')$, while the inner one $\beta(x')$ remains circular. To this end, we use the following function:

$$\alpha(x') = R_a - \sqrt{(R_{\text{out}} - \gamma x')^2 - x'^2} \tag{12}$$

where γ is a parameter determining the changing rate of outer boundary radius. Given that this function is applied to the transformed variables where $x' = 0$ in all samples, the value of $\alpha(x')$ will not change, i.e. the outer circle remains as right boundary. However, the changes in its first and second derivatives, appearing in the feedback law, encourages vehicle move towards the center. If we want to accurately determine the right boundary, we can replace R_{out} in (11) with an updated value after one sampling period, while taking into account the vehicle speed. A sample of such navigation is depicted in Fig. 16, where the allowed space gradually changes from the whole width of roundabout to a narrower band.

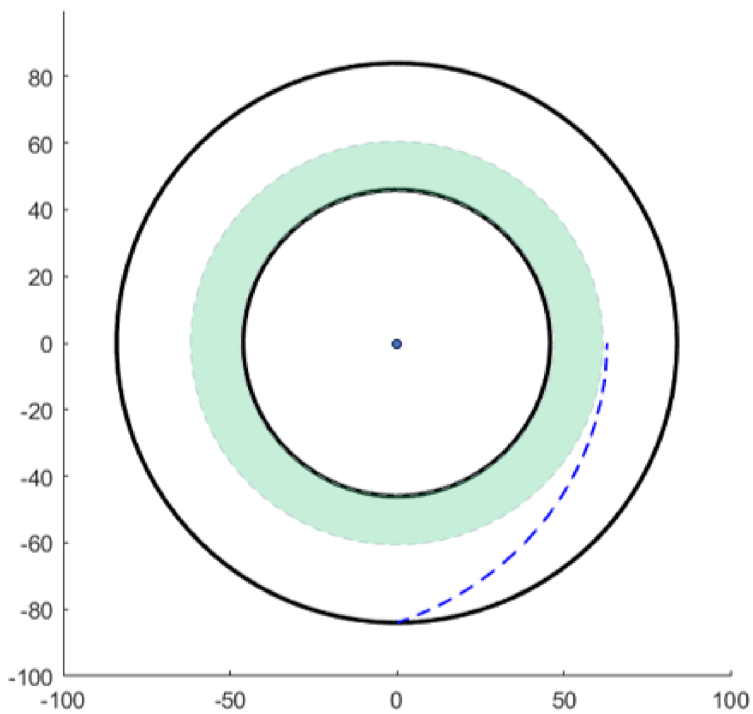

Figure 16 – Navigating vehicles towards the inner boundary

4.3. Guiding vehicles towards the outer boundary

Similarly, to implement smooth exit for vehicles we need to gradually guide them to the outer part of the roundabout. To this end, we consider the outer circle as the right boundary while the left one is as below:

$$\beta(x') = R_a - \sqrt{(R_{\text{in}} + \gamma x')^2 - x'^2} \tag{13}$$

As mentioned in previous section, we can replace R_{in} by the updated radius of left boundary after one sample to have accurate control. A typical guidance of a vehicle towards the outer boundary can be seen in Fig. 17.

To investigate the effectiveness of the suggested approach to guide vehicles toward inner or outer parts, the simulation presented in section 3.3 is repeated. This time, 5 vehicles, shown by black (red), are guided towards the outer (inner) part while others, depicted with light blue, are allowed to freely navigate the entire roundabout as before. A recorded video of the results can be watched at https://bit.ly/4gFwOKp. Also, in Fig. 18 the initial and final position of vehicles are illustrated. The blue vehicles can start and finish at any radius while black (red) vehicles ended up in a narrow band close to outer (inner) boundary. Vehicle speeds, accelerations, steering angles, and minimum inter-vehicle distance are shown in Fig. 19 to 22. The results are almost similar to the ones of section 3.3., meaning that the new maneuvers do not cause performance degradation.

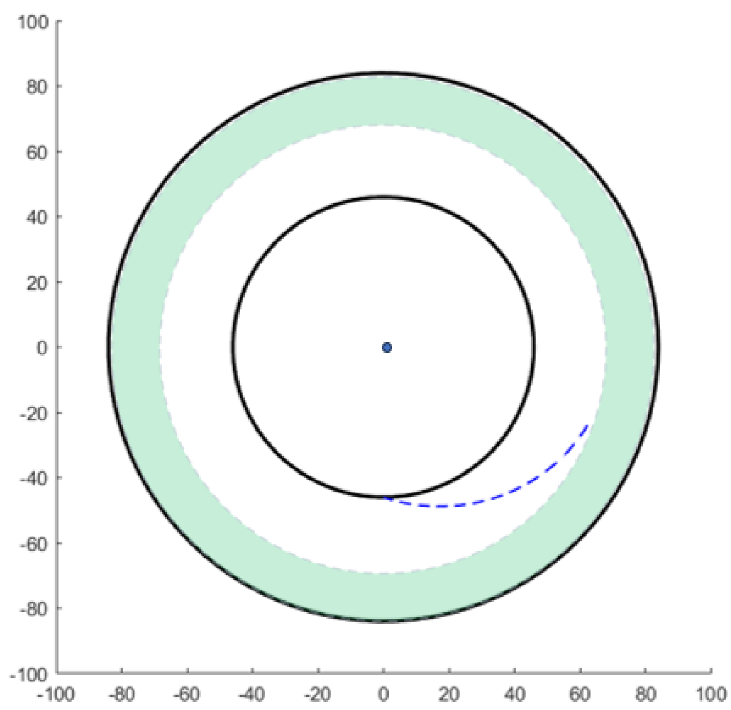

Figure 17– Navigating vehicles towards the outer boundary

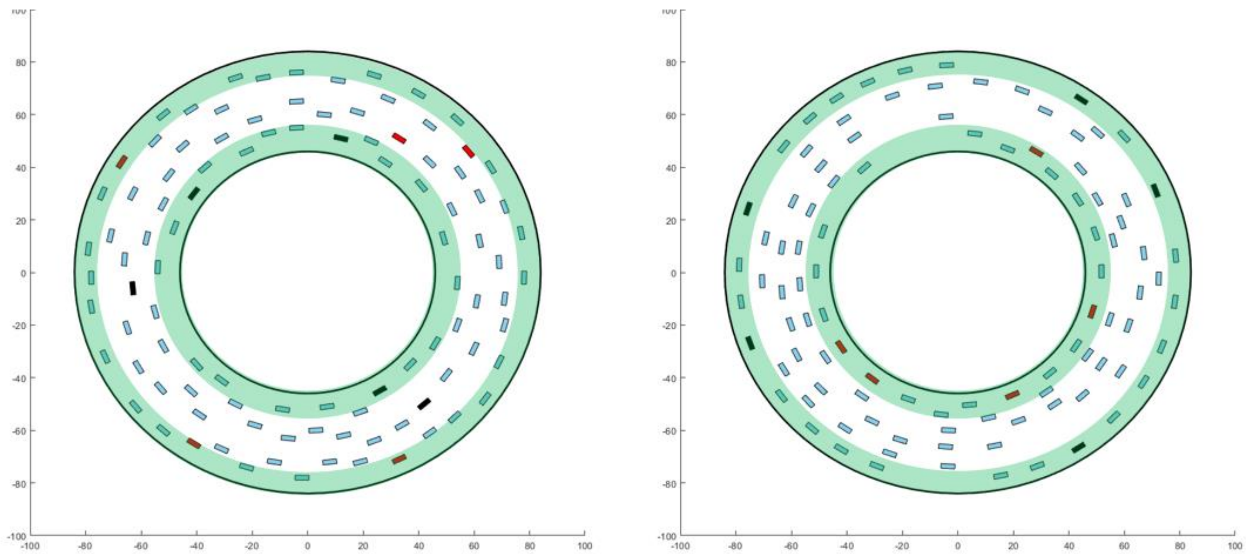

Figure 18 – Initial and final position of vehicles

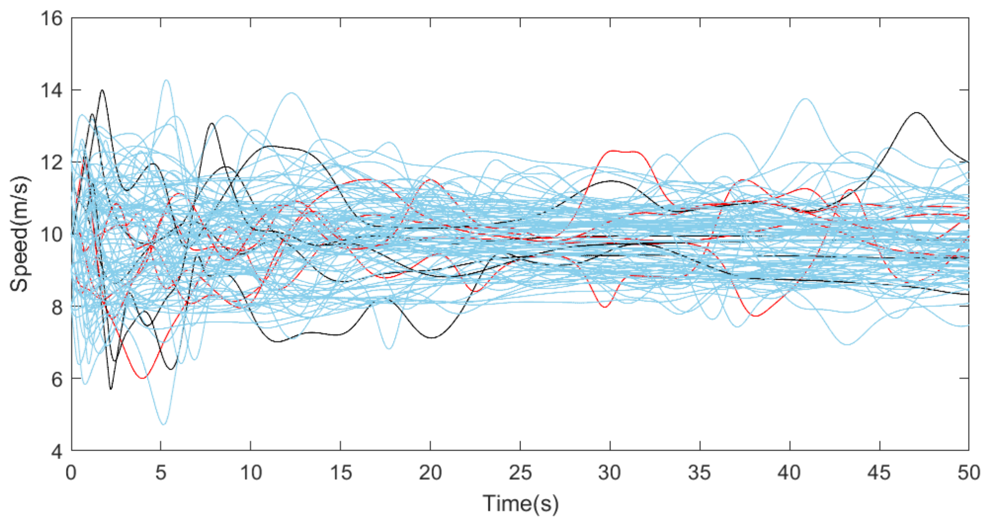

Figure 19 – Vehicle speeds

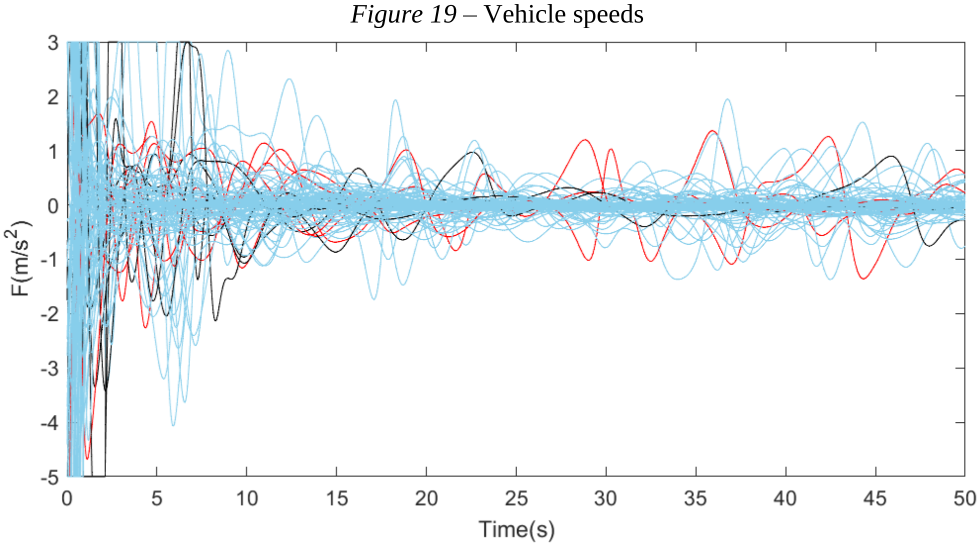

Figure 20- Vehicle accelerations

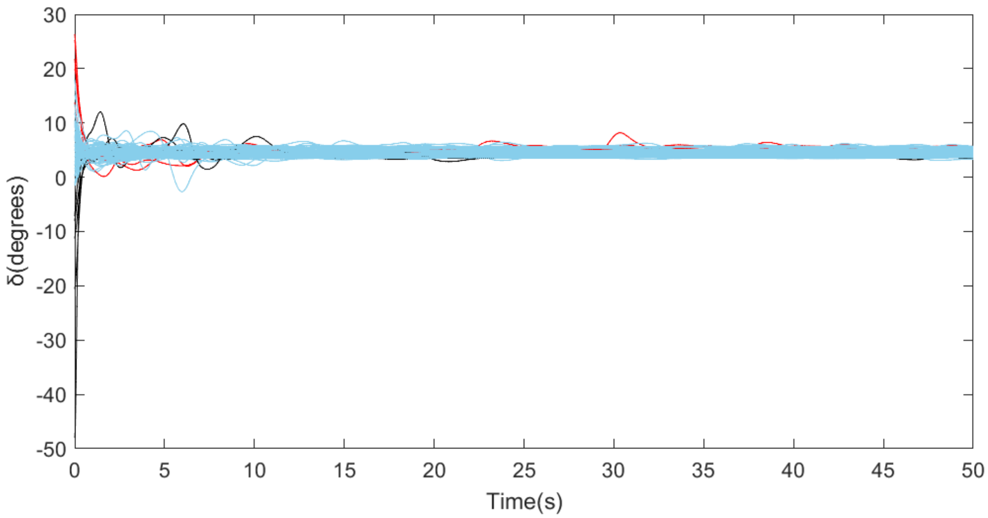

Figure 21- Vehicle steering angles

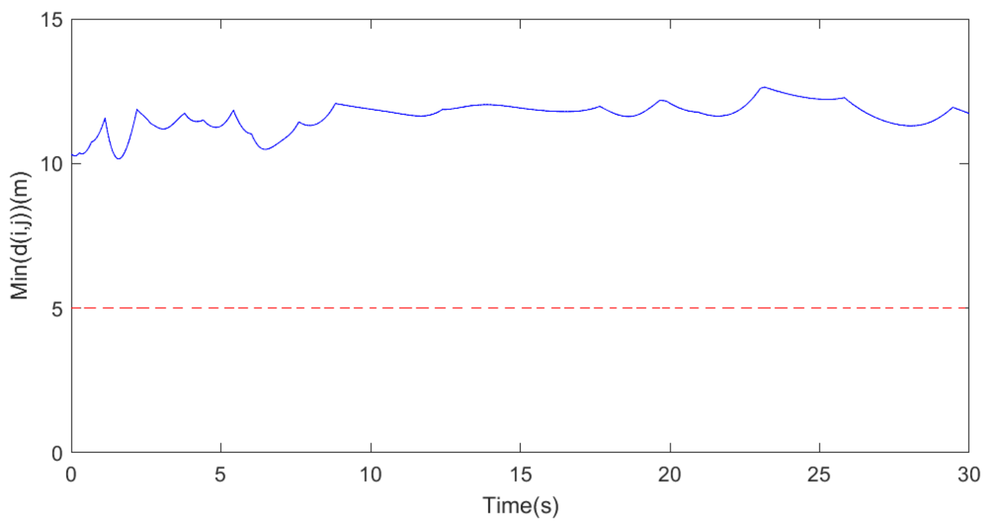

Figure 22- Minimum inter-vehicle distance

4.4. OD corridors

Now, employing the boundaries mentioned above, we can define appropriate Origin-Destination (OD) corridors in which vehicles with specific OD are allowed to drive. We will examine two distinct situations, namely visible and invisible ODs.

4.4.1. Visible OD corridors

For OD couples of a roundabout where the origin and destination are relatively close to each other, e.g. when the destination is visible from the origin, the shortest and simplest way to get there is to take a short path, which is close to or on the outer roundabout boundary, without passages through the inner part of the roundabout. The selection of ODs belonging to this type depends on the specific roundabout geometry. For the case of Place Charles de Gaulle roundabout, the destination is visible from the origin if is up to 3 branches away from the origin. For instance, assuming branch 1 to be an origin, the ODs (1, 2), (1, 3), (1, 4) belong to visible class. In other word, the angular distance between entrance and exit branch should be less than 90˚, i.e. $\Delta\varphi \leq 90$, see Fig. 23 that shows considered corridor for a visible OD. The corridor's right-hand side coincides with the outer roundabout boundary; while its left-hand boundary is a straight line connecting the left boundary of origin branch with the left-most point of the destination branch. The left boundary function can be obtained by writing the line equation using its beginning and end point position in the moving coordinates of each vehicle, see Fig. 24:

$$\beta_{en}(x') = m(x' - x_3') + y_3' \qquad (14)$$

$$m = \frac{y_1' - y_3'}{x_1' - x_3'}$$

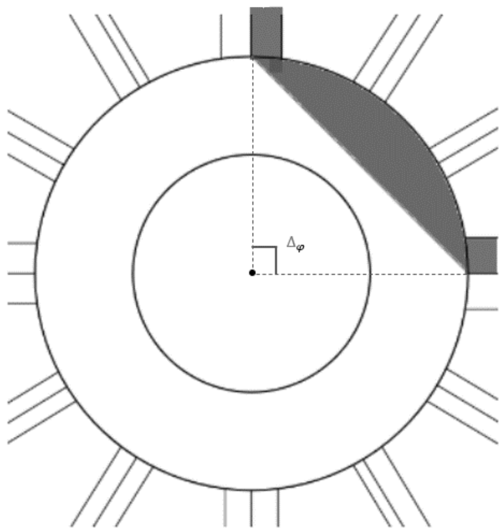

Figure 23- Visible OD

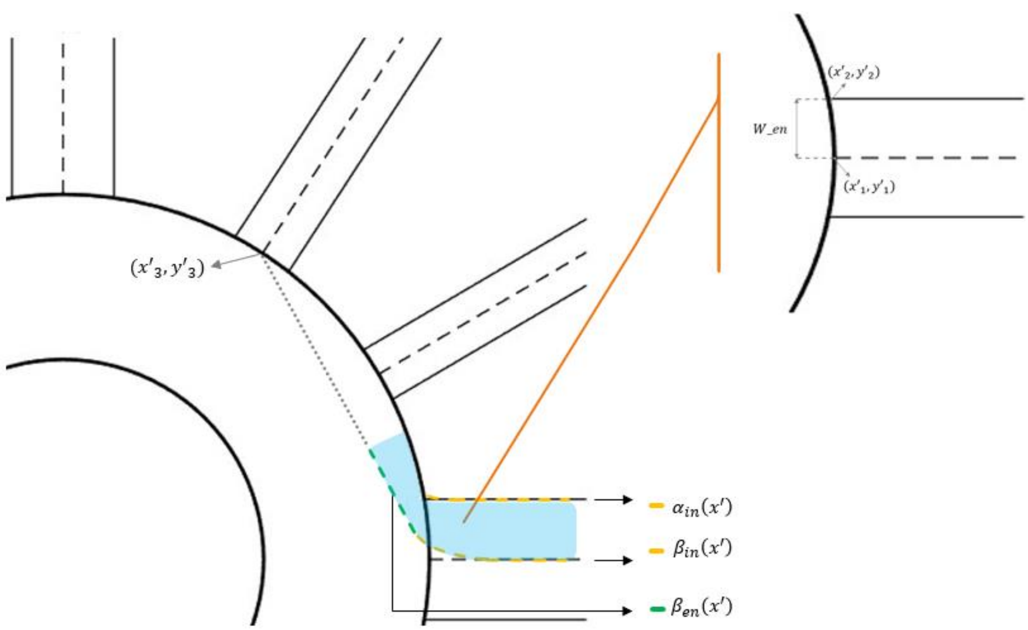

Figure 24- Left boundary of a visible OD

Employing the abovementioned boundaries, a typical trajectory of a vehicle with a visible OD is obtained and can be seen in Fig. 25. The vehicle path is sufficiently smooth and does not interrupt the roundabout traffic unnecessarily. Note that in one of previous works [29], vehicles take a direct line towards the destination while using the approach suggested in this work, they mostly follow circular path. It is because of the fact that we do not use any desired orientation in this work and just determine the OD corridors. Hence, within the corridor vehicles try to drive on circular paths and minimize their deviation from circular angle.

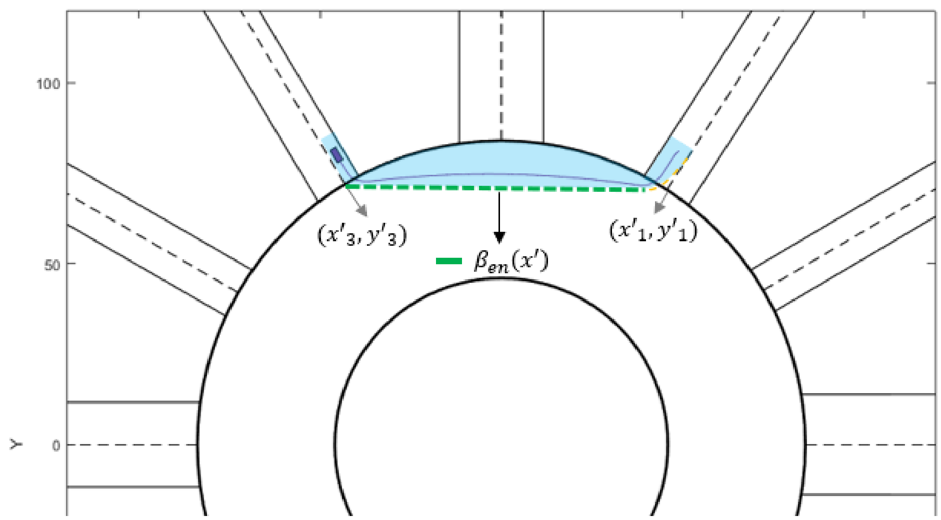

Figure 25- Path of a typical vehicle with visible OD

4.4.2. Invisible OD corridor

For the Place Charles de Gaulle roundabout, ODs having more than 2 branches between origin and destination are called invisible OD. In other words, angular distance between entrance and exit branches is greater than 90°, i.e. $\Delta\varphi > 90$ .For such ODs, as depicted in Fig. 26, since the destination is not visible from the origin, vehicles cannot move on a direct path toward the destination, and a circular motion must be partially pursued.

To address vehicles with invisible ODs, we consider three phases. First, in order to avoid all vehicles to drive close to the outer boundary causing frequent major conflicts with entering vehicles, vehicles are guided towards the inner parts using the right boundary introduced in (12). The longer distance from destination, the closer to the center they could drive. Second, using circular left and right boundaries, introduced by (9), vehicles are allowed to use the whole width of roundabout. In case they do not have conflicts with other vehicles, they will follow a completely circular path. Finally, vehicles should gradually get close to the outer boundary to be prepared for exiting. This is done with changing inner boundary as (13). This boundary gets activated once vehicle has an angular distance of 90° from its corresponding exit.

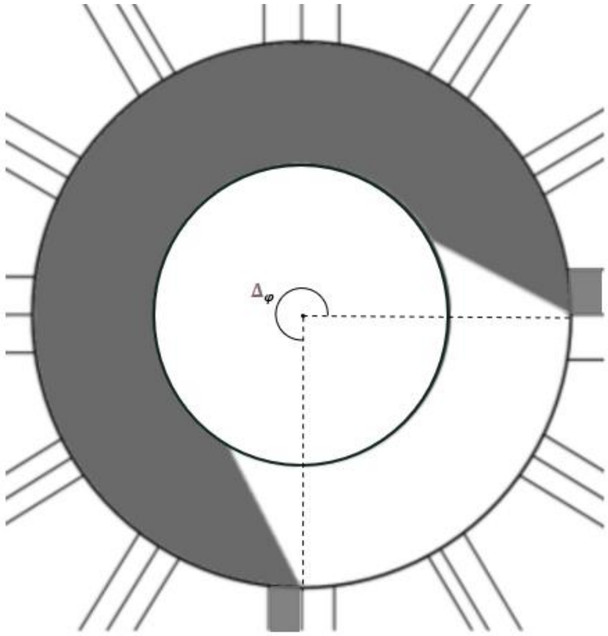

Figure 26 – Invisible OD

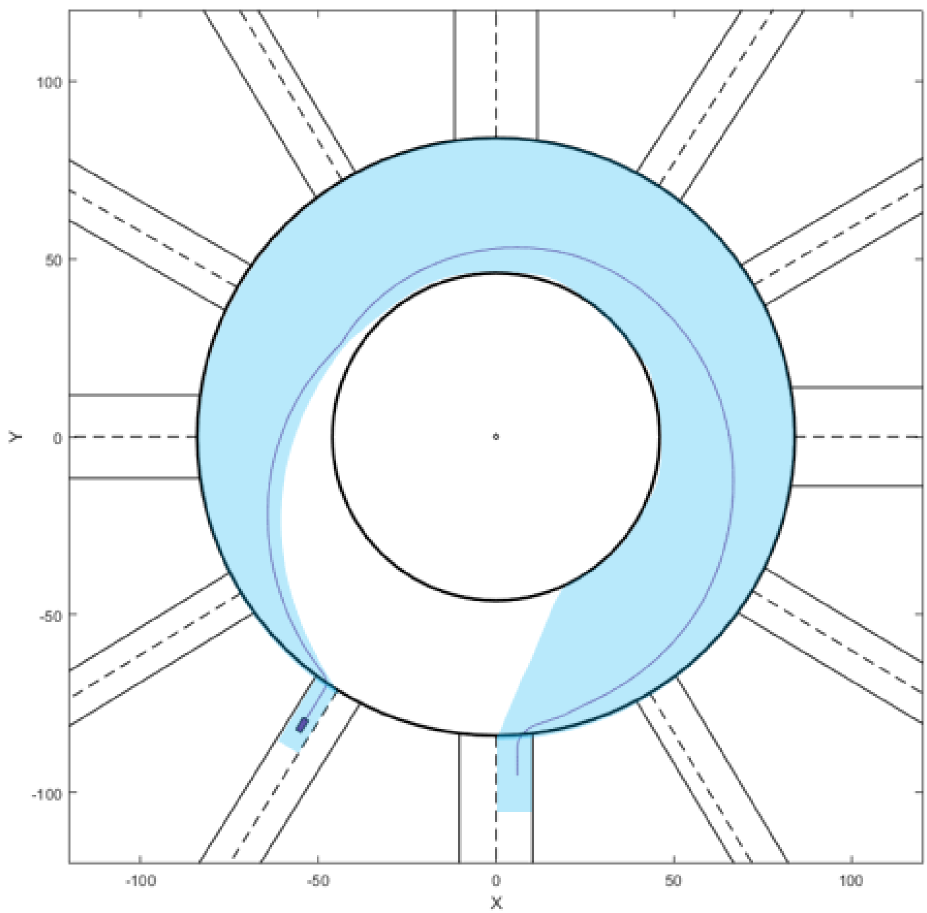

Figure 27 – OD corridor for invisible

4.4.3. Simulation results

In order to investigate the effectiveness of proposed entrance, navigation, and exit scenarios, a simulation of 50 vehicles is conducted. In this simulation, various ODs, including both visible and invisible, are considered. Proper corridors are considered for each vehicle at each sample and necessary transformation is applied for its variables. A video of the results can be watched at https://bit.ly/4eMSySv. Also, vehicle paths are illustrated in Fig. 28 where vehicles with either visible or invisible destinations drive within the assigned corridors and do not miss their exit. Their trajectories are relatively smooth, providing passenger comfort. They may have some conflicts with other vehicles to handle which, they may change their path or speed. They drive safely as confirmed by Fig. 29 where the minimum inter-vehicle distance is greater than the safety distance.

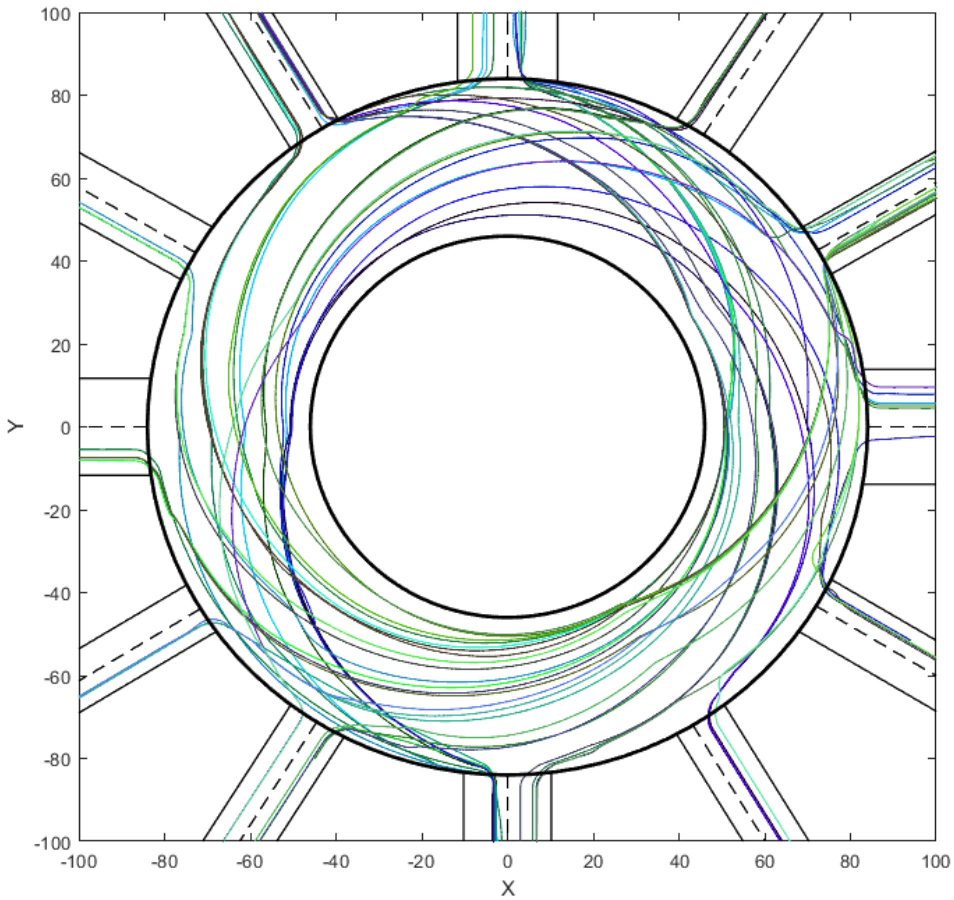

Figure 28 – Vehicle paths

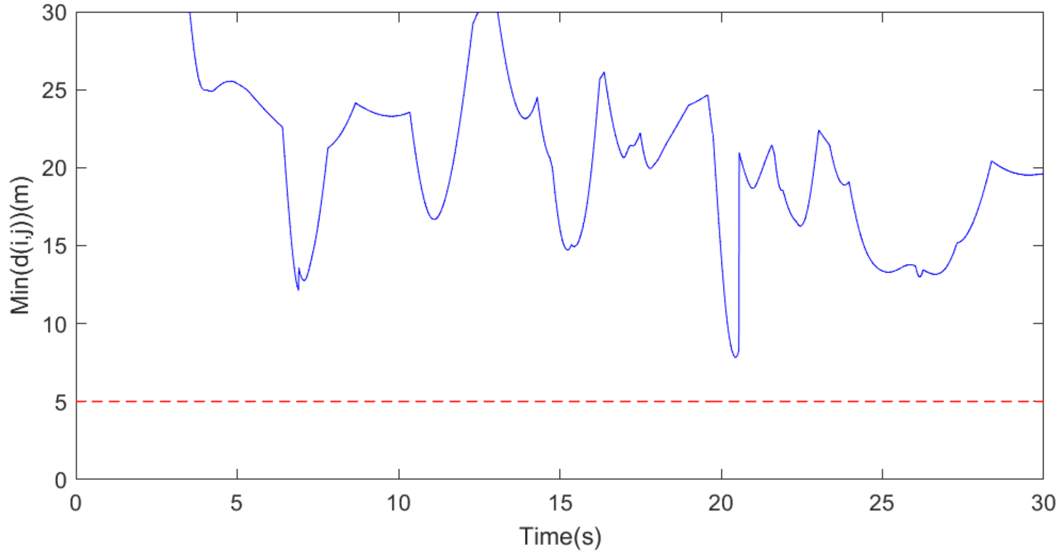

Figure 29 – Minimum inter-vehicle distance

5. Conclusion and future work

5.1. Concluding remarks

In this diploma thesis, we aimed to develop a novel strategy to manage vehicles driving on large lane-free roundabouts. Although a few previous works have tackled this problem, it demands further exploration to improve efficiency and simplify of the solution. In particular, the proposed approach in [32] contains several units like position-dependent desired orientation, adaptive desired speed, boundary and safety controllers, making it complicated for the implementation. Therefore, we decided to employ a new nonlinear controller, designed for roads with curvy boundaries, which integrates the mentioned units and consequently can simplify the implementation of the control system.

However, the new controller could not be directly used to handle vehicles on roundabouts because of some theoretical limitations. Hence, we suggested that each vehicle use moving coordinates, based on its instantaneous position and orientation, and transform its own state variables as well as relative distances from adjacent vehicles to calculate appropriate control signals.

A key element in this controller are road boundaries which are defined via two smooth functions. In order to develop efficient entrance, rotation, and exit scenarios, suitable boundaries are defined for each step. For entrance and exit branches, we considered straight boundaries except for last few meters of entrance branch where the left boundary is curved to decrease conflicts between entering and rotating vehicles. Within the roundabout, boundary functions are suggested to address two different Origin-Destination (OD) types, namely visible and invisible. To prevent vehicles with close destinations from approaching the center, a corridor is designated in the outer part of the roundabout. Its left boundary is defined by a skewed line. On the other hand, through defining appropriate boundaries, vehicles with invisible destinations, first are navigated towards the inner parts. Then, after passing some circular movements, they are guided to the outer boundary to not miss the exit. It is worth mentioning that all boundaries are introduced in the moving coordinates.

Several simulations have confirmed the effectiveness of the proposed strategies. The resulting paths are sufficiently smooth, the whole space of roundabout is properly exploited, and the safety and comfort are provided. However, further tests are necessary to investigate the controller performance in more crowded situations.

5.2. Future work

The current thesis can be followed in several aspects some of which are briefly explained below:

1. **Fine-tuning controller parameters**: Given that the controller and designed corridors have several parameters, it is necessary to find the most efficient values that could be a time-consuming task. Some heuristic optimization approach may be used to this end.
2. **Implementation of the proposed approach in TrafficFluid-Sim**: All simulations presented in this work are conducted in Matlab. Implementing the control strategy in TrafficFluid-Sim, which is based on C/C++, could significantly enhance the computational time and also provide us with several features like defining continuous demands and measuring flows.
3. **Testing high demand conditions**: The proposed strategy can be used in scenarios with higher demands resulting in more crowded situations. Then, the efficiency of the presented approach can be compared with the previous one, more accurately.
4. **Applying the strategy to non-circular roundabouts**: As compared with previous method, the current approach can be applied to non-circular roundabouts much more easily. We just need to define the roundabout boundaries properly.

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